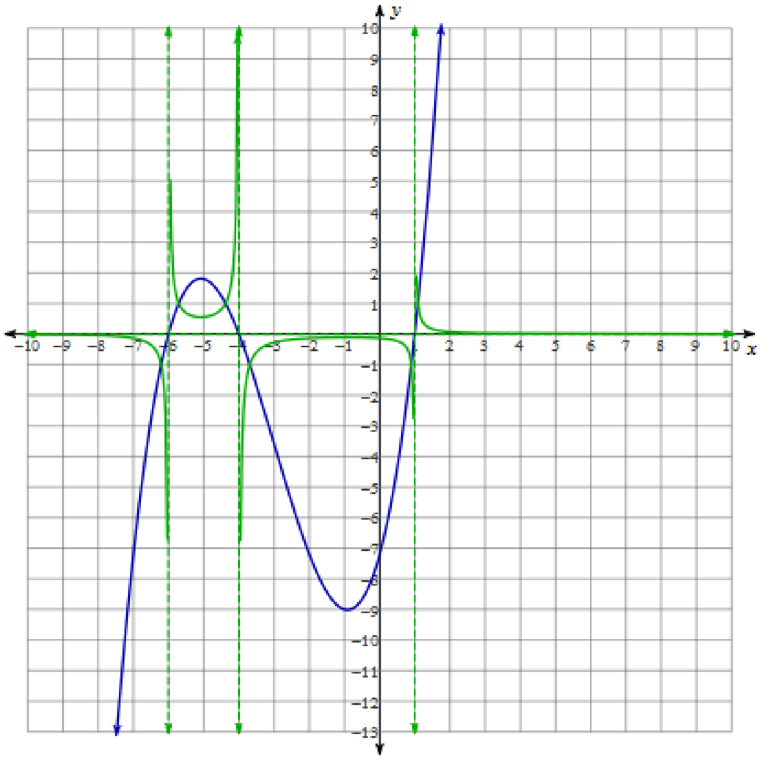
Function Toolkit #3 – Reciprocals, Inverses & Function Operations

1. Given the following graph of $f(x)$, sketch the graph of $y = \frac{1}{f(x)}$ showing all asymptotes, intercepts and invariant points. State the domain and range of the reciprocal function.

