Foundations of Math 110 - Exam Review

Formulas

PROPERTIES OF ANGLES & TRIANGLES

$$S = 180^o(n-2)$$

$$a = \frac{180^\circ\left(n-2\right)}{n}$$

TRIGONOMETRY

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

QUADRATIC FUNCTIONS & EQUATIONS

Standard form: $y = ax^2 + bx + c$

Factored form: $y = a\left(x-r\right)\left(x-s\right)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

FINANCIAL MATHEMATICS

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$FV = \frac{Rn}{r}\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]$$

$$rate\,of\ return = \frac{\text{interest earned}}{\text{total contributions}}$$

$$PV = \frac{Rn}{r}\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]$$

Chapter 1: Inductive & Deductive Reasoning

1. Which number should appear in the centre of Figure 4?

1		2	
	24		
4		3	

Figure 1

4		6	
	192		
4		2	

Figure 2

3		5	
	450		
6		5	

Figure 3

2		2	
2		3	

Figure 4

2. A motorcycle dealer in Fredericton has ordered 25 sport bikes and 12 cruisers for the upcoming riding season. Write a conjecture that describes the trend of motorcycle sales in Fredericton.

3. Write a counterexample for each of the following statements.
   a. As you travel farther north, the climate gets colder.
   b. If a polygon has four right angles, then it is a square.

4. Consider this number trick:

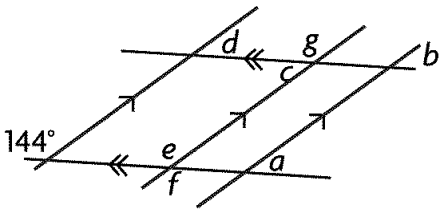
   - Choose any number
   - Multiply by 3
   - Add 12
   - Multiply by 2
   - Subtract 24
   - Divide by 6

   a. Use inductive reasoning to help you make a conjecture about the result of this number trick.
   b. Write your conjecture.
   c. Use deductive reasoning to prove your conjecture.

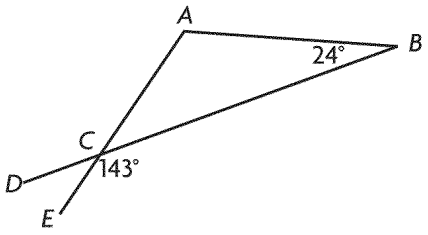
5. Prove, using deductive reasoning, that the sum of two odd integers is even.

Chapter 2: Properties of Angles & Triangles

6. What are the values of the following indicated angles (a-g)?

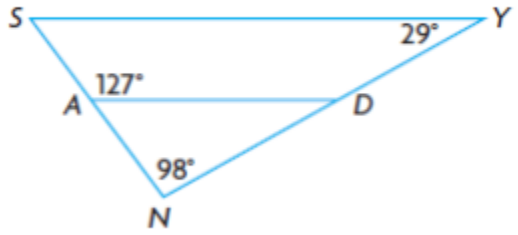


7. What are the measures of angle *DCE* and angle *CAB*?

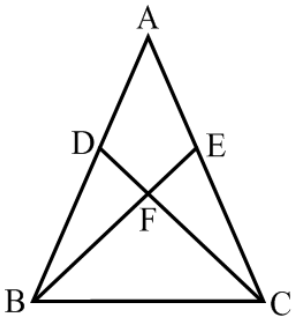


8. Prove that SY is parallel to AD.

Statement	Reason

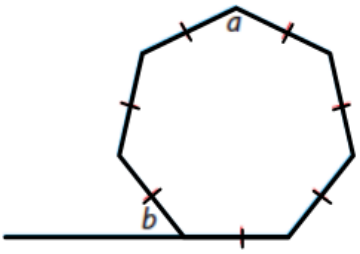


9. In ΔABC, AD = AE and DB = EC. Prove that BE = CD.



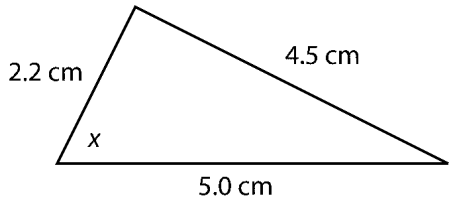
Statement	Reason

10. Calculate the measures of angles *a* and *b.*

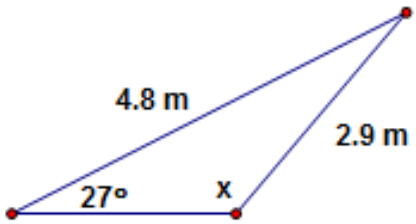


Chapters 3 and 4: Trigonometry

11. Which law could you use to directly determine the unknown angle in this triangle? Calculate x.



12. a. Three ropes were tied together to make an obtuse triangle. From the information given in the diagram shown, calculate the value of angle x.

