Pre-Calculus 120A Exam Review

SHORT RESPONSE: Place your answer for each of the following questions in the box provided.

1. The graph of $y=|x|$ is reflected across the x-axis, stretched vertically by a factor of $\frac{1}{3}$, translated 2 units to the right and 4 units up to form the transformed function $y=g(x)$. Determine the equation of the function $y=g(x)$.

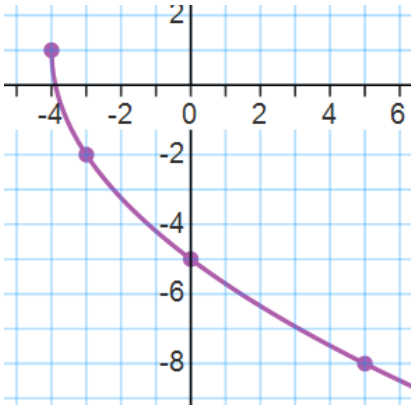
2. The point $(3,-7)$ is on the graph of $y=f(x)$. Determine the coordinates of its image point under the transformation $y=5f(-2(x+1))-6$.

3. The domain of $y=f(x)$ is $\{-2\le x\le 8, x\in R\}$. Determine the domain of the transformed function $g(x)=-4f\left(\frac{1}{2}(x-6)\right)+1$.

4. Which of the following transformations of $f(x)$ produces a graph that has the same x-intercept as $f(x)$? Assume $(0,0)$ is NOT a point on $f(x)$.

   a. $f(x-1)$ b. $-4f(x)$ c. $f(x)-3$ d. $f(5x)$

5. Determine an equation for the following graph.



6. What are the domain and range of the function $y=\sqrt{x^2-8x+12}$?

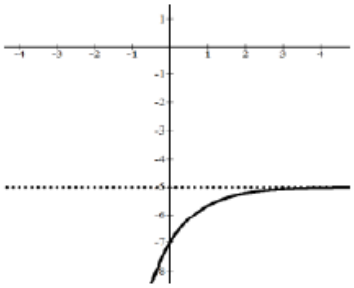
7. What are the domain and range of the function $y=4\sqrt{-2(x+5)}+6$?

8. What are the domain and range of the function $y = 7\left(\frac{4}{5}\right)^{(x+2)} - 3$?

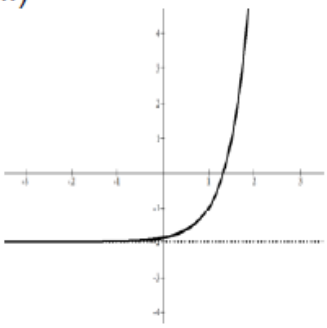
9. Match each function with the corresponding graph.

a. $y = 3^{2(x-1)} - 2$ b. $y = 2^{x-2} + 1$ c. $y = -4^{x+2}$ d. $y = -2(3)^{-x} - 5$

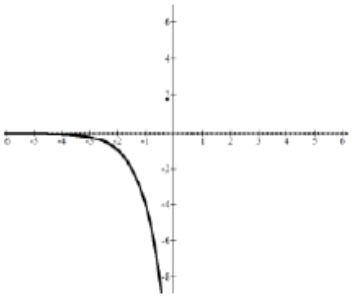
i)



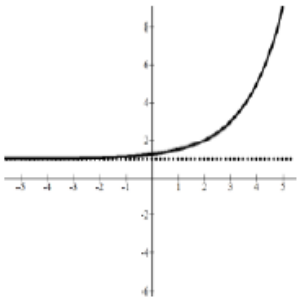
ii)



iii)



iv)



10. Write the expressions 9^{2x+5} and $\left(\frac{1}{27}\right)^{x-1}$ with the same base.

11. State the equation of the asymptote for $y = 4\log_3(x-2)+7$.

12. What is the value of $\log_5 8$ rounded to two decimal places?

13. Solve $\log_{27} x = \frac{4}{3}$.

14. Solve $\log_x \frac{1}{16} = -4$.

15. Write $\log_4 x + \log_4 7 - \frac{1}{2}\log_4 25$ as a single simplified logarithmic expression.

16. Write $\log_7\left(\frac{4}{\sqrt{x}}\right)$ in expanded form.

17. State the domain of the function
$f(x)=\log_2\left(x^2-2x-24\right)$

18. Evaluate the expression $\ln e^5+2\ln 1-e^{4\ln 2}$.

19. Convert 144^o to radians.

20. Convert $\frac{13\pi}{9}$ radians to degrees.

21. State one positive angle coterminal with -579°.

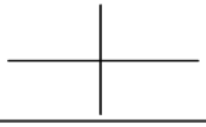
22. Express the angles coterminal with 350° in general form.

23. State one negative angle coterminal with $\frac{13\pi}{5}$.

24. State the angles coterminal with $\frac{9\pi}{7}$ in the domain $-3\pi\le\theta\le 3\pi$.

25. An arc has a central angle of 245° in a circle with radius 12.4 m. State the length of the arc to the nearest tenth of a metre.

26. Sketch $-\frac{17\pi}{6}$ in standard position on the axes provided.



27. Give an angle, in radians rounded to two decimal places, whose terminal arm contains the point $(-3,11)$.

28. The point $(-4,-9)$ is located on the terminal arm of $\angle A$ in standard position. What is the exact value of $\csc A$?

29. Convert the polar coordinates $\left(3,\frac{7\pi}{4}\right)$ to exact rectangular coordinates.

30. State another pair of polar coordinates for the point $\left(3,\frac{7\pi}{4}\right)$.

31. State the approximate value, rounded to four decimal places, for $\cos 6$.

32. Determine a *positive* measure (in degrees) for angle θ if $P(\theta)=\left(-\frac{1}{2},\ \frac{\sqrt{3}}{2}\right)$.

33. State the *exact* value for $\sec 240^\circ$.

34. Determine the missing coordinate for the point $\left(\frac{7}{8},y\right)$ in quadrant 4 on the unit circle.

35. A trigonometric function in the form $y=a\cos b(x-h)+k$ has range $\{y\,|\,-7\le y\le 1;\ y\in R\}$. State the values of *a* and *k*.

36. State the general solution, in degrees, for $\tan x=-1$.

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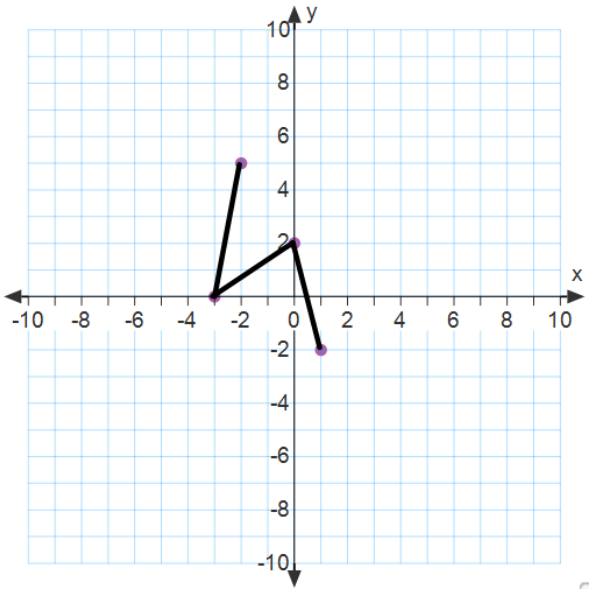
OPEN RESPONSE:

1. Given the graph of $f(x)$, sketch the graph of $g(x)=-\frac{1}{2}f\left(\frac{1}{3}(x-5)\right)-4$.

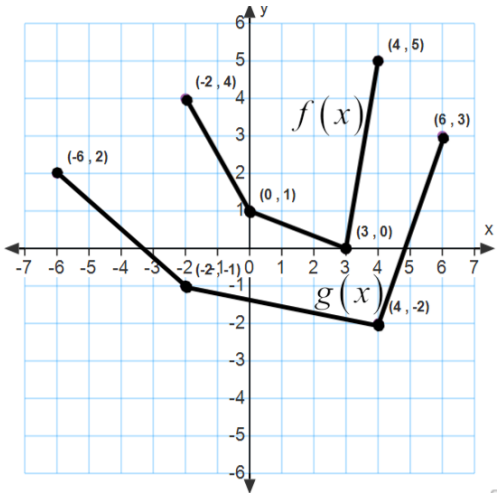
   State the domain and range of $g(x)$.

