

Sums and Differences of Functions

Two functions, $f(x)$ and $g(x)$, can be added to form the combined function $h(x) = (f + g)(x)$. They can also be subtracted to form the combined function $h(x) = (f - g)(x)$.

Sum of Functions

$$h(x) = f(x) + g(x)$$

$$h(x) = (f + g)(x)$$

Difference of Functions

$$h(x) = f(x) - g(x)$$

$$h(x) = (f - g)(x)$$

Example 1: Determine the Sum of Two Functions

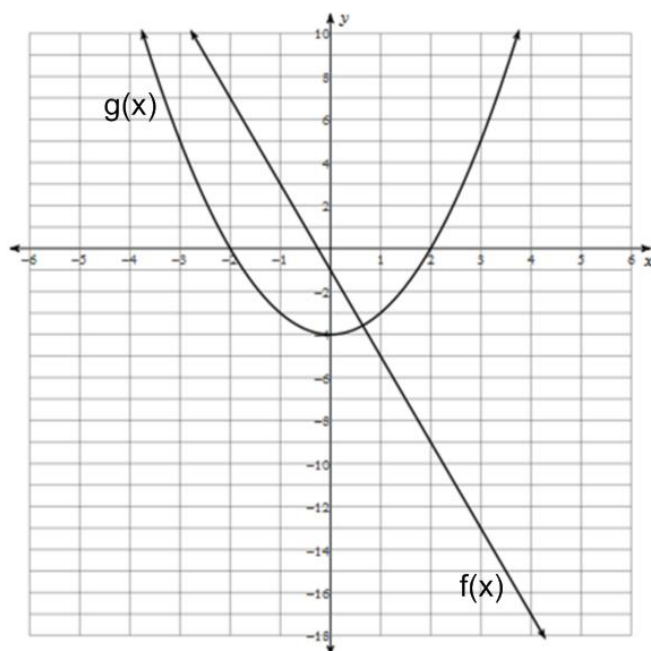
Consider the functions $f(x) = -4x - 1$ and $g(x) = x^2 - 4$.

- Determine the equation of the function $h(x) = (f + g)(x)$.
- Sketch the graph of $h(x)$ on the same set of coordinate axes as $f(x)$ and $g(x)$.
- State the domain and range of $h(x)$.

Solution:

- Add $f(x)$ and $g(x)$ to determine the equation of the function $h(x) = (f+g)(x)$.
- Complete the table of values below and sketch the graph of $h(x)$ on the same grid as $f(x)$ and $g(x)$.

x	$f(x) = -4x - 1$	$g(x) = x^2 - 4$	$h(x) = (f+g)(x)$
-2			
-1			
0			
1			
2			
3			
4			



- Determine the domain of the functions $f(x)$ & $g(x)$.

$f(x)$: _____ $g(x)$: _____

The domain of $h(x)$ consists of all values that are *common* in both the domain of $f(x)$ and $g(x)$.

Domain of $h(x)$: _____

The range of $h(x)$ is: _____

Example 2: Determine the Difference of Two Functions

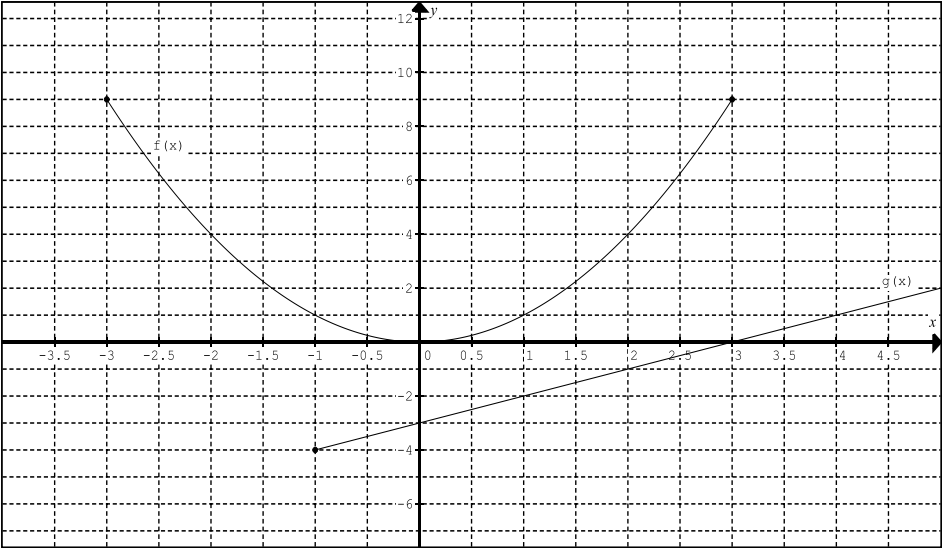
Consider the functions: $f(x) = x^2$ with a domain of $\{x \mid -3 \leq x \leq 3, x \in R\}$ and $g(x) = x - 3$ with a domain of $\{x \mid x \geq -1, x \in R\}$