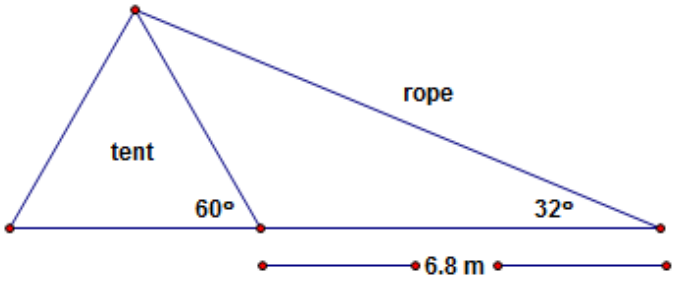


b. In ΔABC, $\angle A = 36°$, a = 5.9 cm, and b = 7.8 cm.

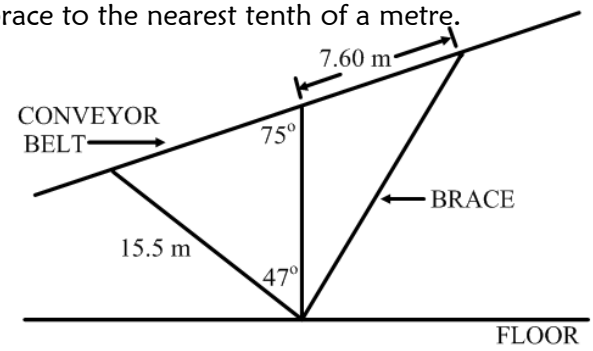
i) There are two possible triangles for this situation. Verify that this is an accurate statement with appropriate sketches / calculations / explanation.

ii) In each case above, determine the length, to the nearest tenth of a cm, of the third side of ΔABC.

13. A rope attached to a stake in the ground is used to support one side of a tent. Use the information given in the diagram to determine the length of the rope.



14. An engineer is working with a cross-section diagram that represents a conveyor belt used to move pulp into the plant, as shown below. The brace indicated on the diagram has to be replaced. Determine the length of the brace to the nearest tenth of a metre.

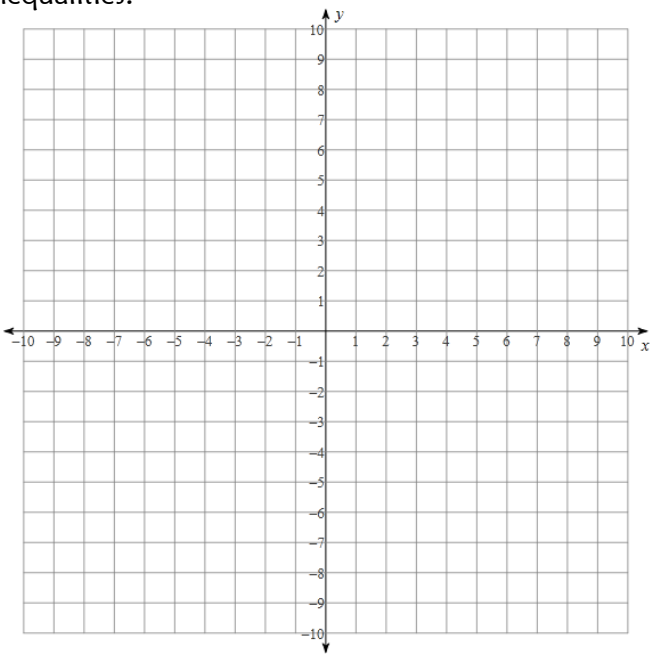


Chapter 5: Systems of Linear Inequalities

15. Is the point (–3, 1) in the solution set for the linear inequality $10y - 12x > 5$?

16. Graph the solution set for the following system of inequalities.

$\{(x, y) \mid x + y > 7, x \in I, y \in I\}$
$\{(x, y) \mid 2x - 8 \geq y, x \in I, y \in I\}$



17. A vending machine sells pop and juice. The vendor would like to maximize revenue from the machine.
    - The machine holds, at most, 200 drinks.
    - Sales from the vending machine show that at least 3 bottle of juice are sold for each can of pop.
    - Each bottle of juice sells for $1.50, and each can of pop sells for $1.00.