   Domain:

   Range:



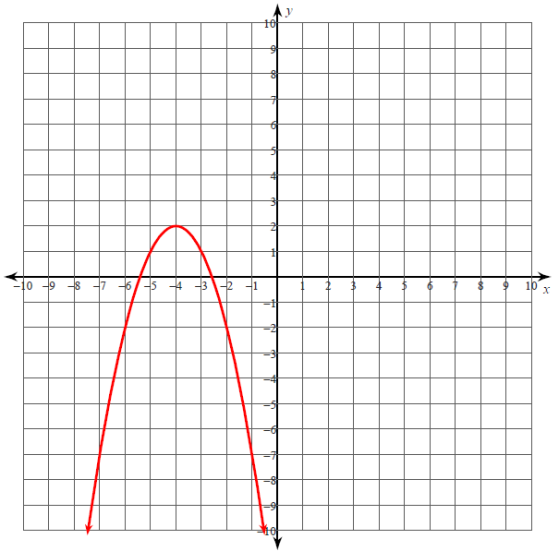
2. State the equation of $g(x)$ as a transformation of $f(x)$ in the form $g(x)=af(b(x-h))+k$.



3. The graph of $f(x)=-(x+4)^2+2$ is illustrated to the right.

   a. Draw the inverse of f(x) on the same grid.

   b. Write the equation of the inverse.



c. State the domain and range of the function, f(x), and its inverse.

Domain of f(x): Range of f(x):

Domain of inverse of f(x): Range of inverse of f(x):

d. How would you restrict the domain of the original function f(x) so that the inverse is a function?

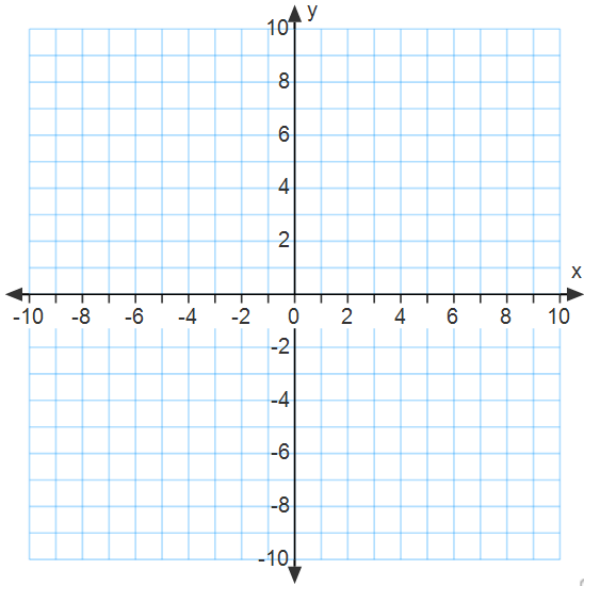
4. Sketch the graph of $y = 2\sqrt{-(x-3)} - 5$. Determine the x-intercept, y-intercept, domain and range.

x-intercept:

y-intercept:

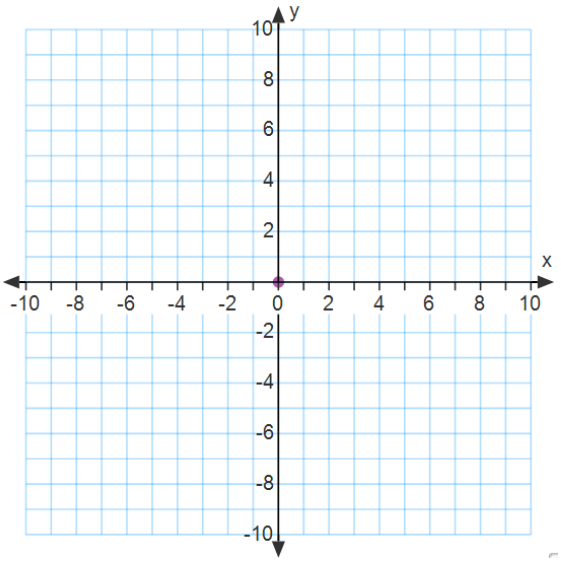
Domain:

Range:

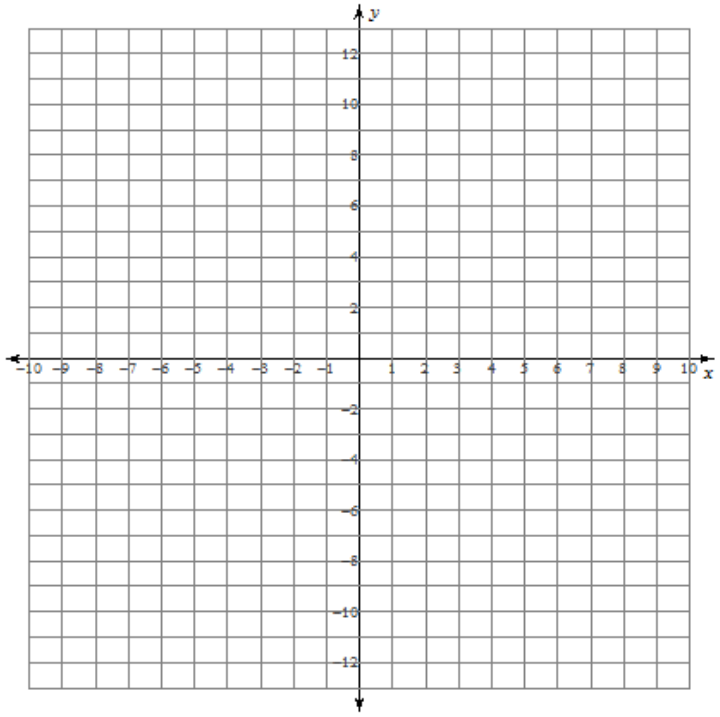


5. Solve the following radical equation *graphically* and *algebraically*. Be sure to check for extraneous solutions.

$x + 2 = \sqrt{x+7} - 3$



6. Graph the exponential function $y = 4\left(\frac{1}{2}\right)^{x-3} + 1$. Identify the domain, range, equation of the asymptote, x-intercept, and y-intercept.



Domain: _____

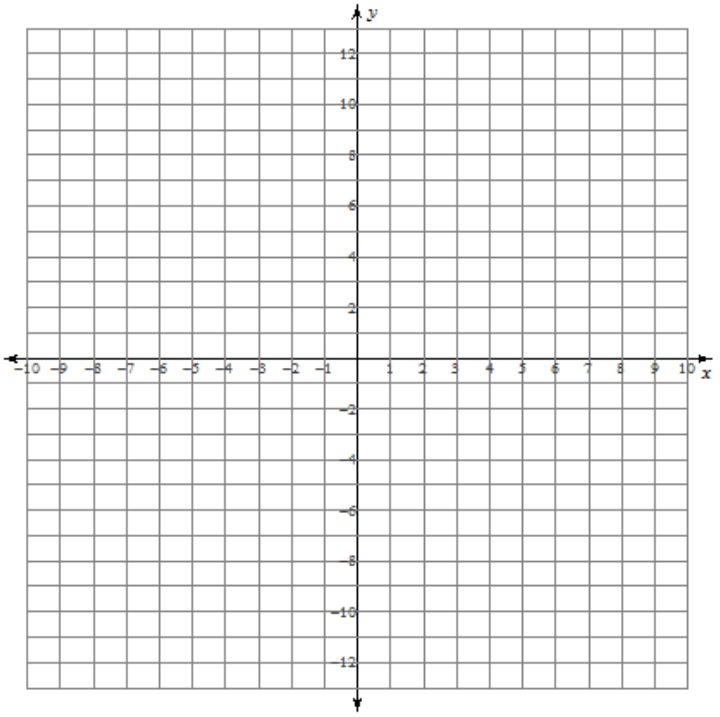
Range : _____

Equation of the asymptote : _____

x-intercept: _____

y-intercept : _____

7. Graph the logarithmic function $y = -\log_2(x+4) - 3$. Identify the domain, range, equation of the asymptote, x-intercept, and y-intercept.



