Equations & Graphs of Polynomial Functions

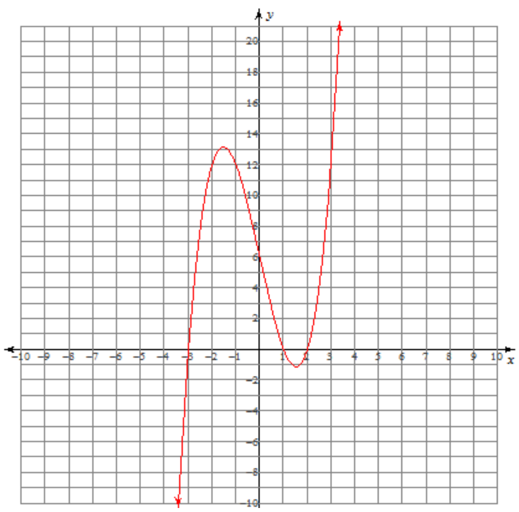
The zeros of any polynomial function y = f(x) correspond to the x-intercepts of the graph and to the roots of the corresponding equation, f(x)=0. For example, the zeros of the function $f(x)=(x-2)(x+3)(x-5)$ are x = 2, –3 and 5. Thus, the x-intercepts of the graph of the function will be at 2, –3 and 5. These will also be the roots of the corresponding equation $(x-2)(x+3)(x-5)=0$.

Example 1: Analyze Graphs of Polynomial Functions

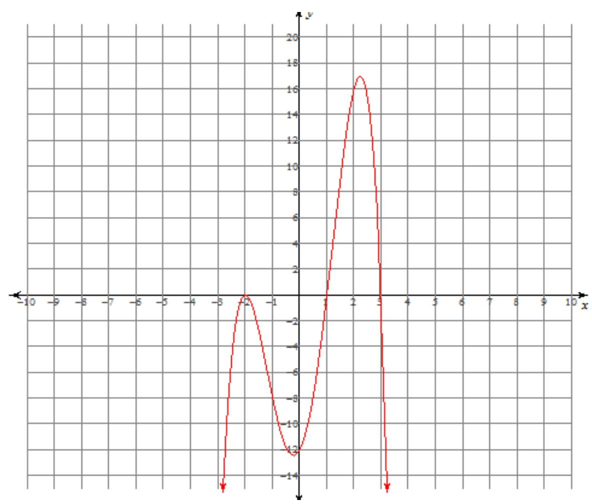
For each graph of a polynomial function shown, determine:

a. the least possible degree of the function
b. the sign of the leading coefficient
c. the x-intercepts and the factors of the function with least possible degree
d. the intervals where the function is positive and the intervals where it is negative

Graph A



Graph B



Solution:

Graph A	**Graph B**
a. least possible degree: _____	a. least possible degree: _____
b. sign of the leading coefficient:______________	b. sign of the leading coefficient:______________
c. x-intercepts: ______________________	c. x-intercepts: ______________________
factors of the function: ________________	factors of the function: ________________
d. intervals where function is positive:	d. intervals where function is positive:
intervals where function is negative:	intervals where function is negative:

Multiplicity of a Zero

- The *multiplicity* of a zero refers to the number of times a zero of a polynomial function occurs. For example, the function $f(x)=(x+3)^2(x-1)$ has a zero of multiplicity 2 at x = –3 and a zero of multiplicity 1 at x = 1.

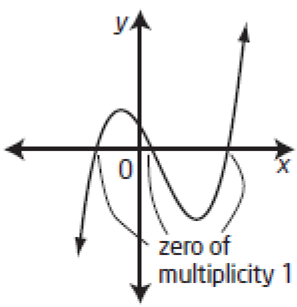
- The multiplicity of a zero or root can also be referred to as the *order* of the zero or root.

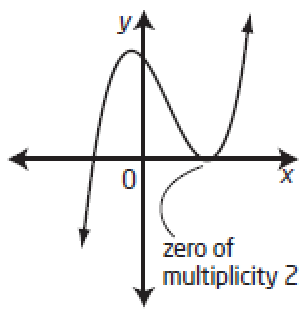
- The shape of the graph of a polynomial function close to a zero of x = a (multiplicity n) is similar to the shape of the graph of a function with degree equal to n of the form $y=(x-a)^n$. For example, the graph of a function with a zero at x=1 (multiplicity 3) will look like the graph of the graph of the cubic function $y=(x-1)^3$ in the region close to x=1.