    If *x* represents the number of cans of pop and *y* represents the number of bottles of juice, write the *constraints*, identify the *restrictions*, and state the *objective function* for this optimization problem.

18. Jan volunteers to fold origami frogs and swans for a display. Each frog requires 1 square of green paper and each swan requires 1 square of white paper. She wants to determine the maximum time it will take her.
    - She has 8 squares of green paper for the frogs and 12 squares of white paper for the swans.
    - There must be at least twice as many swans as frogs.
    - It takes her 4 minutes to fold an origami frog and 3 minutes to fold an origami swan.

    If *x* represents the number of frogs, and *y* represents the number of swans, write the *constraints*, identify the *restrictions*, and state the *objective function* for this optimization problem.

19. A toy company makes two types of stuffed animals, cats and dogs, and it wants to maximize profit. A cat takes 1.5 hours to assemble and 0.5 hours to package. A dog takes 2 hours to assemble and 0.25 hours to package. A maximum of 60 hours is spent on assembly, and a maximum of 20 hours is spent on packaging. The company makes a profit of $5 on the cat and $6 on the dog.
    a. Define the variables and state the restrictions for this situation.
    b. Write the constraints as a system of linear inequalities.
    c. Write the objective function.

20. The following model represents an optimization problem. Graph the constraints and determine the values of x and y that will maximize the value of the objective function, R.

    Restrictions:

    $x \in W,\ y \in W$

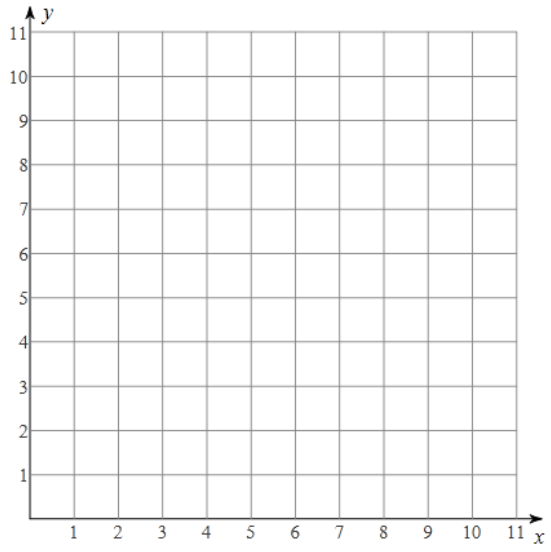
    Constraints:

    $$x \leq 6$$

    $$x + 2y \leq 10$$

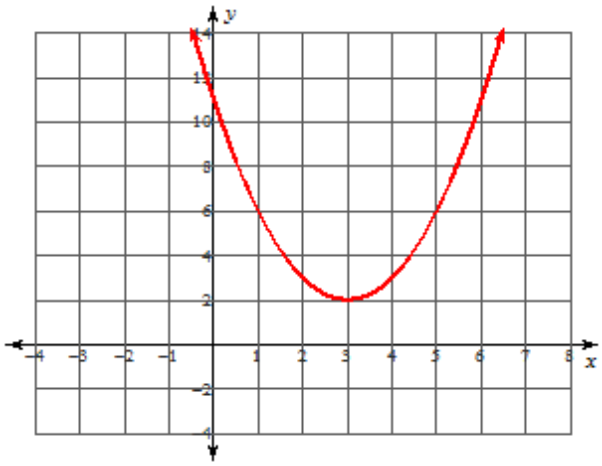
    Objective function:

    $$R = 12x + 11y$$



Chapter 6: Quadratic Functions & Equations

21. For the following graph, state the equation of the axis of symmetry, vertex, domain and range.



22. What are the *x*- and *y*-intercepts for the function $f(x) = x^2 - 7x + 12$?

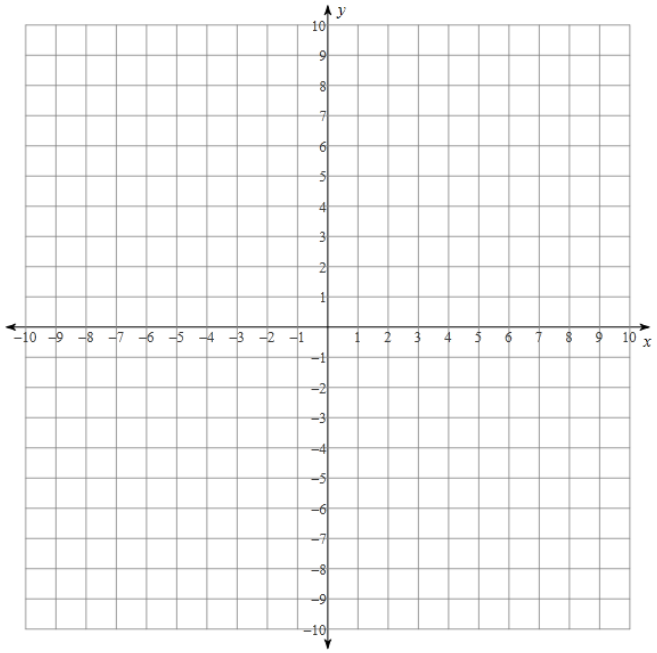
23. Solve by factoring.