Domain: _____

Range : _____

Equation of the asymptote : _____

x-intercept: _____

y-intercept : _____

8. A rabbit population quadruples every three years. There were 250 rabbits to start with.

   a. How many rabbits are there after seven years?

   b. Calculate the number of years it will take for the population to reach 16 000.

9. In a nuclear disaster at Chernobyl in April 1986, approximately 12 600 kg of radioactive iodine-131 was released into the atmosphere. Iodine-131 is known to decay by half every 8.04 days.

   a. Determine the approximate mass of iodine-131 remaining after 30 days.

   b. When was there 126 kg remaining?

10. Simplify and evaluate the following expression:

$$4\log_2 4 - \log_2 8$$

11. Solve the following equations algebraically.

   a. $4(2)^{2x+1} = 8$

   b. $9^{x-1} = 81\sqrt{3^x}$

   c. $3^{2x-1} = 6$

   d. $\log_x 9 = \frac{2}{3}$

   e. $\log_2 x = \log_4 64$

   f. $x = \log_4 3 + \frac{1}{2}\log_4 36 - \log_4 9$

   g. $\log_2(x+3) - \log_2(x-2) = 5$

   h. $e^{x-3} = \ln e^7$

12. A large pizza has a diameter of 40 centimeters. A piece of that pizza has a central angle of $\frac{\pi}{7}$. How long is its crust? An extra-large pizza has a diameter of 46 centimeters. What is the central angle of a piece with the same length of crust as the piece in the large pizza?

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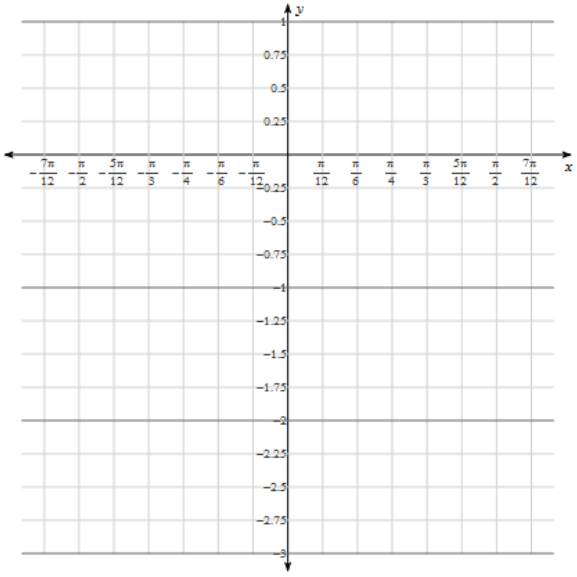
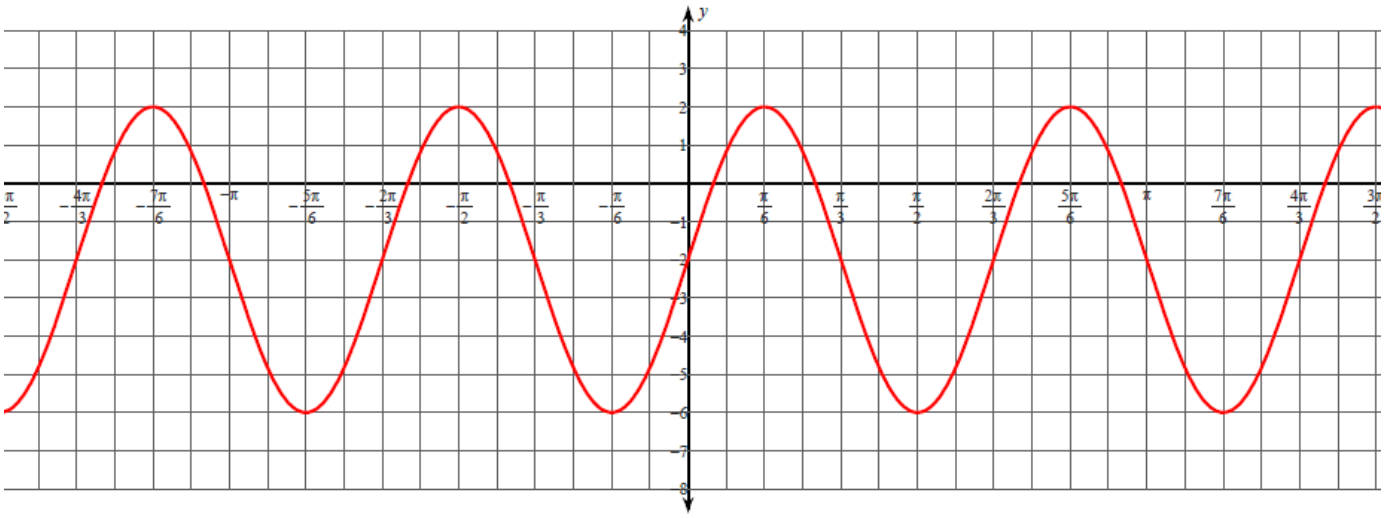
13. Sketch the function $y=-\frac{1}{2}\cos 4(x+\frac{\pi}{3})-2$ on the grid provided, then state its characteristics.



Equation of Sinusoidal Axis: **Amplitude:** **Period:** **Domain:** **Range:**

14. Determine two possible equations (one sine, one cosine) for the sinusoidal function shown below.



Sine equation: Cosine equation:

15. Solve the following equations algebraically for the given interval.

a. $2\cos^2\theta-1=0;\ 0\le\theta\le 360^\circ$

b. $2\sin\theta\cos\theta-\sqrt{2}\cos\theta=0;\ -\pi\le\theta\le\pi$

c. $\tan\theta+\tan^2\theta=0;\ \pi\le\theta\le 2\pi$

d. $\sin 6\left(\theta+10^\circ\right)=-\frac{\sqrt{3}}{2};\ -270^\circ\le\theta\le 90^\circ$

16. Determine the general solution, in radians, for each equation.

a. $\cos 2x = \cos x$

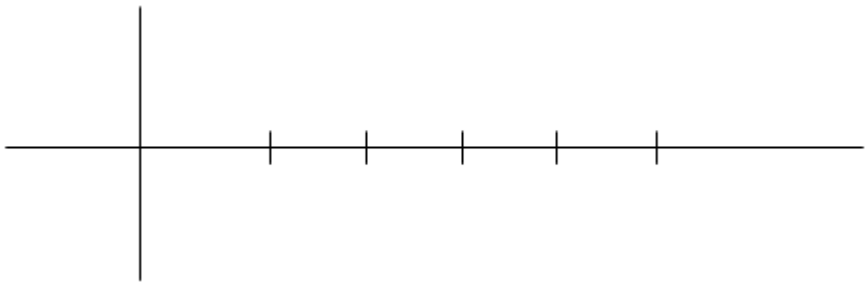
b. $2\sin^2 x = -\sin x + 1$

c. $\cos\left(x + 60^\circ\right) = \sin\left(x + 30^\circ\right)$

d. $2\sin x = 2\cos 2x$

17. Mr. Murdock is distracted by shiny objects, like the hands on a clock. He decides to watch the second hand rotate around. The second hand is 8 cm long and its tip is 2 m from the floor when it's at the top of the clock. Mr. Murdock started watching the clock when the second hand was at the bottom.

a. Sketch a graph to represent this situation.



b. Determine an equation to represent this situation.

c. Determine the height of the second hand after 20 seconds.

d. When will the height be 193 cm? Determine all possible answers.

