

Logarithms Formula Sheet

Definition of Logarithm :

The **logarithm** of a positive number is the **power** of the **base** that produces the number.

A logarithm whose **base** is 10 is called a **common logarithm**. It has the default form $\log N$.

A logarithm whose **base** is e is called a **natural logarithm**. It has the default form $\ln N$.

$\log_b x = y$ if and only if $b^y = x$	or	$\ln x = y$ if and only if $e^y = x$
<i>“logarithmic form”</i>		<i>“logarithmic form”</i>
<i>“exponential form”</i>		<i>“exponential form”</i>

Properties of Logarithms : (obtained by applying the definition)

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| 1. | $\log_b 1 = 0$ | or | $\ln 1 = 0$ | “A logarithm of 1 equals 0.” |
| 2. | $\log_b b = 1$ | or | $\ln e = 1$ | “A logarithm of its base equals 1.” |
| 3. | $\log_b b^n = n$ | or | $\ln e^n = n$ | “A logarithm of a power of its base equals that power.” |
| 4. | $b^{\log_b N} = N$ | or | $e^{\ln N} = N$ | “A base, to a logarithm power with the same base, equals the antilogarithm.” |

Laws of Logarithms : (used to expand or simplify/condense log expressions)

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| 1. | $\log_b MN = \log_b M + \log_b N$ | $\ln MN = \ln M + \ln N$ | Product/Sum Law |
| 2. | $\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$ | $\ln \left(\frac{M}{N} \right) = \ln M - \ln N$ | Quotient/Difference Law |
| 3. | $\log_b x^n = n \log_b x$ | $\ln x^n = n \ln x$ | Power/Coefficient Law |
| 4. | $\log_b M = \log_b N$ if and only if $M = N$ | | Equal Logs Law |

Change-of-Base Formula :

$\log_b N = \frac{\log_a N}{\log_a b}$	* where a is the new base
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$\log_b N = \frac{\log N}{\log b} = \frac{\ln N}{\ln b}$	* changed to common log or natural log
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