a) $36x^2 = 64$ b) $2x^2 - 30x + 112 = 0$ c) $3x^2 - x - 10 = 0$

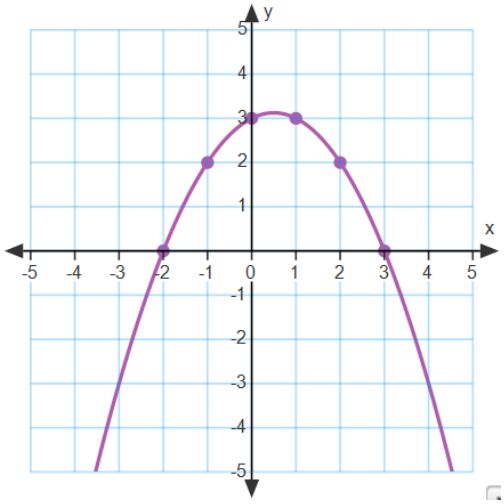
24. For the following function: $f(x) = x^2 + 4x - 3$

a. Use partial factoring to determine two points that are equidistant from the axis of symmetry.

b. Determine the y-intercept, the coordinates of the vertex, the equation of the axis of symmetry, the domain and range, and sketch the graph of the function.



25. Determine the equation of the following quadratic function in factored form and standard form.



26. A water park currently sells day passes for $60. At this price, the park sells 700 passes every day. The owners have determined that they can sell 100 more passes per day for each price decrease of $5.
    a. Write a function that can be used to calculate the daily revenue.
    b. What should the owners charge for the day pass to maximize their revenue?
    c. What will be their maximum revenue?

27. The daily revenue of R dollars for a ski resort can be modeled by the equation $R = -14t^2 - 280t + 4200$, where t represents the temperature in degrees Celsius.
    a. What will the revenue be when the temperature is 0°C?
    b. At what temperature(s) will the revenue be $0?
    c. What temperature will maximize the revenue?
    d. What is the maximum revenue?

Chapter 8: Financial Mathematics – Investing Money

28. Estimate the amount of time it will take for a $700 investment to double if it is invested at 1.7%, compounded weekly.

29. Determine the interest earned on a 15-year investment with an interest rate of 3.88%, compounded semi-annually, if the future value is $120 000.

30. Determine the present value of a 8 year GIC with an interest rate of 6.4%, compounded monthly, if the future value is $12 000.

31. Determine the future value of monthly payments of $800 into an account that pays 2.03% interest, compounded monthly, for 10 years.

32. Determine the regular monthly payment required to have $10 000 at the end of 5 years if the investment earns 3.5% interest, compounded monthly.

33. This portfolio was started 15 years ago:
    - A 15-year $4000 GIC that earns 2.75%, compounded monthly.
    - Weekly deposits of $75 into an account earning 1.85%, compounded weekly.

    a. Determine the current value of the portfolio.

b. What is the current rate of return in these investments?

Chapter 9: Financial Mathematics – Borrowing Money

34. Kate had her wedding photographs framed, and the bill came to $1250. She arranged to make payments every month for a year, at an interest rate of 15%, compounded monthly.
    a. How much will each payment need to be?
    b. How much interest will Kate end up paying?

35. Paul wants to buy a business and has negotiated a loan at 4.9%, compounded quarterly, with regular payments of $3500 every three months. He plans to repay the loan in six years.
    a. How much can Paul borrow?
    b. How much interest will Paul pay?

36. Danielle is buying a house that costs $275 000. She will finance the purchase with a 25 year mortgage with an interest rate of 2.9%, compounded monthly. She must make a down payment of 15% of the purchase price.
    a. What is the amount of the down payment?
    b. How much will each monthly payment be?
    c. How much will Danielle pay in monthly payments over the 25 year period?
    d. How much interest will Danielle end up paying by the time she has paid off the loan?
    e. How much will she pay altogether for the house?