18. Determine the non-permissible values, in radians, for each expression.

a. $\cot x \sin x$

b. $\dfrac{\tan x}{1 - \cos^2 x}$

19. Simplify the following trigonometric expressions.

a. $\sec x \csc x \cot x$

b. $\dfrac{\sin 2x}{1 - \cos 2x}$

c. $\cos\left(x - 180^\circ\right)$

20. Verify that the identity $\frac{\cos x - \sin x}{\cos x} = 1 - \tan x$ is true for $x = 60^\circ$ and for $x = \frac{3\pi}{4}$.

21. Use a sum or difference identity to find the exact value of each expression.

a. $\sin 285^o$

b. $\cos \frac{11\pi}{12}$

22. Prove that each identity holds for all permissible values of x.

a. $\sin x + \cot x \cos x = \csc x$

b. $2\cos x + 2\tan^2 x \cos x = 2\sec x$

c. $\frac{\sec x}{\sec x - \tan x} = \sec^2 x + \sec x \tan x$

d. $\cos 2x + \frac{2\tan^2 x}{\sec^2 x} = 1$

e. $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$

f. $\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$

Pre-Calculus 12A Exam Review Solutions

SHORT RESPONSE:

1. $y=-\frac{1}{3}|x-2|+4$

2. $(3,-7)\rightarrow\left(-\frac{5}{2},-41\right)$

3. $\{2\le x\le 22, x\in R\}$

4. b. $-4f(x)$

5. $y=-3\sqrt{(x+4)}+1$

6. $y=\sqrt{x^2-8x+12}=\sqrt{(x-6)(x-2)}$
   Domain: $x\in(-\infty,2]\cup[6,\infty)$ Range: $y\in[0,\infty)$

7. $y=4\sqrt{-2(x+5)}+6$
   Domain: $x\in(-\infty,-5]$ Range: $y\in[6,\infty)$

8. $y=7\left(\frac{4}{5}\right)^{(x+2)}-3$
   Domain: $x\in R$ Range: $y\in(-3,\infty)$

9. i) D ii) A iii) C iv) B

10. $9^{2x+5}=\left(3^2\right)^{2x+5}=3^{4x+10}$ $\left(\frac{1}{27}\right)^{x-1}=\left(3^{-3}\right)^{x-1}=3^{-3x+3}$

11. Vertical asymptote: $x=2$

12. $\log_5 8=\frac{\log 8}{\log 5}=1.29$

13. $x=27^{\frac{4}{3}}=\left(\sqrt[3]{27}\right)^4=3^4=81$

14.

$$x^{-4} = \frac{1}{16}$$

$$x = \left(\frac{1}{16}\right)^{-\frac{1}{4}} = 16^{\frac{1}{4}} = 2$$

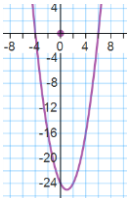
15. $\log_4 x + \log_4 7 - \log_4 \sqrt{25} = \log_4 \left(\frac{7x}{5}\right)$

16. $\log_7 4 - \log_7 \sqrt{x} = \log_7 4 - \frac{1}{2}\log_7 x$

17.

$$\log_2 \left(x^2 - 2x - 24\right)$$

$$x^2 - 2x - 24 > 0$$
$$(x+4)(x-6) > 0$$
$$x < -4 \text{ or } x > 6$$



18.

$$\ln e^5 + 2\ln 1 - e^{4\ln 2}$$
$$= \ln e^5 + 0 - e^{\ln 2^4}$$
$$= 5 - 16$$
$$= -11$$

19. $144^\circ \times \frac{\pi}{180^\circ} = \frac{72\pi}{90} = \frac{4\pi}{5}$

20. $\frac{13(180^\circ)}{90} = 13 \times 20^\circ = 260^\circ$

21. $-579^\circ + 2(360^\circ) = 141^\circ$

22. $\theta = 350^\circ \pm 360^\circ n,\ n \in N$

23. $\frac{13\pi}{5} - 4\pi = \frac{13\pi}{5} - \frac{20\pi}{5} = -\frac{7\pi}{5}$

24.

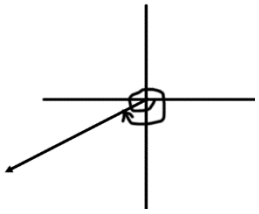
$-\frac{5\pi}{7}$ and $-\frac{19\pi}{7}$

25.

$a = r\theta$

$= (12.4)(245^\circ)\left(\frac{\pi}{180^\circ}\right) = 53.0\,m$

26.

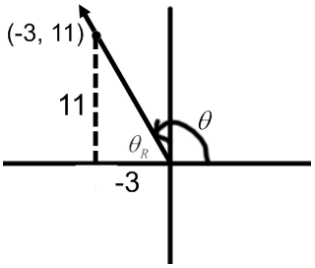


27.

$\tan\theta = -\frac{11}{3}$

$\theta_R = \tan^{-1}\left(\frac{11}{3}\right) = 1.3045$

$\therefore \theta = \pi - 1.3045 = 1.84$



28.

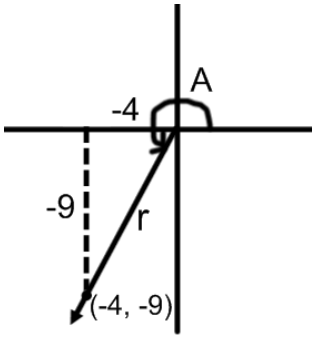
$r^2 = (-4)^2 + (-9)^2$

$= 16 + 81$

$= 97$

$r = \sqrt{97}$

$\csc A = \frac{r}{y} = -\frac{\sqrt{97}}{9}$

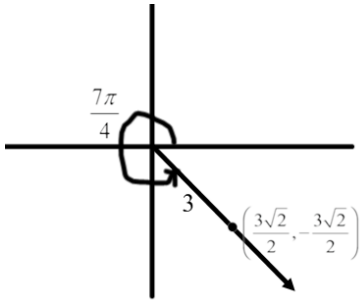


29.

$x = r\cos\theta$ $\quad y = r\sin\theta$

$= 3\cos\left(\frac{7\pi}{4}\right)$ $\quad = 3\sin\left(\frac{7\pi}{4}\right)$ $\quad \therefore \left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

$= \frac{3\sqrt{2}}{2}$ $\quad = -\frac{3\sqrt{2}}{2}$



30. $\left(3, -\frac{\pi}{4}\right)$ or $\left(-3, \frac{3\pi}{4}\right)$ or... (there are others)

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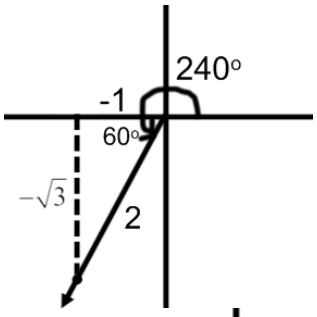
31.

$\cos 6 = 0.9602$

32.

