Unit 1 Test Review:

p.35 # 2, 3, 8, 9, 10, 11

p.58 # 1 - 7

p.61-62 # 6, 7, 8, 10

TEST REVIEW - ANSWERS

2. Evaluate the following squares.

67^2 667^2 6667^2 66667^2

What pattern do you notice? Make a conjecture about the 25th term in the pattern.

e.g., The squares follow a pattern of $t + 1$ fours, t eights, and 1 nine, where t is the term number. For example, the second term, $t = 2$, is $667^2 = 444889$

The 25th term in the pattern will be 25 sixes and 1 seven, squared, and the result will be 26 fours, 25 eights, and 1 nine.

3. Part of Pascal's triangle is shown below. The column on the right represents the sums of the numbers in the rows of Pascal's triangle.

1	1
1 1	2
1 2 1	4
1 3 3 1	8
1 4 6 4 1	16
1 5 10 10 5 1	32
1 6 15 20 15 6 1	64

a) Based on the evidence in the column on the right, make a conjecture about the sum of the numbers in the 10th row.

b) Make a conjecture about the sum of any row.

e.g.,

a) The sum of the numbers in the 10th row will be 512.

b) The sum of any row is $2^{(r-1)}$, where r is the row number.

8. Use inductive reasoning to make a conjecture about each number trick below. Then use deductive reasoning to prove your conjecture.

a) Choose a number. Add 3. Multiply by 2. Add 4. Divide by 2. Subtract the number you started with. What is the result?

b) Choose a number. Double it. Add 9. Add the number you started with. Divide by 3. Add 4. Subtract the number you started with. What is the result?

e.g.,

a) If 5 is chosen, the result is 5. If 2 is chosen, the result is 5.
Conjecture: The number trick always has a result of 5.

n	n
$+3$	$n+3$
$\times 2$	$2n+6$
$+4$	$2n+10$
$\div 2$	$n+5$
$-n$	5

b) If 7 is chosen, the result is 7. If 4 is chosen, the result is 7.
Conjecture: The number trick always has a result of 7.

n	n
$\times 2$	$2n$
$+9$	$2n+9$
$+n$	$3n+9$
$\div 3$	$n+3$
$+4$	$n+7$
$-n$	7

9. Prove that the sum of four consecutive natural numbers is always even.

e.g.,
Let n, $n + 1$, $n + 2$, and $n + 3$ represent any four consecutive natural numbers.
$n + (n + 1) + (n + 2) + (n + 3) = 4n + 6$
$n + (n + 1) + (n + 2) + (n + 3) = 2(2n + 3)$
Since 2 is a factor of the sum, the sum of four consecutive natural numbers is always even.

10. Consider the following statement: The square of the sum of two positive integers is greater than the sum of the squares of the same two integers. Test this statement inductively with three examples, and then prove it deductively.

e.g.,

$(7 + 11)^2 = 324$	$(1 + 10)^2 = 121$	$(3 + 5)^2 = 64$
$7^2 + 11^2 = 170$	$1^2 + 10^2 = 101$	$3^2 + 5^2 = 34$
$(7 + 11)^2 > 7^2 + 11^2$	$(1 + 10)^2 > 1^2 + 10^2$	$(3 + 5)^2 > 3^2 + 5^2$

Let n and m be any two positive integers.

The square of the sum of two positive integers:

$(n + m)^2 = n^2 + 2mn + m^2$

The sum of the squares of two positive integers:

$n^2 + m^2$

Since $2mn > 0$ for all positive integers,

$n^2 + 2mn + m^2 > n^2 + m^2$

The square of the sum of two positive integers is greater than the sum of the squares of the same two integers.

11. Prove that the difference between the square of any odd integer and the integer itself is always an even integer.

e.g.,
Let $2n + 1$ represent any odd integer.
$(2n + 1)^2 - (2n + 1) = (4n^2 + 4n + 1) - (2n + 1)$
$(2n + 1)^2 - (2n + 1) = 4n^2 + 2n$
$(2n + 1)^2 - (2n + 1) = 2(2n^2 + n)$
Since the difference has a factor of 2, the difference between the square of an odd integer and the integer itself is always even.