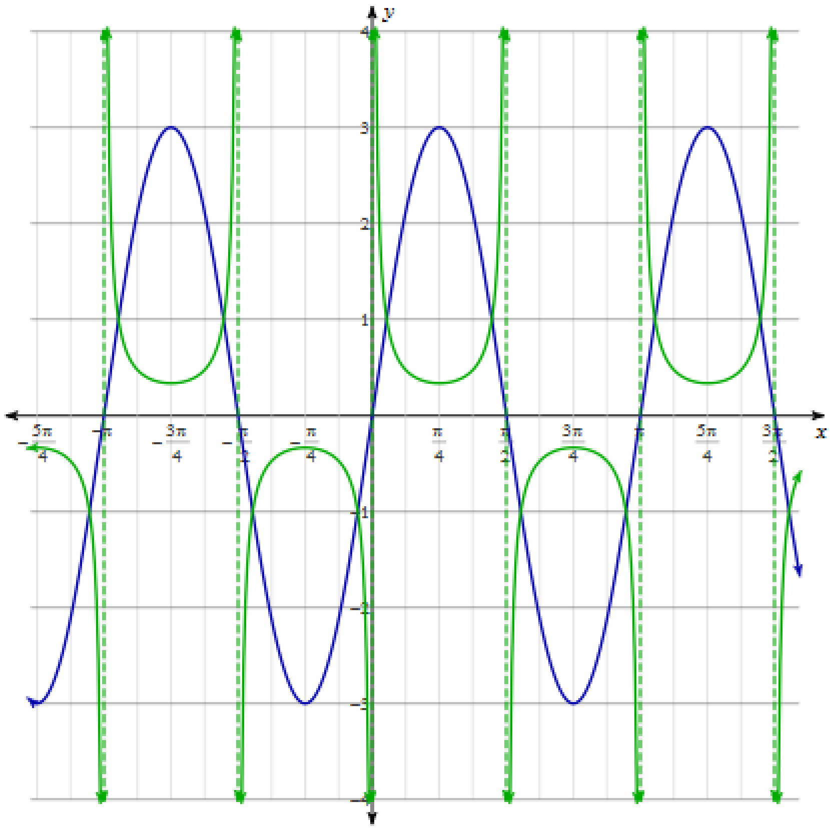


Domain: $x \in R, x \neq -6, -4, 1$

Range: $y \in R, y \neq 0$

2. Given $f(x) = \log x$, sketch the graph of $y = \dfrac{1}{\log x}$.

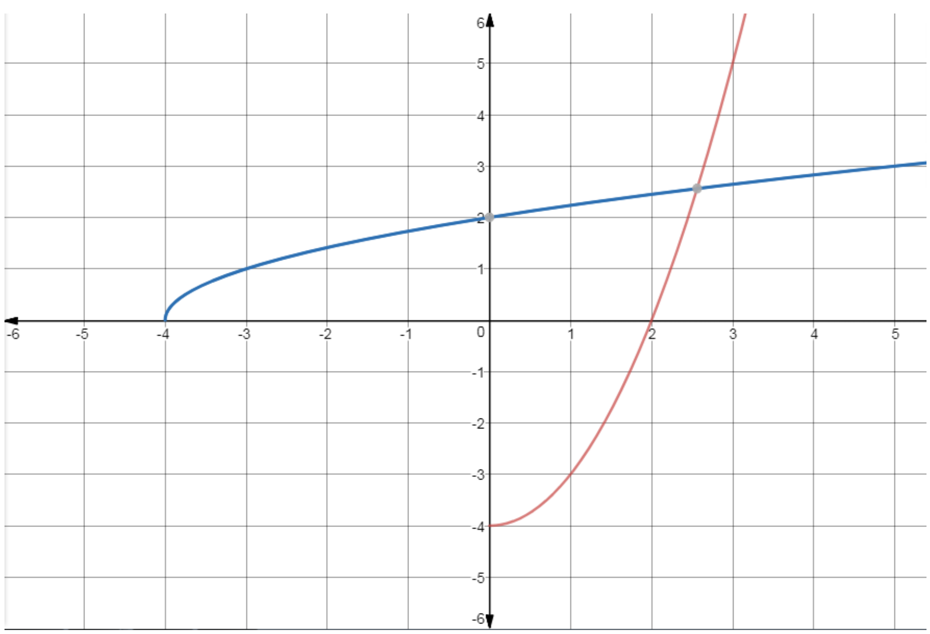
3. Given $f(x) = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$, sketch the graph of $y = \frac{1}{f(x)}$ showing all x- and y-intercepts and invariant points. State the domain and range of the reciprocal function.



Domain: $x \in R,\ x \neq \frac{\pi}{2}n,\ n \in I$

Range: $y \in (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, \infty)$

4. Given $f(x) = x^2 - 4, x \geq 0, x \in R$, sketch the graph of $y = f^{-1}(x)$. State the equation, domain and range of the inverse function. Explain why the domain of the original function must be restricted so the inverse is a function.