$\theta = 120^\circ$

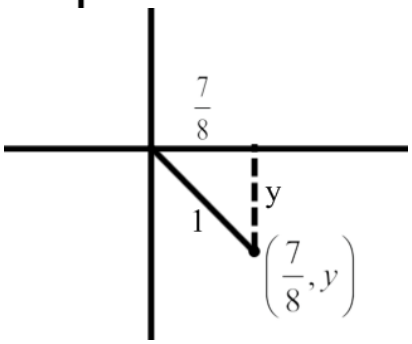
33. $\sec 240^\circ = \frac{r}{x} = \frac{2}{-1} = -2$



34. Quadrant IV, so y<0

$y^2 = 1^2 - \left(\frac{7}{8}\right)^2 = 1 - \frac{49}{64} = \frac{15}{64}$

$y = -\frac{\sqrt{15}}{8}$



35.

$k = \frac{-7+1}{2} = -3$

$a = -3 - (-7) = 4$

36.

$x = 135^\circ \pm 180^\circ n,\ n \in W$

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OPEN RESPONSE:

1. $g(x)=-\frac{1}{2}f\left(\frac{1}{3}(x-5)\right)-4$

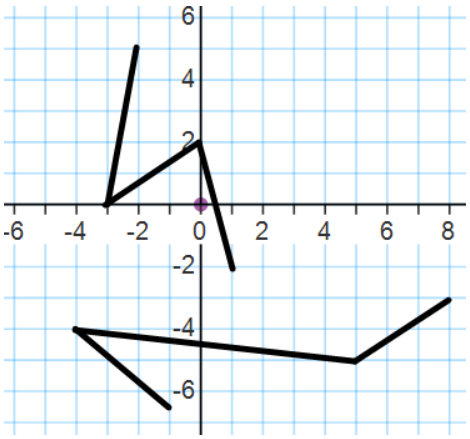
$(x,y)\rightarrow\left(3x+5,-\frac{1}{2}y-4\right)$

$(-2,5)\rightarrow(-1,-6.5)$

$(-3,0)\rightarrow(-4,-4)$

$(0,2)\rightarrow(5,-5)$

$(1,-2)\rightarrow(8,-3)$



Domain: $\{-4\le x\le 8, x\in R\}$ Range: $\{-6.5\le y\le -3, y\in R\}$

2. Width of $f(x)=4-(-2)=6$, Width of $g(x)=6-(-6)=12$, so horizontal stretch of 2

Height of $f(x)=5-0=5$, Height of $g(x)=3-(-2)=5$, so no vertical stretch

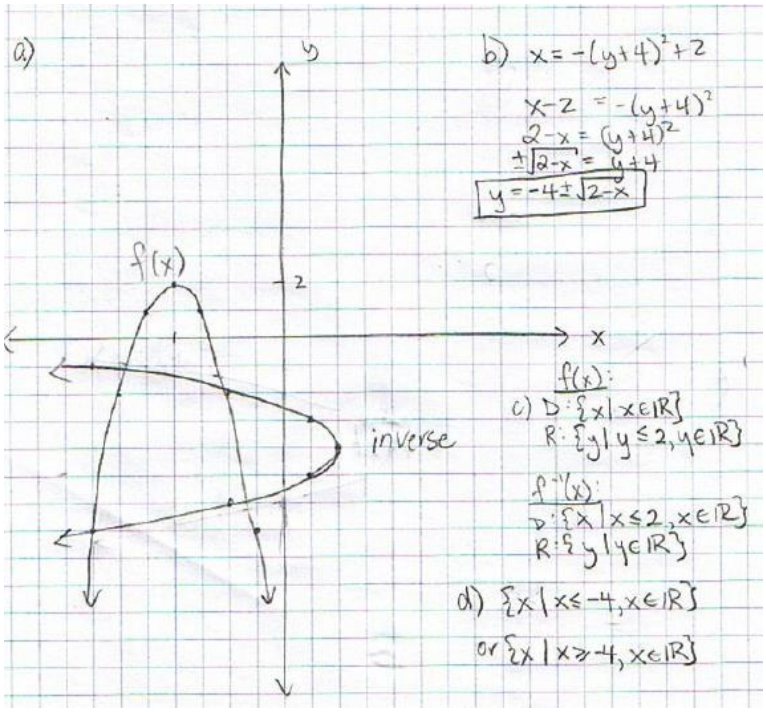
No reflections

$(0,1)\rightarrow(-2,-1)$, so $h=-2$

$(3,0)\rightarrow(4,-2)$, so $k=-2$

Thus, $g(x)=f\left(\frac{1}{2}(x+2)\right)-2$

3.



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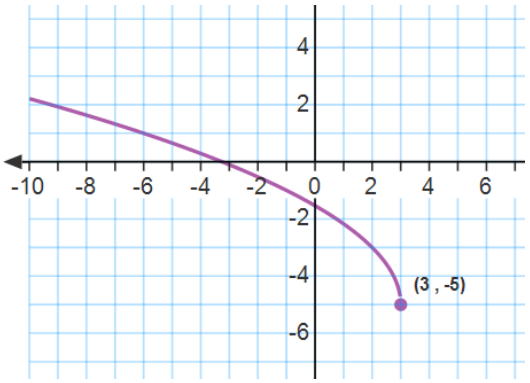
4.

x-intercept: y-intercept: Domain: $x \in (-\infty, 3]$ Range: $y \in [-5, \infty)$

$$0 = 2\sqrt{-(x-3)} - 5$$
$$5 = 2\sqrt{-(x-3)}$$
$$\left(\frac{5}{2}\right)^2 = \left(\sqrt{-(x-3)}\right)^2$$
$$\frac{25}{4} = -(x-3)$$
$$-\frac{25}{4} = x - 3$$
$$-\frac{25}{4} + \frac{12}{4} = -\frac{13}{4} = x$$

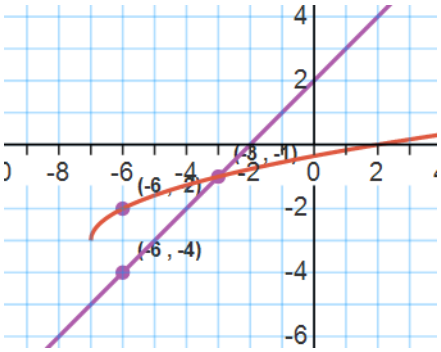
$$y = 2\sqrt{-(0-3)} - 5$$
$$= 2\sqrt{3} - 5$$



5.

$$x + 2 = \sqrt{x+7} - 3$$
$$(x+5)^2 = \left(\sqrt{x+7}\right)^2$$
$$x^2 + 10x + 25 = x + 7$$
$$x^2 + 9x + 18 = 0$$
$$(x+6)(x+3) = 0$$
$$x = -6,\ x = -3$$

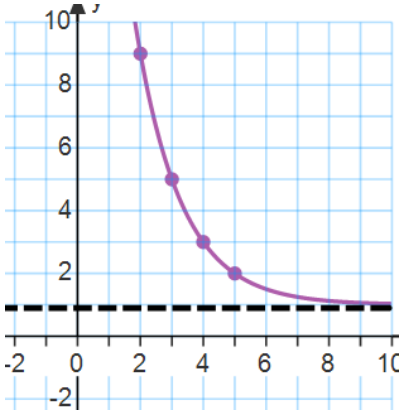
when $x = -6$:
$LS = x + 2 = -4$
$RS = \sqrt{x+7} - 3 = -2$
$\therefore\ x = -6$ *is extraneous*
when $x = -3$:
$LS = x + 2 = -1$
$RS = \sqrt{x+7} - 3 = -1$
$\therefore\ x = -3$ *is a solution*



