9) a) 2 _ _ 4 ← Case ① 1 x 3 x 1 = 3

3 _ 2 or 4 ← case ② 1 x 3 x 2 = 6

4 _ 2 ← Case ③ 1 x 3 x 1 = 3

5 _ 2 or 4 ← Case ④ 1 x 3 x 2 = 6

18 possibilities

b) B _ _ A, E, U ← case ① 1 x 4 x 3 x 3 = 36

E _ _ A, U ← case ② 1 x 4 x 3 x 2 = 24

60 possibilities

15) $40 \cdot 40 \cdot 40 = 64000$ codes

$$\frac{64000 \text{ codes}}{4 \text{ codes/min}} = 16000 \text{ min} = 266\text{h } 40\text{min.}$$

15 s per code

11b) $\frac{7!}{2!} = 2520$

17b)

3 l's together

$$\frac{6!}{2!} = 360$$

2 a's

19) 3000 to 8999 inclusive, no 7's

$5 \cdot 9 \cdot 9 \cdot 9 = 3645$

(3 to 8, not 7)

not 7

20) a) $\underset{L}{26} \cdot \underset{D}{10} \cdot \underset{L}{26} \cdot \underset{D}{10} \cdot \underset{L}{26} \cdot \underset{D}{10}$

$= \boxed{17\,576\,000}$ possible postal codes

22c) $${}_nP_3 = 4({}_{n-1}P_2)$$

$$\frac{n!}{(n-3)!} = 4\,\frac{(n-1)!}{(n-1-2)!}$$

$$\frac{n!}{(n-3)!} = \frac{4(n-1)!}{(n-3)!}$$

$$n! = 4(n-1)!$$

$$\frac{n!}{(n-1)!} = 4$$

$$\frac{n(n-1)!}{(n-1)!} = 4$$

$$\boxed{n = 4}$$

25) One-digit: 1, 3, or 5 — 3

two-digits: not 0 — 1, 3, or 5 — + 4 × 3

three digits: not 0 — — 1, 3, or 5 + 4 × 4 × 3

0, 1, 2, 3, 4, 5

3 + 12 + 48 = 63

(odd numbers of at most three digits using the digits 0, 1, 2, 3, 4, 5 with no repetitions)

26) four-digits: 0, 1, 2, 3, 5

① ___ ___ ___ 0 $4 \times 3 \times 2 \times 1 = 24$

② not 0 ___ ___ 2 $3 \times 3 \times 2 \times 1 = 18$

five-digits:

① ___ ___ ___ ___ 0 $4 \times 3 \times 2 \times 1 \times 1 = 24$

② not 0 ___ ___ ___ 2 $3 \times 3 \times 2 \times 1 \times 1 = 18$

$24 + 18 + 24 + 18 = \boxed{84}$

(even numbers of at least 4 digits using the digits 0, 1, 2, 3, 5 with no repetitions)

27) between 1 and 1000, no repeats

one-digit: not 0 or 1 — 8

two digits: not 0 — not first digit — $9 \times 9 = 81$

three digits: not 0 — not 1st digit — not 1st or 2nd digit — $9 \times 9 \times 8 = 648$

$8 + 81 + 648 = \boxed{737}$