Products and Quotients of Functions

Product of Functions

$h(x) = f(x) \bullet g(x)$

$h(x) = (f \bullet g)(x)$

Quotient of Functions

$h(x) = \dfrac{f(x)}{g(x)}$

$h(x) = \left(\dfrac{f}{g}\right)(x)$

The domain of a product or quotient of functions is the domain common to both f(x) and g(x). However, the domain of a quotient of functions must have the restriction that the divisor cannot equal zero. That is, for $h(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$.

Example 1: Determine the Product of Functions

Let $f(x) = x - 3$ and $g(x) = x^2 - 8x + 15$.

a. Determine $h(x) = (f \bullet g)(x)$
b. Complete a table of values for h(x) using f(x) and g(x) values.
c. Sketch the graph of h(x) on the same axes as f(x) and g(x).
d. State the domain and range of h(x).

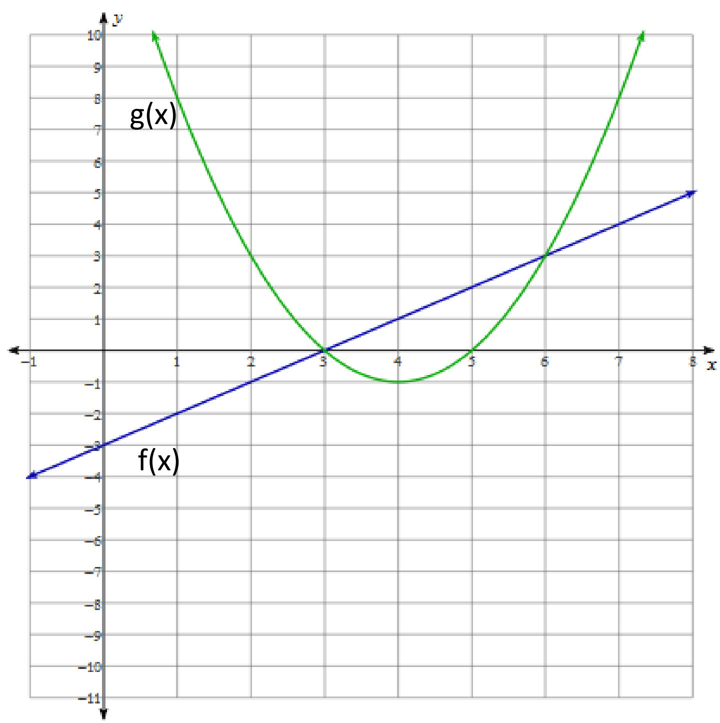
Solution:

a. $h(x) = f(x) \bullet g(x)$

b.

x	f(x)	g(x)	h(x)
1.5	–1.5	5.25	
2	–1	3	
2.5	–0.5	1.25	
3	0	0	
3.5	0.5	–0.75	
4	1	–1	
4.5	1.5	–0.75	
5	2	0	
5.5	2.5	1.25	
6	3	3	

c.



d. The domain of $f(x)$, $g(x)$, and $h(x)$ is _______________.

The range of h(x) is _______________.

Example 2: Determine the Quotient of Functions

Let $f(x) = x - 3$ and $g(x) = x^2 - 8x + 15$.

a. Determine $h(x) = \left(\frac{f}{g}\right)(x)$. Remember to state any non-permissible values.

b. Complete a table of values for h(x) using f(x) and g(x) values.

c. Sketch the graph of h(x) on the same axes as f(x) and g(x).

d. State the domain and range of h(x).

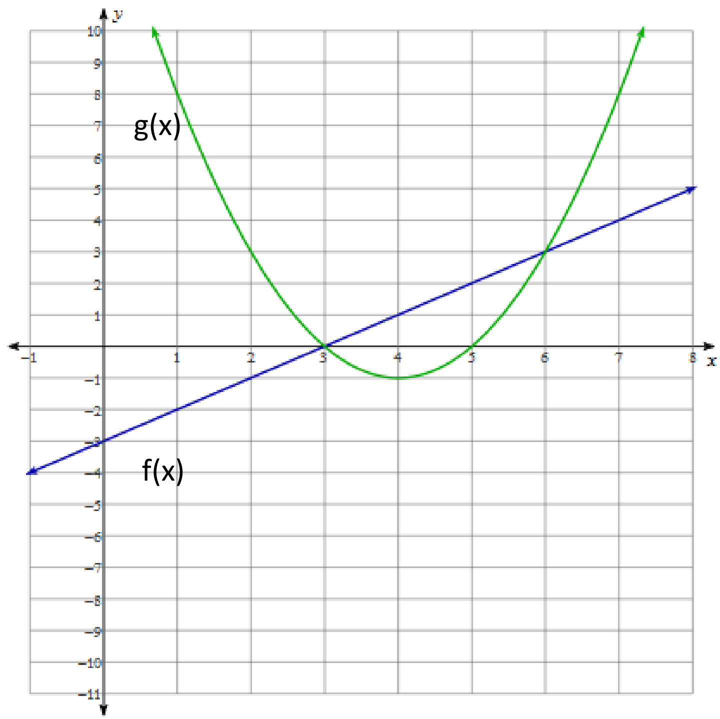
Solution:

a. $h(x) = \frac{f(x)}{g(x)}$

b.

x	f(x)	g(x)	h(x)
1	–2	8	
2	–1	3	
3	0	0	
4	1	–1	
5	2	0	
6	3	3	
7	4	8	

c.



d. Domain of f(x): _______________

Domain of g(x): _______________

The domain of h(x) consists of all values that are in both the domain of f(x) and g(x), *excluding* the values of x for which g(x) = 0. These values of x are _____ and _____.

Domain of h(x): ____________________

Range of h(x): ____________________

Example 3: Application of Products and Quotients of Functions

Last Saturday, an electronics store held a one-day promotion. At 8:00 a.m., the selling price of a 3-D television was $1020. Every hour, starting at 9:00 a.m., the selling price was reduced by 5% of the original price. This promotion continued on a first-come-first-served basis until all stock had been sold. The number of televisions, N, sold t hours after 8:00 a.m. was modeled by the function $N(t) = 6t$.

a. Write the function for the selling price per television, P(t), at t hours.

b. Write the function for the store's revenue, R(t), at t hours.

c. If the store's cost for each television was $705, write the function for cost, C(t), at t hours.

d. What was the store's revenue at 12:00 p.m.? What was the store's cost?

e. Calculate what the store's percent gain or loss in profit was at 12:00 p.m.

f. Write an expression for the percent gain or loss in profit at any time, t hours.

g. Use the expression in (f) to calculate what the percent gain or loss was at 3:00 p.m.