6. $y = 4\left(\frac{1}{2}\right)^{x-3} + 1$

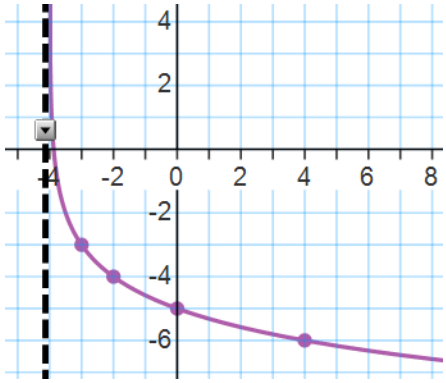
$(x, y) \to (x+3, 4y+1)$
$(-2, 4) \to (1, 17)$
$(-1, 2) \to (2, 9)$
$(0, 1) \to (3, 5)$
$\left(1, \frac{1}{2}\right) \to (4, 3)$
$\left(2, \frac{1}{4}\right) \to (5, 2)$

Domain : $x \in R$
Range : $y > 1, y \in R$
Equation of the asymptote : $y = 1$
x – int : *none*



y – int : $y = 4\left(\frac{1}{2}\right)^{-3} + 1 = 4(2)^3 + 1 = 4(8) + 1 = 33$

7. $y = -\log_2(x+4) - 3$



$(x, y) \rightarrow (x-4, -y-3)$

$(1,0) \rightarrow (-3,-3)$

$(2,1) \rightarrow (-2,-4)$

$(4,2) \rightarrow (0,-5)$

$(8,3) \rightarrow (4,-6)$

Domain : $x > -4, x \in R$

Range : $y \in R$

Equation of the asymptote : $x = -4$

$x-\text{int}: 0 = -\log_2(x+4) - 3$

$3 = -\log_2(x+4)$

$-3 = \log_2(x+4)$

$2^{-3} = x+4$

$\frac{1}{8} = x + \frac{32}{8}$

$\frac{-31}{8} = x$

$y-\text{int}: y = -\log_2(4) - 3 = -2 - 3 = -5$

8.

$P(t) = 250(4)^{\frac{t}{3}}$

a. $P(7) = 250(4)^{\frac{7}{3}}$

$P(7) = 6349\, rabbits$

**rounded to a whole number*

b. $16000 = 250(4)^{\frac{t}{3}}$

$64 = 4^{\frac{t}{3}}$

$4^3 = 4^{\frac{t}{3}}$

$3 = \frac{t}{3}$

$9\, years = t$

9.

$A(t) = 12600\left(\frac{1}{2}\right)^{\frac{t}{8.04}}$

a. $A(30) = 12600\left(\frac{1}{2}\right)^{\frac{30}{8.04}}$

$A(30) = 948.69\, kg$

b. $126 = 12600(0.5)^{\frac{t}{8.04}}$

$0.01 = (0.5)^{\frac{t}{8.04}}$

$\ln(0.01) = \frac{t}{8.04}\ln(0.5)$

$t = \frac{8.04\ln(0.01)}{\ln(0.5)} = 53.4\, days$

10.

$4\log_2 4 - \log_2 8 = \log_2\left(\frac{4^4}{8}\right) = \log_2\left(\frac{2^8}{2^3}\right) = \log_2 2^5 = 5$

11.

a.
$$4(2)^{2x+1}=8$$
$$(2)^{2x+1}=2$$
$$2x+1=1$$
$$2x=0$$
$$x=0$$

b.
$$9^{x-1}=81\sqrt{3^x}$$
$$(3^2)^{x-1}=(3^4)\left(3^{\frac{x}{2}}\right)$$
$$(3)^{2x-2}=(3)^{4+\frac{x}{2}}$$
$$2x-2=4+\frac{x}{2}$$
$$4x-4=8+x$$
$$3x=12$$
$$x=4$$

c.
$$3^{2x-1}=6$$
$$2x-1=\log_3 6=1.6309$$
$$2x=2.6309$$
$$x=1.3154$$

d.
$$\log_x 9=\frac{2}{3}$$
$$x^{\frac{2}{3}}=9$$
$$x=9^{\frac{3}{2}}=3^3=27$$

e.
$$\log_2 x=\log_4 64$$
$$\log_2 x=3$$
$$x=2^3=8$$

f.
$$x=\log_4 3+\frac{1}{2}\log_4 36-\log_4 9$$
$$=\log_4\left(\frac{3\bullet 36^{\frac{1}{2}}}{9}\right)$$
$$=\log_4 2$$
$$=\frac{1}{2}$$

g.
$$\log_2(x+3)-\log_2(x-2)=5$$
$$\log_2\left(\frac{x+3}{x-2}\right)=5$$
$$2^5=\frac{x+3}{x-2}$$
$$32=\frac{x+3}{x-2}$$
$$32(x-2)=x+3$$
$$32x-64=x+3$$
$$31x=67$$
$$x=\frac{67}{31}$$

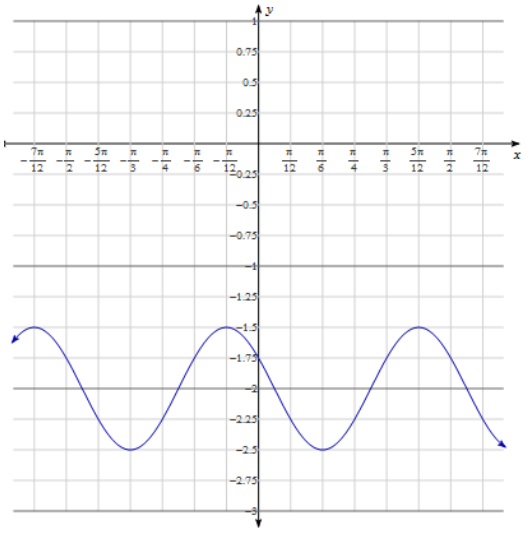
h.
$$e^{x-3}=\ln e^7$$
$$e^{x-3}=7$$
$$(x-3)\ln(e)=\ln(7)$$
$$x-3=\ln(7)$$
$$x=\ln(7)+3\approx 4.946$$

12.
$$d=40$$
$$r=20$$
crust: $a=r\theta=20\left(\frac{\pi}{7}\right)=\frac{20\pi}{7}cm$

$$d=46$$
$$r=23$$
Central angle: $\theta=\frac{a}{r}=\frac{\frac{20\pi}{7}}{23}=\frac{20\pi}{161}$

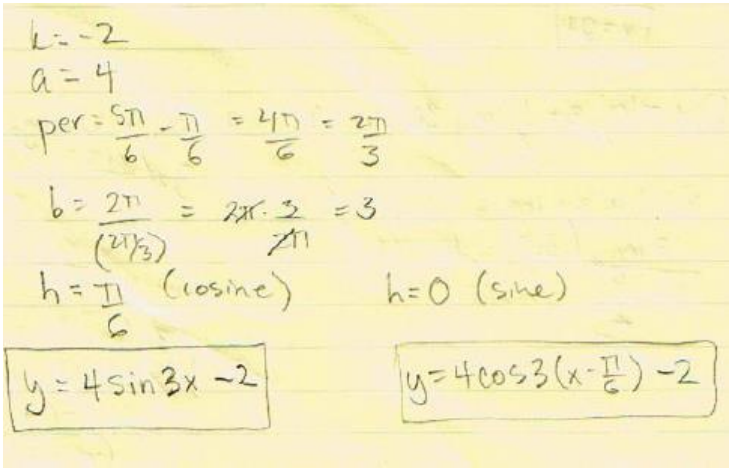
Pre-Calculus 12A Exam Review

13. $y = -\frac{1}{2}\cos 4(x + \frac{\pi}{3}) - 2$



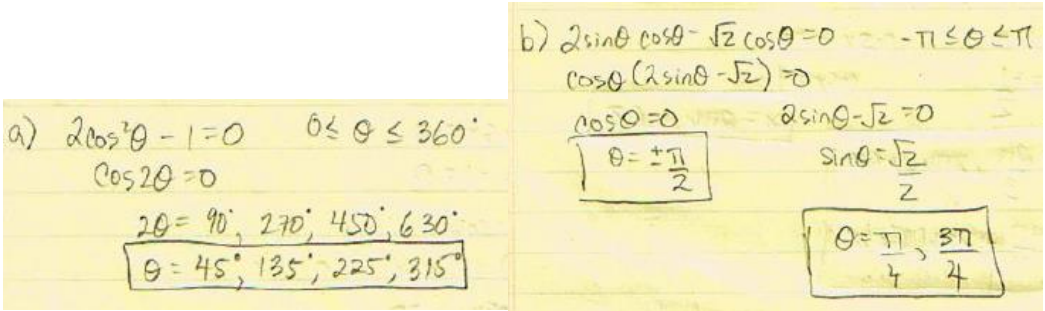
Equation of Sinusoidal Axis: y = –2 Amplitude: 0.5 Period: $\frac{\pi}{2}$ Domain: $x \in R$ Range: $-2.5 \le y \le -1.5$

14.