37. Sasha got a job and now needs a place to live. She can rent an apartment for $500 per month. She can also buy a house for $240 000. She has negotiated a mortgage with the bank for 90% of the purchase price at an interest rate of 4.6%, compounded monthly. Sasha will pay off the mortgage by making regular monthly payments for 17 years. The house also appreciates at 1.5% annually.
    a. What are the costs of renting an apartment over 17 years? Show your work.
    b. What are the costs of purchasing a house over 17 years? Show your work.
    c. Which option would you recommend for Sasha? Explain.

Solutions

1. Multiply the outside numbers in the square: $2\times 2\times 2\times 3=24$

2. Frank expects to sell more sport bikes than cruisers. In fact, he expects to sell about twice as many sport bikes than cruisers.

3. Answers will vary
    a. If you are in the southern hemisphere and you travel north, the climate gets hotter.
    b. A rectangle is a quadrilateral with 4 right angles.

4.
    a. For example, choose $n=5$
    $5\times 3=15$
    $15+12=27$
    $27\times 2=54$
    $54-24=30$
    $30\div 6=5$

b. The answer of this number trick is always the chosen number.

c. $(n)\times 3 = 3n$

$(3n)+12 = 3n+12$

$(3n+12)\times 2 = 6n+24$

$(6n+24)-24 = 6n$

$(6n)\div 6 = n$

5. Let $2a+1$ and $2b+1$ be any two odd integers.
$(2a+1)+(2b+1) = 2a+2b+2 = 2(a+b+1)$ which is always an even number.

6. $e = 144°$ - corresponding angle with $144°$
$f = 144°$ - vertically opposite angle to e
$c = 36°$ - co-interior angle to e
$d = 36°$ - alternate-interior angle to c
$g = 144°$ - supplementary angle to c
$a = 36°$ - alternate-interior angle to c
$b = 36°$ - corresponding angle with a

7. $\angle DCE = 37°$ - supplementary angle to $143°$
$\angle ACB = 37°$ - supplementary angle to $143°$
$\angle CAB = 119°$ - sum of the angles of a Δ

8.

Statements	Reasons
$\angle SNY = 98°$	given
$\angle SYN = 29°$	given
$\angle SAD = 127°$	given
$\angle NSY = 53°$	The sum of the angles of a triangle
$\angle DAN = 53°$	Supplementary to $\angle SAD$
$SY \parallel AD$	Corresponding angles are equal

9.

Statements	Reasons
$AD = AE$	Given
$DB = EC$	Given
$AB = AC$	Congruent parts
ΔABC is isosceles	Two equal sides
$\angle ABC = \angle ACB$	**I**sosceles **T**riangle **T**heorem
$BC = BC$	Common side
$\Delta BDC \cong \Delta CEB$	**S**ide **A**ngle **S**ide
$\therefore BE = CD$	Corresponding sides of congruent triangles

10. $a = \dfrac{180°(n-2)}{n} = \dfrac{180°(7-2)}{7} \approx 128.6°$; $b = 180° - 128.6° = 51.4°$

11. The law of cosines (all 3 sides are known). x = 64°

12. a. $\dfrac{\sin x}{4.8} = \dfrac{\sin 27°}{2.9}$

$\sin x = \dfrac{4.8\sin 27°}{2.9} = 0.7514$

$x = \sin^{-1}(0.7514) \doteq 48.7°$

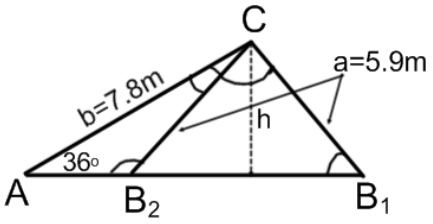
But, since angle x is obtuse, then $x = 180° - 48.7° = 131.3°$.

b. i)

$h = b\sin A$ Since h < a < b, then there are two triangles possible.

