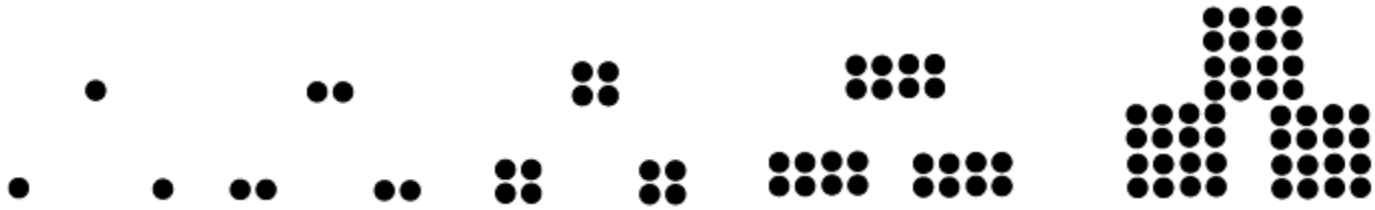
Geometric Sequences



The number of dots in each arrangement forms the sequence: _____, _____, _____, _____, _____

A **geometric sequence** is a sequence in which each term, after the first term, is found by multiplying the previous term by a non-zero constant, r, called **the common ratio.** The common ratio can be found by dividing each term by the previous term, $r = \frac{t_n}{t_{n-1}}$.

In the sequence 3, 6, 12, 24, …, the common ratio, r, is _____.

Determining the General Term for a Geometric Sequence

Consider the geometric sequence 3, 6, 12, 24, … from above.

Using the *first term* and the *common ratio*, we can write the term values of this sequence as follows:

$t_1 = 3 \times 2^0$
$t_2 = 3 \times 2^1$
$t_3 = 3 \times 2^2$
$t_4 = 3 \times 2^3$
...
$t_n =$ _____

FORMULA FOR THE *GENERAL TERM* OF A *GEOMETRIC* SEQUENCE

$t_n =$ _____ $t_n =$ general term $t_1 =$ first term $r =$ common ratio $n =$ term number

Example 1: Graph a Geometric Sequence

Round 1 of a tennis tournament starts with 128 players. After each round, half the players are eliminated from the tournament.

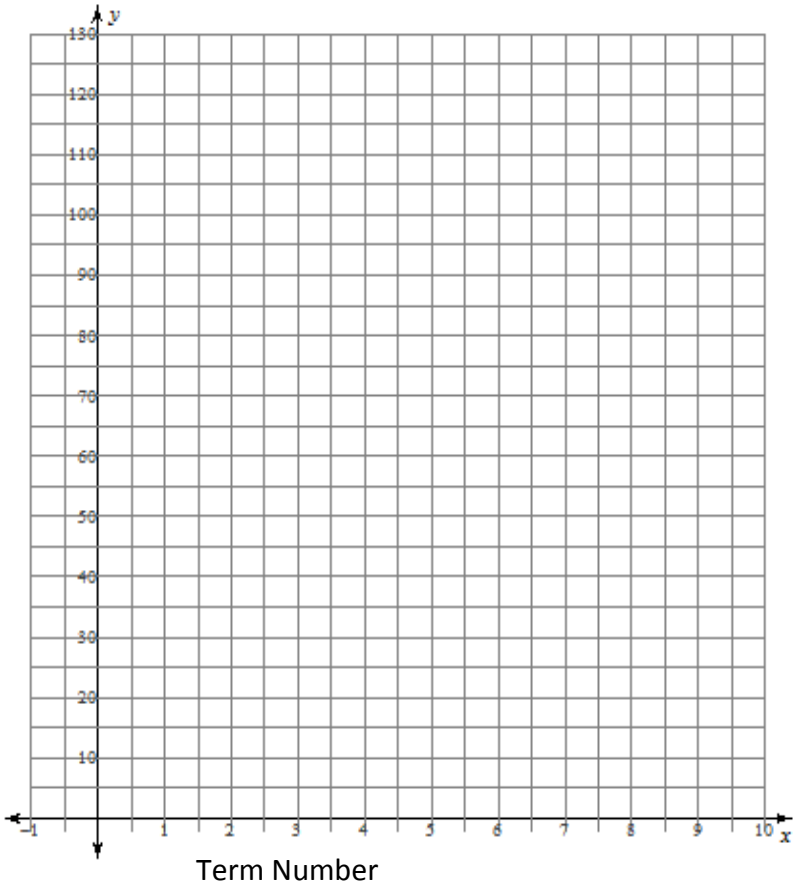
a. Generate the sequence relating the number of players, t_n, to the round number, n.
b. Sketch a graph of t_n versus n.
c. What type of relationship is exhibited in the graph?
d. Determine the explicit formula for this geometric sequence.

Solution:

a. The sequence relating the number of players, t_n, to the round number, n.

Round Number, n	1	2	3	4	5
Number of Players, t_n	128				

b.



c. The type of relationship exhibited in the graph above is _____.

d. Explicit formula for this geometric sequence: _____

Example 2: Determine r and t_n

There are 10 bacterial cells initially present in a bacteria sample. The bacteria reproduce by splitting into two so that one cell becomes 2, then 4, then 8 cells, and so on.

a. State the first 6 terms of the sequence generated by the doubling of the bacteria and determine the common ratio, r.

b. Determine the general term in this situation.

Solution:

a. t_1 = _____, t_2 =_____, t_3 =_____, t_4 =_____, t_5 =_____, t_6 =_____, …

Determine the common ratio by dividing any two consecutive terms, $r = \frac{t_n}{t_{n-1}}$.

r = _____

b. $t_n = t_1 r^{n-1}$

The general term of the sequence is t_n =_____

Example 3: Determine a Particular Term

A company stores 5 kg of a radioactive material. After one year, 92% of the radioactive material remains. How much radioactive material will be left after ten years? State your answer to the nearest tenth of a kilogram.

t_1 =_____ r = _____ n = _____

The general term is: _____

After ten years, approximately _____ kg of the radioactive material remains.

Example 4: Determine r and t_1

In a geometric sequence, the fifth term is 5250 and the seventh term is 131 250.

a. Find the possible values of r and t_1.
b. List the first five terms of the possible sequences.

Solution:

a. Find the possible values of r and t_1.

b. List the first five terms of the possible sequences.

When r = 5, the first five terms of the sequence are _____, _____, _____, _____, _____, …

When r = –5, the first five terms of the sequence are _____, _____, _____, _____, _____, …

Example 5: Apply Geometric Sequences

Listeria monocytogenes are bacteria that are sometimes found in food. It takes about 7 hours for the number of these organisms to double when the temperature is $10°C$. Suppose the initial bacteria count in a sample of food is 100.

a. Write the first five terms of the geometric sequence that models this situation.
b. How long will it take for the bacteria count to reach 1 638 400?

Solution:

a. First five terms: 100, 200, _____, _____, _____

b. Determine the time it will take for the count to reach 1 638 400.

It will take _____ hours for the bacteria count to reach 1 638 400.