15.

A



Pre-Calculus 12A Exam Review

c) $\tan\theta + \tan^2\theta = 0 \quad \pi \le \theta \le 2\pi$

$\tan\theta(1+\tan\theta) = 0$

$\tan\theta = 0 \qquad 1+\tan\theta = 0$

$\theta = \pi, 2\pi \qquad \tan\theta = -1$

$\theta = \frac{7\pi}{4}$

d) $\sin 6(\theta + 10^\circ) = -\frac{\sqrt{3}}{2}, \quad -270^\circ \le \theta \le 90^\circ$

$6(\theta + 10^\circ) = -120^\circ \qquad 6(\theta + 10^\circ) = -60^\circ$

$\theta + 10^\circ = -20^\circ + 60^\circ k, k \in \mathbb{Z} \qquad \theta + 10^\circ = -10^\circ + 60^\circ k, k \in \mathbb{Z}$

$\theta = -30^\circ + 60^\circ k, k \in \mathbb{Z} \qquad \theta = -20^\circ + 60^\circ k, k \in \mathbb{Z}$

$\theta = -30^\circ, 30^\circ, -90^\circ, -150^\circ, -210^\circ, -270^\circ \qquad \theta = -20^\circ, 40^\circ, -80^\circ, -140^\circ, -200^\circ, -2$

16.

A

a) $\cos 2x = \cos x$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$2\cos^2 x - 2\cos x + \cos x - 1 = 0$

$2\cos x(\cos x - 1) + 1(\cos x - 1) = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0 \qquad \cos x - 1 = 0$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$x = 2\pi k, k \in \mathbb{Z}$

$x = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}$

$x = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}$

b) $2\sin^2 x = -\sin x + 1$

$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1 = 0$

$2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \qquad \sin x + 1 = 0$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = \frac{\pi}{6} + 2\pi k, k \in \mathbb{Z} \qquad x = \frac{3\pi}{2} + 2\pi k, k \in \mathbb{Z}$

$x = \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$

c) $\cos(x + 60^\circ) = \sin(x + 30^\circ)$

$\cos x \cos 60^\circ - \sin x \sin 60^\circ = \sin x \cos 30^\circ + \cos x \sin 30^\circ$

$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$

$0 = \frac{2\sqrt{3}}{2}\sin x$

$0 = \sin x$

$x = \pi k, k \in \mathbb{Z}$

d) $2\sin x = 2\cos 2x$

$2\sin x = 2(1 - 2\sin^2 x)$

$2\sin x = 2 - 4\sin^2 x$

$4\sin^2 x + 2\sin x - 2 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1 = 0$

$2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \qquad \sin x + 1 = 0$

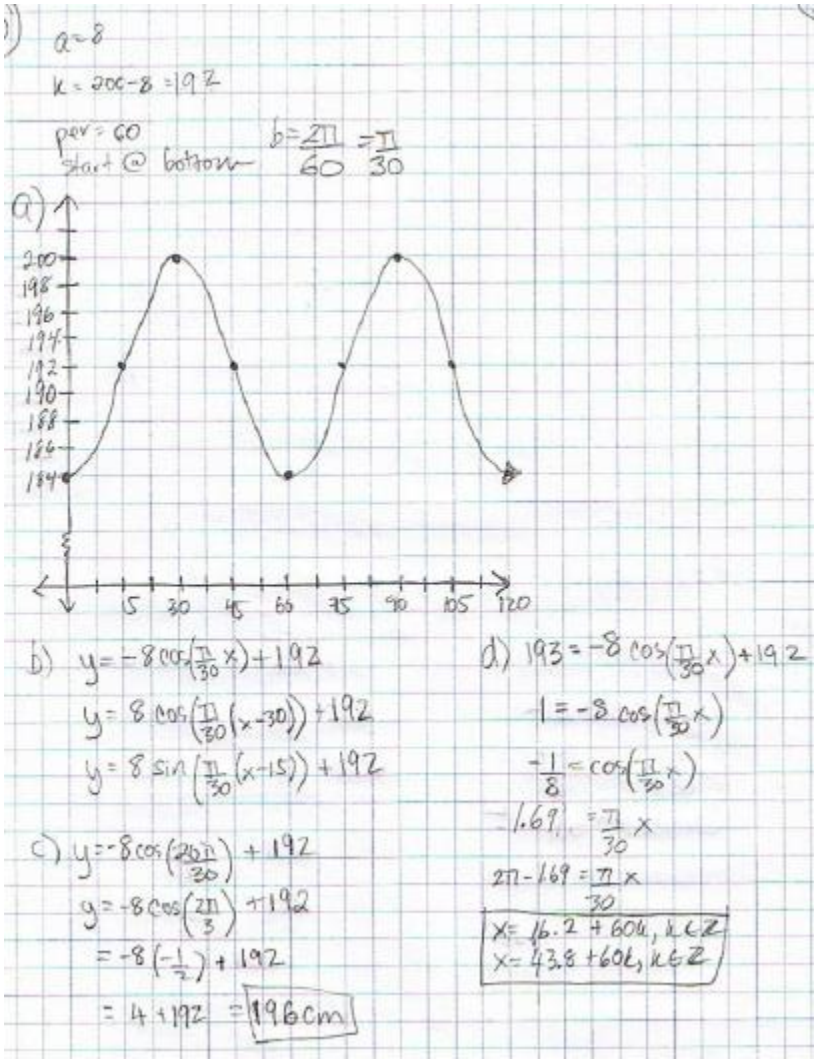
$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = \frac{\pi}{6} + 2\pi k, k \in \mathbb{Z} \qquad x = \frac{3\pi}{2} + 2\pi k, k \in \mathbb{Z}$

$x = \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$

17.

A



18. *a*) $\cot x \sin x$

$\cot x$ is undefined for $x = 0 \pm \pi n,\ n \in W$

so, $x \neq 0 \pm \pi n,\ n \in W$

b) $\dfrac{\tan x}{1-\cos^2 x}$

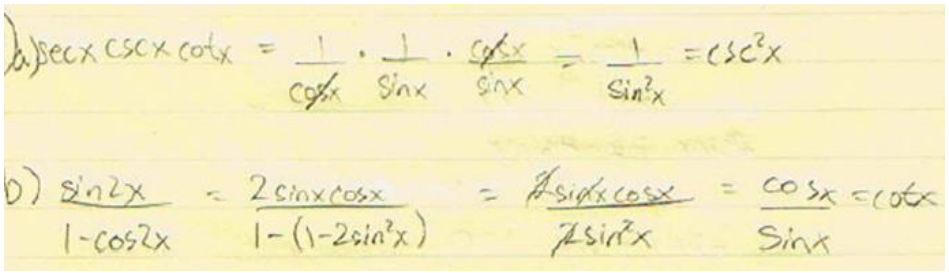
$\tan x$ is undefined for $x = \dfrac{\pi}{2} \pm \pi n,\ n \in W$

$1-\cos^2 x = 0 \rightarrow \cos^2 x = 1 \rightarrow \cos x = \pm 1 \rightarrow x = 0 \pm \pi n,\ n \in W$

so, $x \neq \dfrac{\pi}{2} \pm \pi n,\ n \in W$ and $x \neq 0 \pm \pi n,\ n \in W$

19.