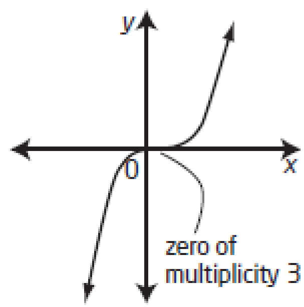
- Polynomial functions change sign at x-intercepts that correspond to odd multiplicity. The graph crosses over the x-axis at these intercepts.

- Polynomial functions do not change sign at x-intercepts of even multiplicity. The graph touches, but does not cross, the x-axis at these intercepts.

To illustrate this concept, consider the following graphs of degree 3 polynomial functions:







Sketching Graphs of Polynomial Functions

To sketch the graph of a polynomial function, determine characteristics such as:

- the degree of the function
- the sign of the leading coefficient
- end behaviour
- the y-intercept
- the x-intercepts and their multiplicity
- intervals where the graph is positive / negative
- other points on the graph

Example 2: Analyze Equations to Sketch Graphs of Polynomial Functions

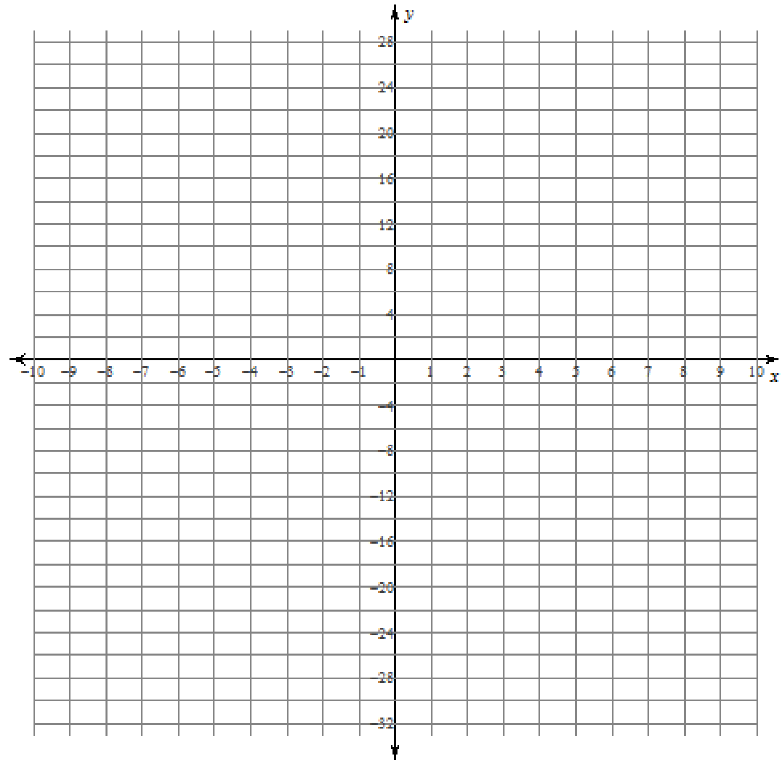
Sketch the graph of each polynomial function.

a. $f(x) = (x+4)(x+2)(x-3)$ b. $f(x) = -(x-1)^3(x+3)$ c. $f(x) = -2x^3 + 6x - 4$

Solution:

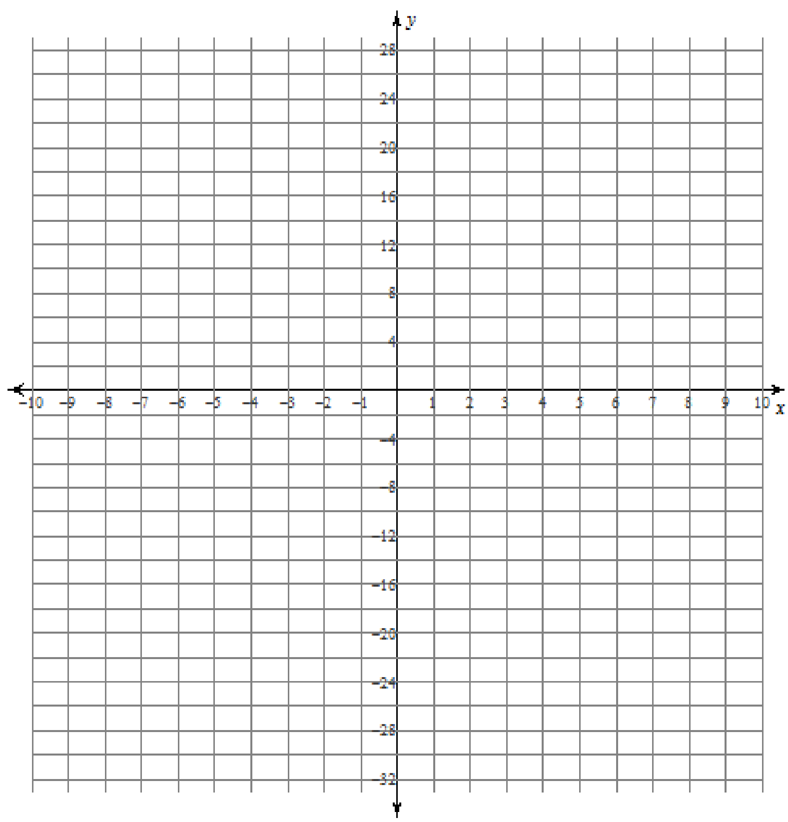
a. $f(x) = (x+4)(x+2)(x-3)$

Degree	
Leading Coefficient	
End Behaviour	
x-intercepts & multiplicity	
y-intercept	
Interval(s) where the function is positive	
Interval(s) where the function is negative	
Other points	



b. $f(x) = -(x-1)^3(x+3)$

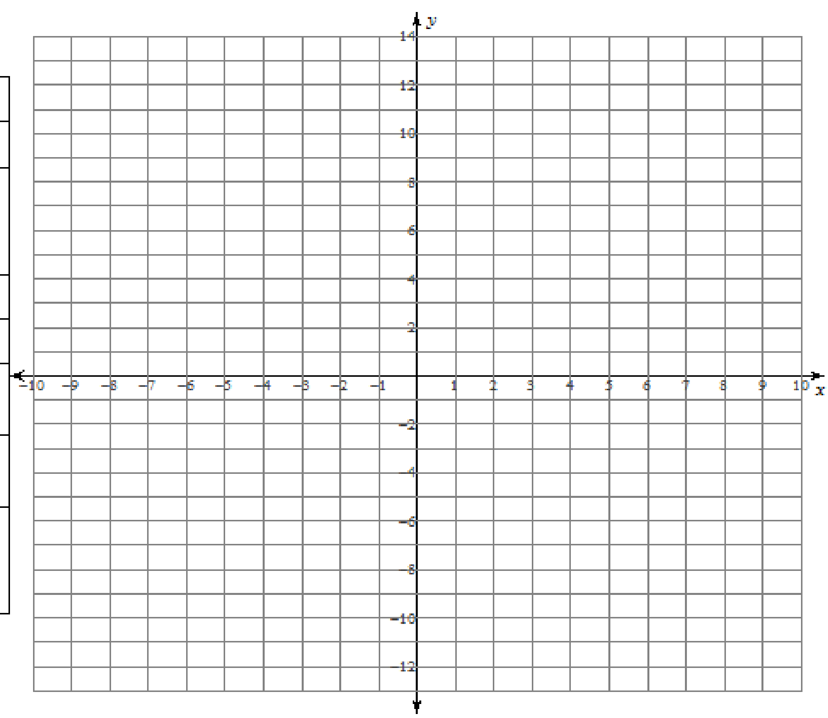
Degree	
Leading Coefficient	
End Behaviour	
x-intercepts & multiplicity	
y-intercept	
Interval(s) where the function is positive	
Interval(s) where the function is negative	
Other points	



c. $f(x) = -2x^3 + 6x - 4$

Determine the factored form of the function:

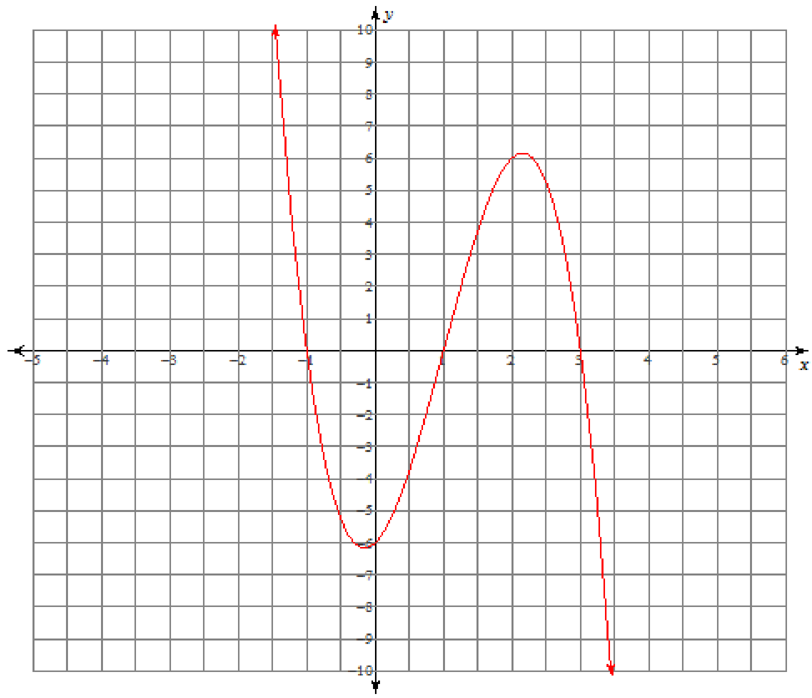
Degree	
Leading Coefficient	
End Behaviour	
x-intercepts & multiplicity	
y-intercept	
Interval(s) where the function is positive	
Interval(s) where the function is negative	
Other points	



Example 3: Determining the Equation of a Polynomial Function from its Graph

Determine the equation of the polynomial function that corresponds to the graph shown.

Solution:



The graph of the function has ______ x –intercepts.
All of the x-intercepts are of _________ multiplicity.
The least possible multiplicity of each x-intercept is ______, so the least possible degree is ______.

The graph extends up into quadrant ______ and down into quadrant ______, so the leading coefficient is ____________________.

The y-intercept is ______; this is the _______________ term in the expanded form of the equation of the function.

The x-intercepts are ______, ______, and ______.

Use the above information to determine the factored form of the equation of the polynomial function:

f(x) = ______(_________)(_________)(_________)

Graphing Polynomial Functions Using Transformations

The graph of a function of the form $y = a(b(x-h))^n + k$ is obtained by applying transformations to the graph of the general polynomial function $y = x^n$, where $n \in N$. The effects of changing parameters in polynomial functions are the same as the effects of changing parameters in other types of functions. Remember to apply the reflections and stretches before the translations.

Example 4: Apply Transformations to Sketch a Graph

The graph of $y = x^3$ is transformed to obtain the graph of $y = \frac{1}{5}(2(x+4))^3 - 5$.

a. Describe the transformations.

b. Sketch the graph of $y = \frac{1}{5}(2(x+4))^3 - 5$.

Solution:

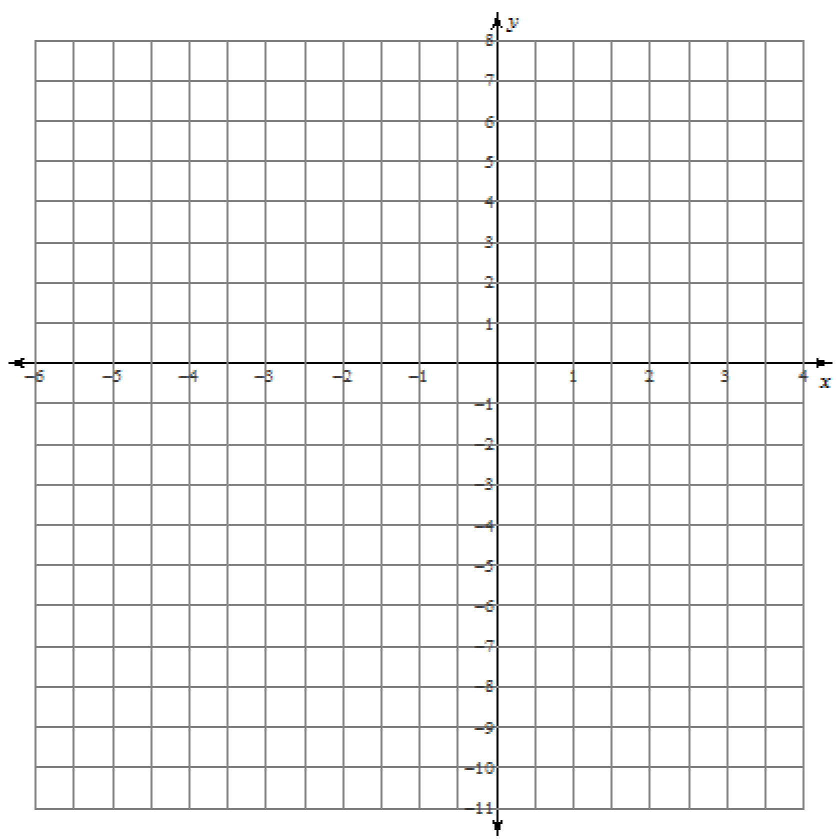
a. Transformations:

b. Sketch of $y = \frac{1}{5}(2(x+4))^3 - 5$:

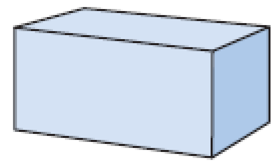
Mapping rule: ________________________________

$y = x^3$	
x	y
–3	
–2	
–1	
0	
1	
2	
3	

$y = \frac{1}{5}(2(x+4))^3 - 5$	
x	y



Example 5: Model and Solve Problems Involving Polynomial Functions



Alicia is preparing to make an ice sculpture. She has a block of ice that is 4 ft wide, 5 ft long, and 6 ft high. She wants to reduce the size of the block of ice by removing the same amount from each of the three dimensions such that the reduced volume of the ice block is $24 ft^3$.

a. Write an equation that models this situation.
b. How much should Alicia remove from each dimension?

Solution:

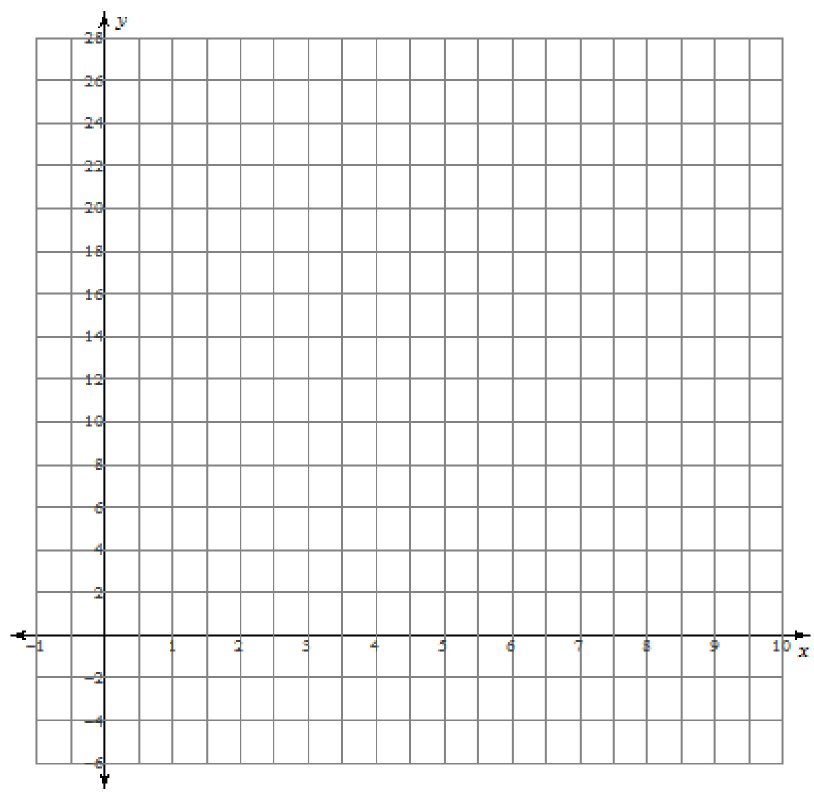
a. Let x represent ____.

Then, V(x) = ________________________ = __________

b. How much should Alicia remove from each dimension?

Method 1: Graphical Solution

Enter the equations y_1 = ________________________ and y_2 = ________ into the graphing calculator and determine the x-coordinate of the point of intersection of the two graphs.



______ feet must be removed from each dimension of the block of ice.

Method 2: Algebraic Solution

_______ feet must be removed from each dimension of the block of ice.