= 7.8sin36°

= 4.6 m



ii)

Case 1:

$\dfrac{\sin B}{7.8} = \dfrac{\sin 36^o}{5.9}$

$\sin B = 0.7771$

$\angle B = 51^o$

$\angle C = 180^o - 36^o - 51^o = 93^o$

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 5.9^2 + 7.8^2 - 2(5.9)(7.8)\cos 93^o$

$c = 10.0\ m$

Case 2:

$\angle B$ is obtuse, so

$\angle B = 180^o - 51^o = 129^o$

$\angle C = 180^o - 36^o - 129^o = 15^o$

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 5.9^2 + 7.8^2 - 2(5.9)(7.8)\cos 15^o$

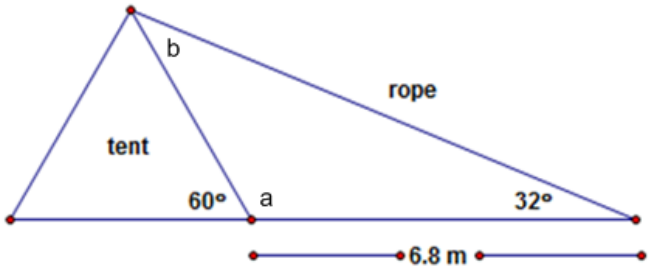
$c = 2.6\ m$

13. $\angle a = 180° - 60° = 120°$

$\angle b = 180° - 32° - 120 = 28°$

$\dfrac{\text{rope}}{\sin 120°} = \dfrac{6.8}{\sin 28°}$

rope $\doteq$ 12.5m



14. $x = 180° - 75° - 47° = 58°$

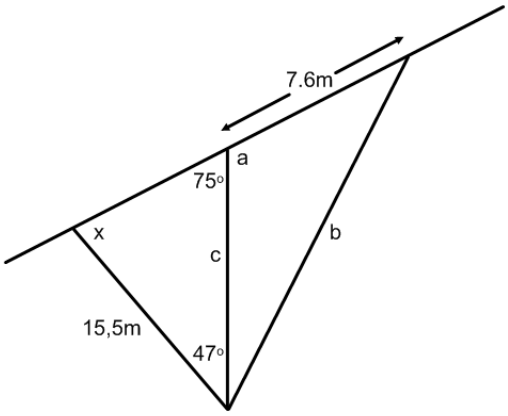
$a = 180° - 75° = 105°$

$\dfrac{c}{\sin 58°} = \dfrac{15.5}{\sin 75°}$

$c = \dfrac{15.5\sin 58°}{\sin 75°} \doteq 13.61\text{m}$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b = \sqrt{(7.6)^2 + (13.61)^2 - 2(7.6)(13.61)\cos 105°} \doteq 17.2\text{m}$



15. Substitute the point $(-3,1)$ in the inequality.

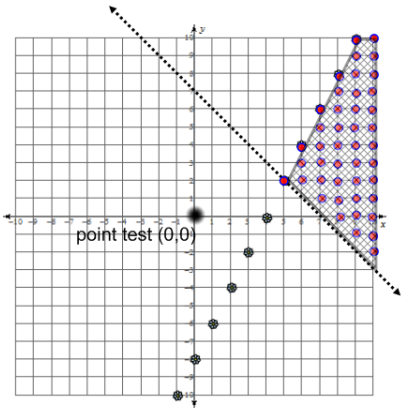
$10y-12x>5$

$10(1)-12(-3)>5$

$10+36>5$

$46>5$

Therefore, the point is in the solution set.