A



a) $\sec x \csc x \cot x = \frac{1}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin^2 x} = \csc^2 x$

b) $\frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x \cos x}{1-(1-2\sin^2 x)} = \frac{2\sin x \cos x}{2\sin^2 x} = \frac{\cos x}{\sin x} = \cot x$

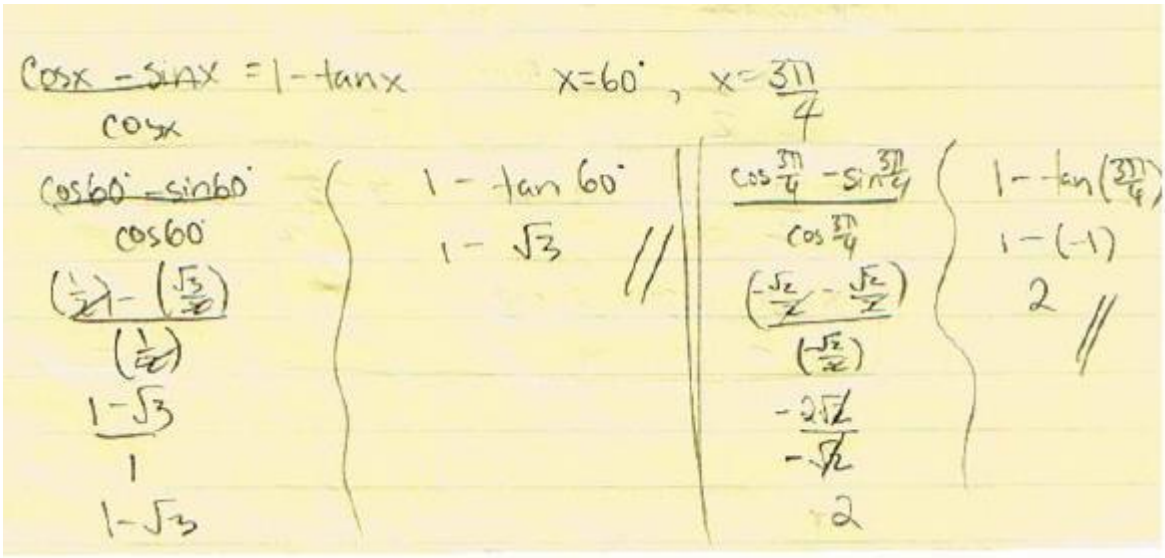
c) $\cos(x-180^\circ) = \cos x \cos 180^\circ + \sin x \sin 180^\circ$

$= \cos x(-1) + \sin x(0)$

$= -\cos x$

20.

A



$\frac{\cos x - \sin x}{\cos x} = 1 - \tan x \qquad x = 60^\circ, \; x = \frac{3\pi}{4}$

$\frac{\cos 60^\circ - \sin 60^\circ}{\cos 60^\circ}$ $\qquad$ $1 - \tan 60^\circ$

$\frac{\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)}$ $\qquad$ $1 - \sqrt{3}$ //

$\frac{1-\sqrt{3}}{1}$

$1-\sqrt{3}$

$\frac{\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}}{\cos\frac{3\pi}{4}}$ $\qquad$ $1 - \tan\left(\frac{3\pi}{4}\right)$

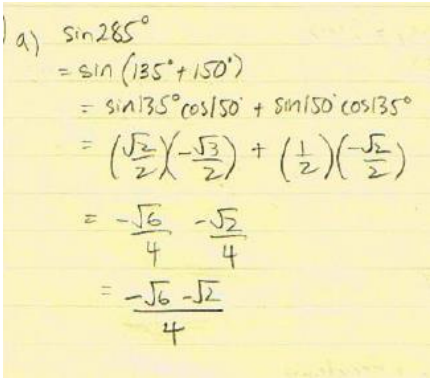
$\frac{\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)}$ $\qquad$ $1-(-1)$

$\frac{-2\sqrt{2}}{-\sqrt{2}}$ $\qquad$ 2 //

2

21.

A



a) $\sin 285^\circ$

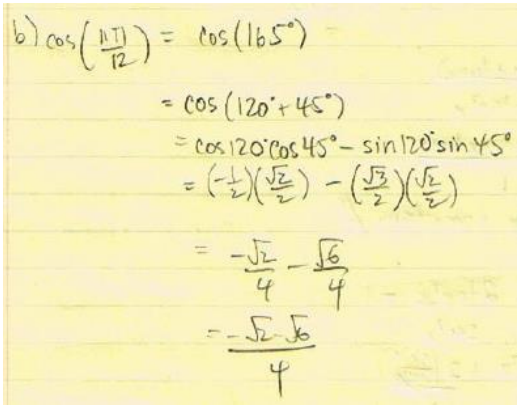
$= \sin(135^\circ + 150^\circ)$

$= \sin 135^\circ \cos 150^\circ + \sin 150^\circ \cos 135^\circ$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{-\sqrt{6}-\sqrt{2}}{4}$



b) $\cos\left(\frac{11\pi}{12}\right) = \cos(165^\circ)$

$= \cos(120^\circ + 45^\circ)$

$= \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{-\sqrt{2}-\sqrt{6}}{4}$

22. Please note: There are correct solutions other than the ones shown.

a.

$$\sin x + \cot x \cos x = \csc x$$

$$LHS = \sin x + \frac{\cos x}{\sin x}\cos x$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

$$= RHS$$

b.

$$2\cos x + 2\tan^2 x \cos x = 2\sec x$$

$$LHS = 2\cos x\left(1 + \tan^2 x\right)$$

$$= 2\cos x \sec^2 x$$

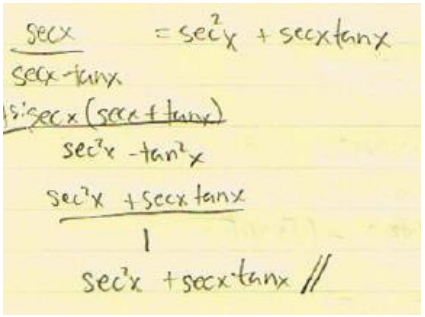
$$= 2\frac{1}{\sec x}\sec^2 x$$

$$= 2\sec x$$

$$= RHS$$

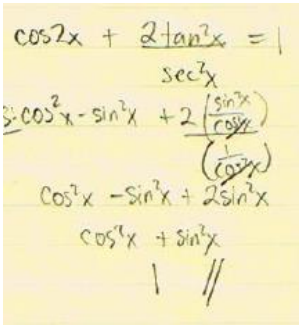
c.

$$\frac{\sec x}{\sec x - \tan x} = \sec^2 x + \sec x \tan x$$



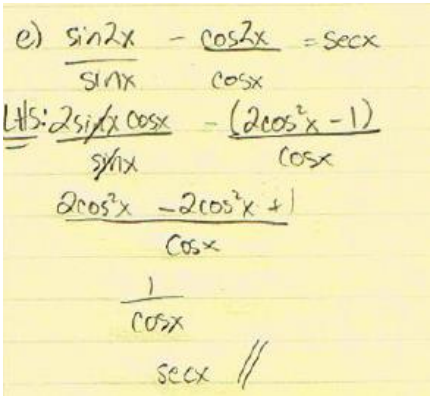
d.

$$\cos 2x + \frac{2\tan^2 x}{\sec^2 x} = 1$$



e.

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$$



f.

$$\sec 2x + \tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x}$$