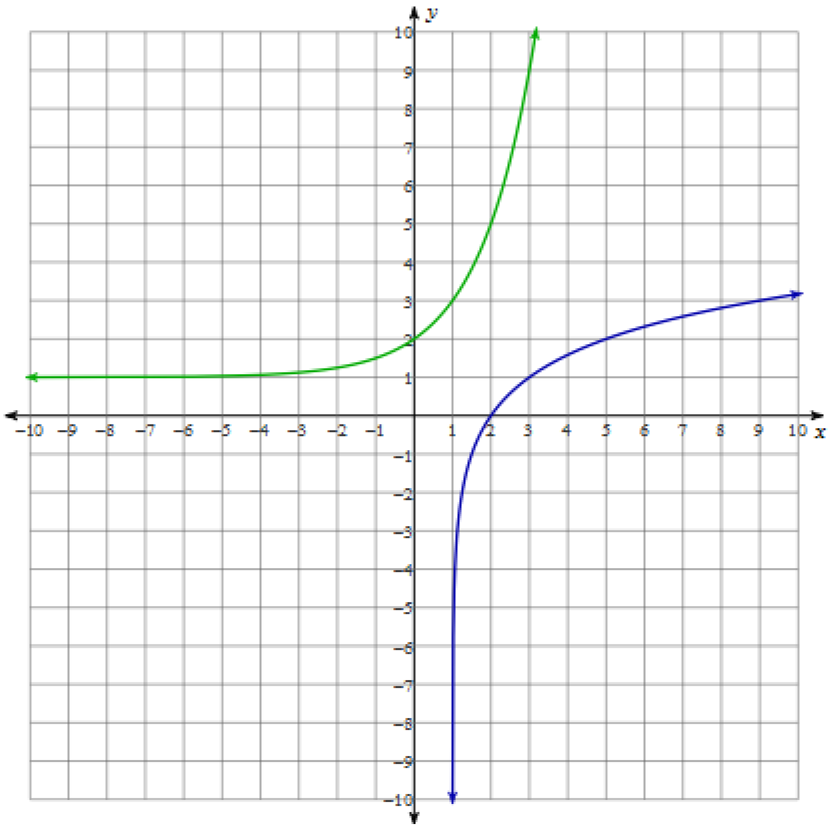
Equation: $f^{-1}(x) = \pm\sqrt{x+4}$

Domain: $x \geq -4,\ x \in R$

Range: $y \geq 0,\ y \in R$

For each y-value in the original function there can only be one x-value so that there will only be one y-value for each x-value for the inverse, which makes the inverse a function.

5. Given $f(x) = \log_2(x-1)$, sketch the graph of $y = f^{-1}(x)$. State the domain and range of the *inverse* function.

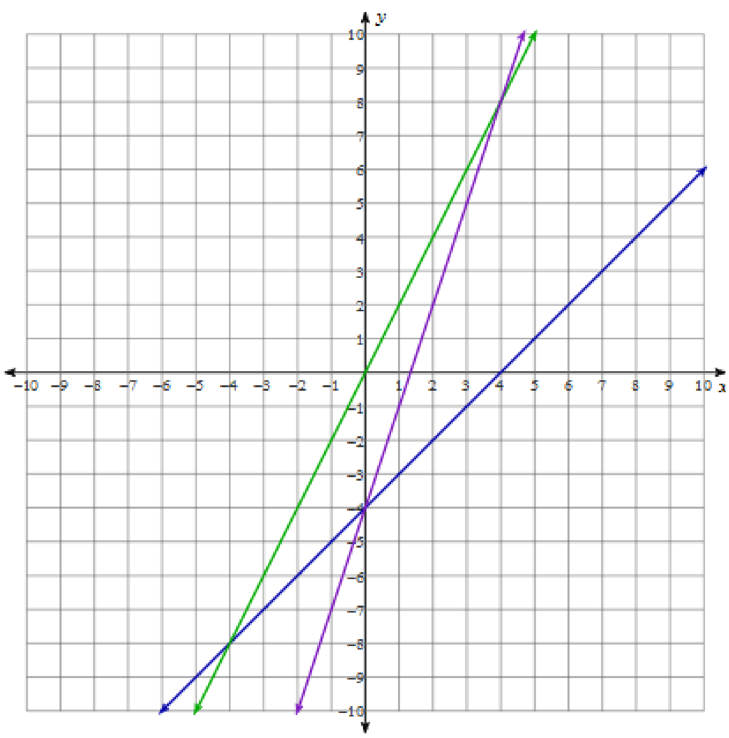


Domain: $x \in R$

Range: $y > 1,\ y \in R$

6. Given the graphs of $f(x) = 2x$ and $g(x) = x - 4$, sketch the following graphs.

a) $y = f(x) + g(x)$



b) $y = \frac{f(x)}{g(x)}$