16. $x+y=7$ $2x-8=y$

x-int: $x=7$ x-int: $x=4$

y-int: $y=7$ y-int: $y=-8$

Test Point $(0,0)$

$0+0>7$ False $2(0)-8\geq 0$ False

17. Constraints: $x+y\leq 200$ and $y\geq 3x$

Restrictions: $x\in W$, $y\in W$

Objective function: $R=x+1.5y$

18. Constraints: $x\leq 8$, $y\leq 12$, and $y\geq 2x$

Restrictions: $x\in W$, $y\in W$

Objective function: $T=4x+3y$

19. a. Let x = the number of stuffed cats, $x\in W$

Let y = the number of stuffed dogs, $y\in W$

b. *assembly*: $1.5x+2y\leq 60$

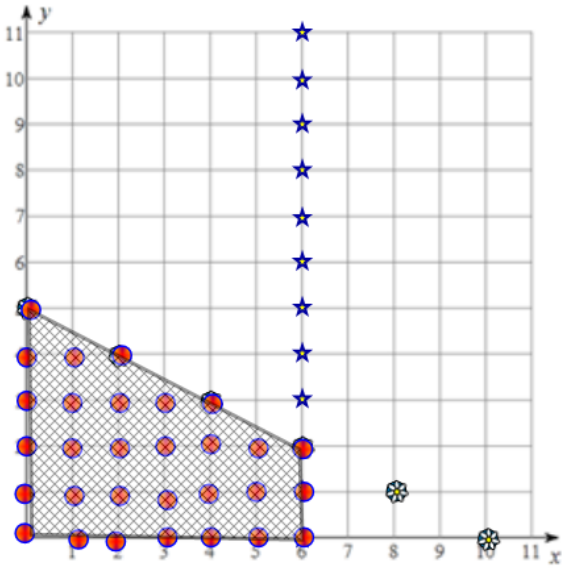
packaging: $0.5x+0.25y\leq 20$

c. $P=5x+6y$

20. $x=6$ $2y+x\leq 10$

x-int: $x=6$ x-int: $x=10$

vertical line y-int: $y=5$



Points	$R=12x+11y$
$(0,0)$	\$0
$(0,5)$	\$55
$(6,2)$	\$94
$(6,0)$	\$72

Therefore, the max value of R is \$94 when x = 6 and y = 2.

21. Vertex: $(3,2)$; axis of symmetry: $x=3$; Domain: $x\in\mathbb{R}$; Range: $\{y\geq 2, y\in\mathbb{R}\}$

22. $f(x)=x^2-7x+12=(x-3)(x-4)=0$

$(x-3)=0$ $\quad$ $(x-4)=0$

$x=3$ $\quad$ $x=4$ $\quad$ $f(0)=12$

$(3,0)$ $\quad$ $(4,0)$ $\quad$ $(0,12)$

23.

a. $36x^2=64$

$36x^2-64=0$

$4(9x^2-16)=0$

$4(3x-4)(3x+4)=0$

$x=\frac{4}{3},\ -\frac{4}{3}$

b. $2x^2-30x+112=0$

$2(x^2-15x+56)=0$

$2(x-7)(x-8)=0$

$x=7,\ 8$

c. $3x^2-x-10=0$

$3x^2-6x+5x-10=0$

$3x(x-2)+5(x-2)=0$

$(x-2)(3x+5)=0$

$x=2,\ -\frac{5}{3}$

24.

a. $let\ f(x)=-3$

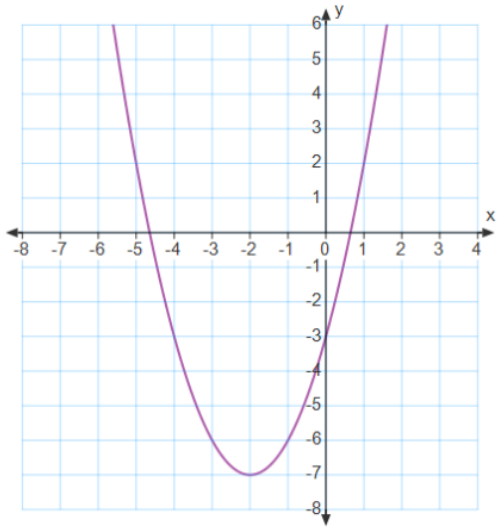
$-3=x^2+4x-3$

$0=x^2+4x$

$0=x(x+4)$

$x=0,-4$

Points: $(0,-3),\ (-4,-3)$



b. y-int: $(0,-3)$; axis of symmetry: $x=-2$;

vertex: $(-2,-7)$; domain: $x\in\mathbb{R}$; range: $\{y\geq -7, y\in\mathbb{R}\}$

25.

$y=a(x-r)(x-s)$

x-intercepts are –2 and 3, so $y=a(x+2)(x-3)$

Substitute the point (0, 3) to find "a":

$3=a(0+2)(0-3)$

$3=-6a$

$a=-\frac{1}{2}$

$y=-\frac{1}{2}(x+2)(x-3)=-\frac{1}{2}(x^2-x-6)$

$\therefore y=-\frac{1}{2}x^2+\frac{1}{2}x+3$

26.

a. $\text{Revenue} = \text{Price} \times \text{Quantity} = (60-5x)(700+100x)$

b. $(60-5x)(700+100x) = 0$

$(60-5x) = 0 \qquad (700+100x) = 0$

$5x = 60 \qquad 100x = -700$

$x = 12 \qquad x = -7$

Thus, the axis of symmetry is $x = \dfrac{12-7}{2} = \dfrac{5}{2} = \$2.50$

$\text{Price} = 60 - 5(2.5) = \$47.50$

c. $\text{Revenue} = (60-5(2.5))(700+100(2.5)) = (47.50)(950) = \$45125$

27.

