# The Remainder Theorem

## Long Division

You can use long division to divide a polynomial by a binomial: $\dfrac{P(x)}{x-a} = Q(x) + \dfrac{R}{x-a}$

The components of long division are:

- the *dividend*, P(x), which is the polynomial that is being divided
- the *divisor*, x–a, which is the binomial that the polynomial is being divided by
- the *quotient*, Q(x), which is the expression that results from the division
- the *remainder*, R, which is the value or expression that is left over after dividing

To check the division of a polynomial, verify the statement $P(x) = (x-a)Q(x) + R$. In other words, multiply the quotient, Q(x), by the divisor, x–a, and add the remainder, R, to the product. The result is the dividend, P(x).

## Example 1: Divide a Polynomial by a Binomial of the Form x–a

a. Divide $P(x) = 9x + 4x^3 - 12$ by $x + 2$. Express the result in the form $\dfrac{P(x)}{x-a} = Q(x) + \dfrac{R}{x-a}$

b. Identify any restrictions on the variable.

c. Write the corresponding statement that can be used to check the division.

### Solution:

a. $x+2\overline{)4x^3 + 0x^2 + 9x - 12}$

b. Restrictions on the variable:

c. $P(x) = (x-a)Q(x) + R$

## Example 2: Apply Polynomial Division to Solve a Problem

The volume, V, in cubic centimeters, of gift boxes is given by $V(x)=2x^3+x^2-27x-36$. The height, h, in centimeters is $x+3$. What are the possible dimensions of the boxes in terms of x?

**Solution:**

Divide the _______________ of the box by the _______________ to obtain an expression for the _______________ of the base of the box. Then, factor this expression to obtain expressions for the _______________ and _______________ of the base.

Expressions for the dimensions, in centimeters, are _____________, _____________, _____________.

## Synthetic Division

- a short form of division that uses only the coefficients of the terms and fewer calculations.

## Example 3: Divide a Polynomial Using Long Division and Synthetic Division

a. Use *long division* to divide $5x^2-x+2x^3-6$ by $x+2$.
b. State any restrictions on x.
c. Use *synthetic division* to divide $5x^2-x+2x^3-6$ by $x+2$.

**Solution:**

# Remainder Theorem

The **remainder theorem** states that when a polynomial, P(x), is divided by a binomial of the form x–a, the remainder is P(a).

- If the remainder is 0, then the binomial x–a is a factor of P(x)
- If the remainder is *not* 0, then the binomial x–a is *not* a factor of P(x)

## Example 4: Apply the Remainder Theorem

a. Use the remainder theorem to verify that $x+2$ a factor of $5x^2-x+2x^3-6$. (See Example 3)

b. Use the remainder theorem to determine the remainder when $P(x)=3x^4-x^3-5$ is divided by $x-3$.

c. Verify your answer using synthetic division.

d. Is $x-3$ a factor of $3x^4-x^3-5$?