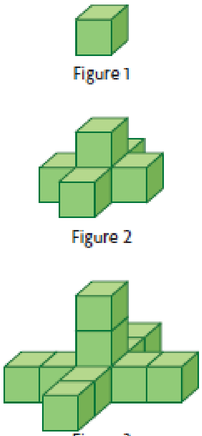
Figure 1
Figure 2
Figure 3

1. Danielle made the cube structures to the left.

a) What would the 4th and 5th structures look like? How many cubes would Danielle need to build each of these structures?

b) Make a conjecture about the relationship between the nth structure and the number of cubes needed to build it.

c) How many cubes would be needed to build the 25th structure? Explain how you know.

e.g.,

a) Figure 4 would have one additional cube at the end of each arm, requiring 16 cubes in all. Figure 5 would have 5 cubes more than Figure 4, with one at the end of each arm, requiring 21 cubes in all.

b) The nth structure would require $5n - 4$ cubes to build it.

c) 121 cubes

2. Frank tosses a coin five times, and each time it comes up tails. He makes the following conjecture: The coin will come up tails on every toss. Is his conjecture reasonable? Explain.

His conjecture isn't reasonable: the chance of the coin coming up heads is 50%.

3. Koby claims that the perimeter of a pentagon with natural number dimensions will always be an odd number. Search for a counterexample to his claim.

e.g., A pentagon with sides of length 2 has a perimeter of 10.

4. Prove that the product of two consecutive odd integers is always odd.

Let $2n + 1$ and $2n + 3$ represent two consecutive odd integers.
Let P represent the product of these integers.

$P = (2n + 1)(2n + 3)$

$P = 4n^2 + 8n + 3$

$P = 2(2n^2 + 4n) + 3$

$2(2n^2 + 4n)$ is an even integer, 3 is an odd integer, and the sum of any even and odd integer is an odd integer, so the product of any two consecutive odd integers is an odd integer.

5. Prove that the following number trick always results in 10:
Choose a natural number. Double it. Add 20. Divide by 2. Subtract the original number.

e.g.,

n	n
$\times 2$	$2n$
$+ 20$	$2n + 20$
$\div 2$	$n + 10$
$- n$	10

6. Andy, Bonnie, Candice, and Darlene are standing in line to buy ice cream. Determine the order in which they are lined up, using these clues:
   - Candice is between Andy and Bonnie.
   - Darlene is next to Andy.
   - Bonnie is not first.

Darlene, Andy, Candice, Bonnie

7. The following proof seems to show that $10 = 9.\overline{9}$. Is this proof valid? Explain.

Let $a = 9.\overline{9}$.

$10a = 99.\overline{9}$	Multiply by 10.
$10a - a = 90$	Subtract a.
$9a = 90$	Simplify.
$a = 10$	Divide by 9.

The proof is valid; all the steps are correct.

6. Harry claims that if opposite sides of a quadrilateral are the same length, the quadrilateral is a rectangle. Do you agree or disagree? Justify your decision.

Disagree. e.g., Rhombuses and parallelograms have opposite sides of equal length.

7. Sadie claims that the difference between any two positive integers is always a positive integer. Do you agree or disagree? Justify your decision.

Disagree. e.g., $5 - 5 = 0$

8. Complete the conclusion for the following deductive argument: If an integer is an even number, then its square is also even. Six is an even number, therefore, ….

Six is an even number; therefore, its square is also even.

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10. Linda came across this number trick on the Internet and tried it:

- Think about the date of your birthday.
- Multiply the number for the month of your birthday by 5. (For example, the number for November is 11.)
- Add 7.
- Multiply by 4.
- Add 13.
- Multiply by 5.
- Add the day of your birthday.
- Subtract 205.
- Write your answer.

a) Try the trick. What did you discover?

b) Prove how this trick works. Let m represent the number for the month of your birthday and d represent the day.

a) The result is the birth month number followed by the birthday, e.g., 415.

b)

m	m
$\times 5$	$5m$
$+ 7$	$5m + 7$
$\times 4$	$20m + 28$
$+ 13$	$20m + 41$
$\times 5$	$100m + 205$
$+ d$	$100m + 205 + d$
$- 205$	$100m + d$

The birth month is multiplied by 100, leaving enough space for a two-digit birthday.

Attachments

1s6e1 finalt.mp4

1s6e2 final.mp4