a. $R(0) = -14(0)^2 - 280(0) + 4200 = 4200$

b. $R(t) = -14t^2 - 280t + 4200 = 0$

$t^2 + 20t - 300 = 0$

$(t-10)(t+30) = 0$

$t = 10°\text{C} \qquad t = -30°\text{C}$

c. $t = \dfrac{10-30}{2} = -10°\text{C}$

d. $R(-10) = -14(-10)^2 - 280(-10) + 4200 = -1400 + 2800 + 4200 = \$5600$

28. Use the rule of 72.

$t = \dfrac{72}{1.7} \doteq 42.4 \text{ years}$

29. $A = P\left(1+\dfrac{r}{n}\right)^{nt}$

$120000 = P\left(1+\dfrac{0.0388}{2}\right)^{(2)(15)}$

$P = \dfrac{120000}{\left(1+\dfrac{0.0388}{2}\right)^{(2)(15)}} = \$67428.32$

thus $I = \$120000 - \$67428.32 = \$52571.68$

30. $A = P\left(1+\frac{r}{n}\right)^{nt}$

$12000 = P\left(1+\frac{0.064}{12}\right)^{(12)(8)}$

$P = \frac{12000}{\left(1+\frac{0.064}{12}\right)^{(12)(8)}} = \$7201.34$

31. $FV = \frac{Rn}{r}\left[\left(1+\frac{r}{n}\right)^{nt}-1\right] = \frac{(800)(12)}{0.0203}\left[\left(1+\frac{0.0203}{12}\right)^{(12)(10)}-1\right] = \$106338.86$

32. $FV = \frac{Rn}{r}\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]$

$10000 = \frac{R(12)}{0.035}\left[\left(1+\frac{0.035}{12}\right)^{(12)(5)}-1\right]$

$10000 = \frac{R(12)}{0.035}[0.190942829]$

$\$152.75 = R$

33.

a. GIC: $A = P\left(1+\frac{r}{n}\right)^{nt} = 4000\left(1+\frac{0.0275}{12}\right)^{(12)(15)} = \$6039.51$

Deposits: $FV = \frac{Rn}{r}\left[\left(1+\frac{r}{n}\right)^{nt}-1\right] = \frac{(75)(52)}{0.0185}\left[\left(1+\frac{0.0185}{52}\right)^{(52)(15)}-1\right] = \$67409.07$

Current Value: $\$6039.51 + \$67409.07 = \$73448.58$

b. Total Contributions: $\$4000 + (\$75)(52)(15) = \$4000 + \$58500 = \$62500$

Interest Earned = $\$73448.58 - \$62500 = \$10948.58$

$\text{Rate of Return} = \frac{\text{interest earned}}{\text{total contributions}} = \frac{\$10948.58}{\$62500} = 0.175 \text{ or } 17.5\%$

34.

a. $PV = \frac{Rn}{r}\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]$

$1250 = \frac{R(12)}{0.15}\left[1-\left(1+\frac{0.15}{12}\right)^{-(12)(1)}\right]$

$R = \frac{(1250)(0.15)}{(12)\left[1-\left(1+\frac{0.15}{12}\right)^{-(12)(1)}\right]} = \$112.82$

b. $I = Rnt - PV = (\$112.82)(12)(1) - \$1250 = \$103.84$

35.

a. $PV = \frac{Rn}{r}\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]$

$= 3500\frac{4}{0.049}\left[1-\left(1+\frac{0.049}{4}\right)^{-(4)(6)}\right]$

$= 3500\frac{4}{0.049}[0.253391123]$

$= \$72397.46$

$I = Rnt - PV = (\$3500)(4)(6) - \$72397.46$

b. $= \$84000 - \$72397.46$

$= \$11602.54$

36. A) $275000 \times 0.15 = \$41250$

B) $233750 = R\frac{12}{0.029}\left(1-\left(1+\frac{0.029}{12}\right)^{-12\times 25}\right)$

$233750 = R(213.207575)$

$R = \$1096.35$

C) $1096.35 \times 12 \times 25 = \$328905$

D) $I = 328905 - 233750 = \$95155$

E) $T = 41250 + 328905 = \$370155$

37. A) $A = 500 \times 12 \times 17 = \$102000$

B) $240000 \times 0.90 = 216000$

$$216000 = R\frac{12}{0.046}\left(1 - \left(1 + \frac{0.046}{12}\right)^{-12\times17}\right)$$

$$216000 = R(141.3457614)$$

$$R = \$1528.17$$

Total monthly payments = 1528.17 x 12 x 17 = $311 746.68
Selling price = 240 000$(1.015)^{17}$ = $309 124.88
Total cost = $24 000 (down payment) + $311 746.68 – $309 124.88 = $26 621.80

C) Answers may vary…
Renting is better because it is a lower monthly bill.
Buying a house is better because you can recover some of the cost as an owner.