- a. Determine the equation of the function $h(x) = (f - g)(x)$.
- b. Sketch the graph of $h(x)$ on the same set of coordinate axes as $f(x)$ and $g(x)$.
- c. State the domain and range of $h(x)$.
- d. Determine the value of $f(x)$, $g(x)$ and $h(x)$ when $x = 6$.

Solution:

- a. Subtract $g(x)$ from $f(x)$ to determine the equation of the function $h(x)$.
- b. Complete the table of values below and sketch the graph of $h(x)$ on the same grid as $f(x)$ and $g(x)$.

x	f(x)	g(x)	h(x)
-3			
-2			
-1			
0			
0.5			
1			
2			
3			
4			



- c. The domain of $h(x)$ consists of all values that are common in both the domain of $f(x)$ and $g(x)$.

Domain of $h(x)$: _____

Range of $h(x)$: _____

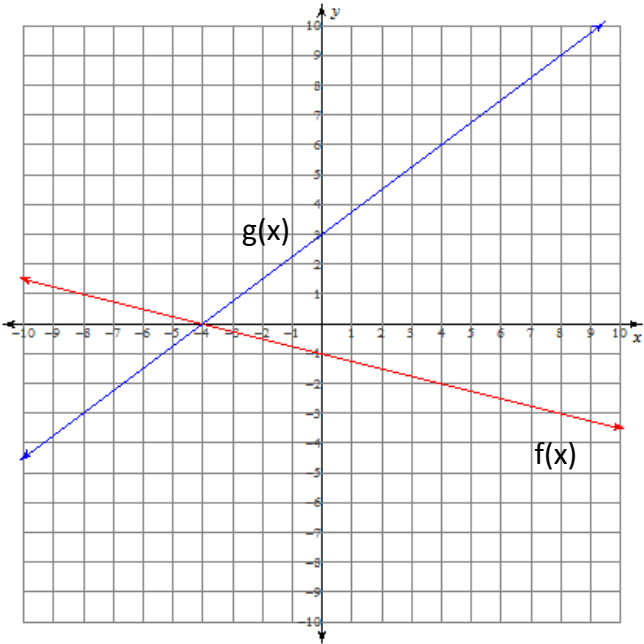
d. $f(6) =$ _____

$g(6) =$ _____

$h(6) =$ _____

Example 3: Determine a Combined Function from Graphs

Use the graphs of $f(x)$ and $g(x)$ to sketch the graph of $h(x) = (f + g)(x)$.



Solution:

Method 1: Add the y-coordinates of points on the graphs of $f(x)$ and $g(x)$ that have the same x-coordinates.

Complete the second and third columns of the table below using the graphs of $f(x)$ and $g(x)$.

x	f(x)	g(x)	h(x) = (f + g)(x)
-8			
-4			
0			
4			

Add the y-values of $f(x)$ and $g(x)$ at each x-value to determine the points on the graph of $h(x)$ and draw the graph of $h(x)$ on the same grid as $f(x)$ and $g(x)$.

Method 2: Determine the equations of $f(x)$ & $g(x)$, then determine the equation of $h(x)$ and graph the function.

Equation of $f(x)$ = _____ Equation of $g(x)$ = _____

Now determine the equation of $h(x)$ algebraically:

$h(x) = f(x) + g(x)$

$h(x) =$ _____

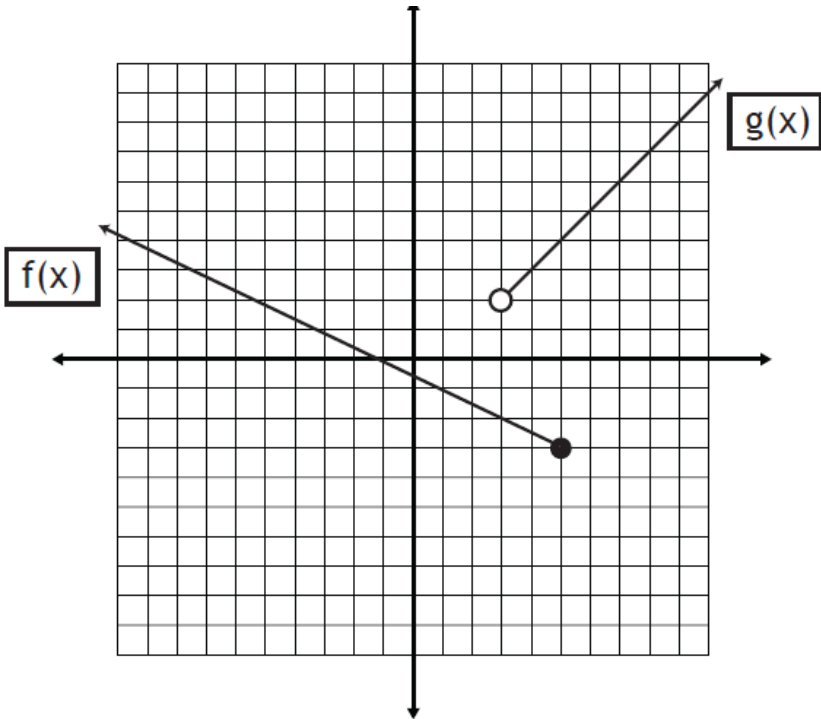
Draw the graph of $h(x)$. * Confirm that this is the graph you drew using method 1

Example 4: Sketch a Combined Function

Sketch the combined function $h(x) = (f + g)(x)$ and state its domain and range.

Domain of $h(x)$: _____

Range of $h(x)$: _____



Example 5: Application of the Difference of Two Functions

The yearbook committee prepares the annual book to sell to students. The fixed cost is \$800 and the printing cost for each yearbook is \$15. The committee plans to sell the yearbooks for \$20 each.

- a. Write an equation to represent:

The total cost, C , as a function of the number of yearbooks printed, n . _____

The revenue, R , as a function of the number of yearbooks sold, n . _____

- b. Graph $C(n)$ and $R(n)$ on the same set of axes.

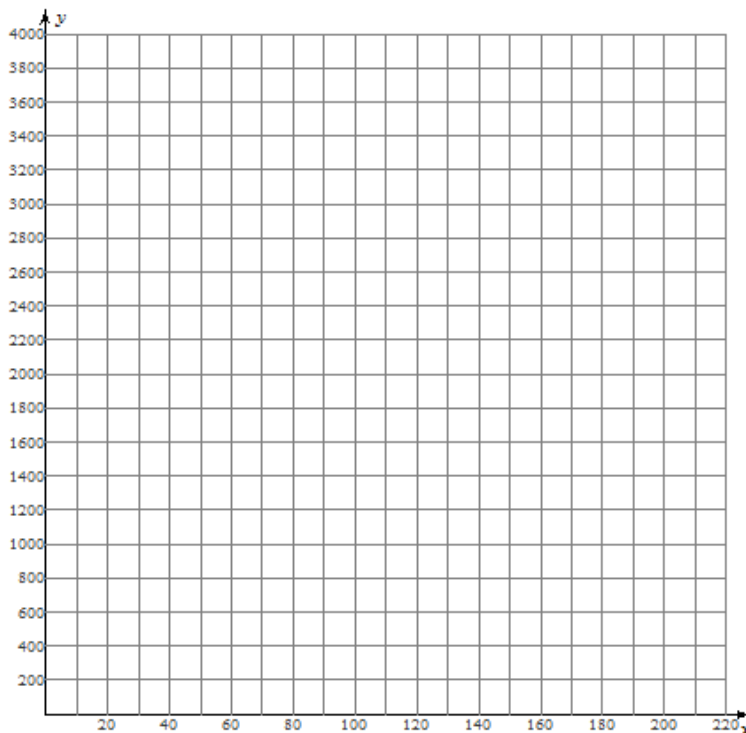
- c. What does the point of intersection represent?

- d. What is the minimum number of yearbooks that need to be sold in order for the committee to make a profit? _____

- e. Write a function, $P(n)$, representing profit.

$$P(n) = R(n) - C(n)$$

$$P(n) =$$



- f. What are the domain and range for $C(n)$, $R(n)$, and $P(n)$?

Domain for $C(n)$, $R(n)$ and $P(n)$:

Range $C(n)$: _____

Range $R(n)$: _____

Range $P(n)$: _____

Extra Practice:

Sketch the combined function $h(x) = (f + g + m)(x)$ and state its domain and range.

Domain of $h(x)$: _____

Range of $h(x)$: _____

