

Combinations

A **combination** is a selection of objects without regard to order. Examples of combinations include choosing five fruits from a possible eight to make a fruit salad, choosing four people from a group of ten to serve on a committee, picking three colors from a color brochure.

Example 1: Formula for Combinations

The FHS Physical Education department has 3 free passes to a school hockey game to give away. Amanda, Brendan, Chloe, Daniel and Emily are all interested in having one of the free passes. How many ways can the Phys Ed department give away the passes?

Solution:

First determine how many ways you can select three people in a particular order from a group of five.

Number of choices for 1 st pass	Number of choices for 2 nd pass	Number of choices for 3 rd pass
5	4	3

There are $5 \times 4 \times 3 = 60$ or ${}_5P_3 = 60$ ways to select three people from a group of five, if the order matters. But consider the following possible selections of three people from the group of five:

ABC	ACB	BAC	BCA	CAB	CBA
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These groups represent 6 of the 60 possible *permutations* of three students from five, but they represent only 1 possible *combination* of three students from five. Similarly, every other possible combination of three from five would be represented in 6 different ways as permutations. So, the 60 permutations would have to be reduced by a factor of 6, or $3!$, to obtain the number of possible combinations:

$$\frac{{}_5P_3}{3!} = \frac{60}{6} = 10 \quad \text{So, there are 10 ways to give the 3 passes to 3 out of 5 students}$$

The notation ${}_nC_r$ is used to represent the number of combinations, without regard to order, of r items taken from a set of n distinct items.

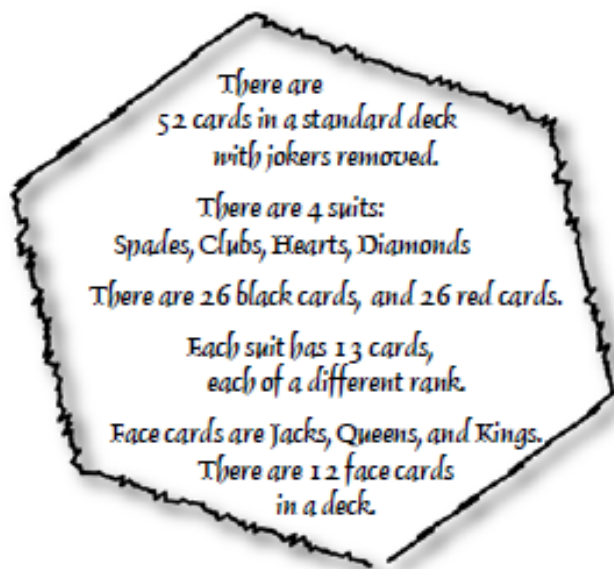
Formula for Combinations: ${}_nC_r = \frac{n!}{(n-r)!r!}$
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Example 1: Basic Combinations

If a pizza can have 3 out of a possible 7 toppings, how many different pizzas can be made?

Solution:

Example 2: Combinations Including Specific Items



From a deck of 52 cards, a five-card hand is dealt. How many distinct five-card hands are there if the queen of spades and the four of diamonds are included in the hand?

Solution:

Example 3: Combinations from Multiple Selection Pools

- a. A committee of 3 boys and 5 girls is to be formed from a group of 10 boys and 11 girls. How many committees are possible?

- b. From a deck of 52 cards, a 7-card hand is dealt. How many distinct hands are there if the hand must contain exactly 2 spades and 3 diamonds?

Example 4: At Least / At Most

A research team of 5 members is to be formed from a selection pool of 8 chemists and 9 biologists. How many research teams are possible consisting of

- a. at least 3 biologists? b. at least 1 chemist? c. at most 1 chemist?

Solution:

- a. at least 3 biologists?

- b. at least 1 chemist?

- c. at most 1 chemist?

Example 5: Permutations and Combinations Together

There are 4 men and 5 women in a committee selection pool. A three-person committee consisting of President, Vice-President, and Treasurer is being formed. How many ways can exactly two men be on the committee?

Example 6: Simplifying Expressions and Solving Equations with Combinations

a. Express as factorials and simplify $\frac{{}_nC_7}{{}_{n-1}C_5}$.

b. Solve for n if $3({}_nC_3) = {}_{n+1}C_4$