Graphing Linear Inequalities in Two Variables

Linear Inequalities

A linear inequality is a relationship between two linear expressions in which one expression is *less than* (<), *greater than* (>), *less than or equal to* (≤), or *greater than or equal to* (≥) the other expression. The solution of an inequality in two variables is a set of ordered pairs.

Example 1:

Tell whether the ordered pair (3, 1) is a solution of the following linear inequalities by substituting the ordered pair into each inequality.

a. $13-3x>4y$

The ordered pair (3, 1) _________ a solution.

b. $2y-5\leq x$

The ordered pair (3, 1) _________ a solution.

c. $y+x<10$

The ordered pair (3, 1) _________ a solution.

d. $y\geq 9$

The ordered pair (3, 1) _________ a solution.

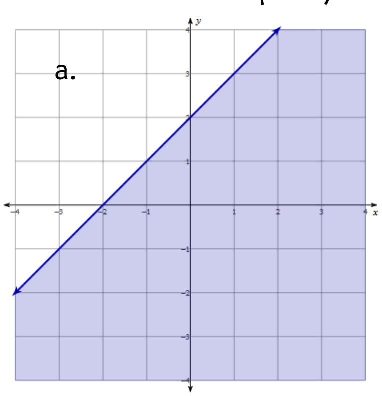
Graphing a Linear Inequality

In the graph of a linear inequality with two variables, the *boundary line* is the graph of the *related equation*. The graph of the inequality is the region on *one* side of the line (called a *half-plane*) and either *includes* points on the boundary line (if the symbol is ≥ or ≤), in which case a *solid line* is drawn, or *does not include* points on the boundary line (if the symbol is > or <), in which case a *dashed line* is drawn.

Example 2:

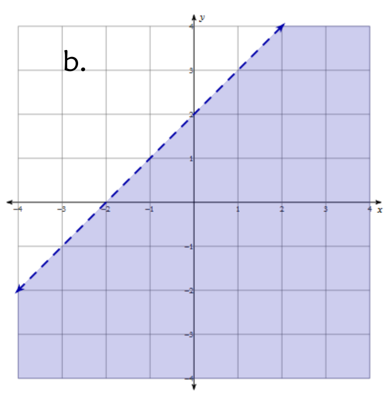
Match each inequality with its graph: $y>x+2$, $y<x+2$, $y\geq x+2$, $y\leq x+2$

a.



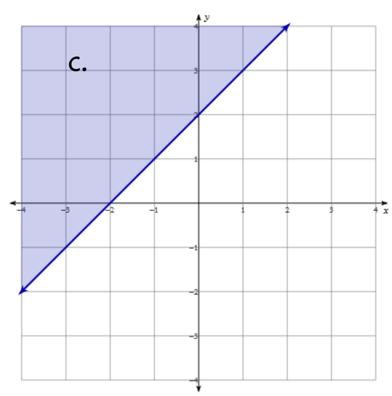
The half-plane *below* the boundary line is shaded and the line is *solid*; _______________.

b.



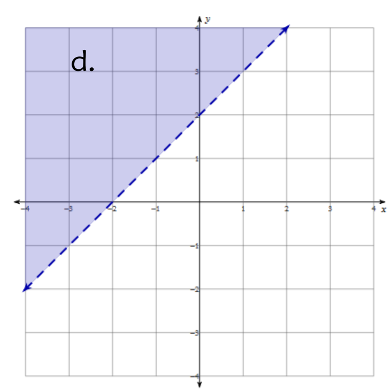
The half-plane *below* the boundary line is shaded and the line is *dashed*; _______________.

c.



The half-plane *above* the boundary line is shaded and the line is *solid*; _______________.

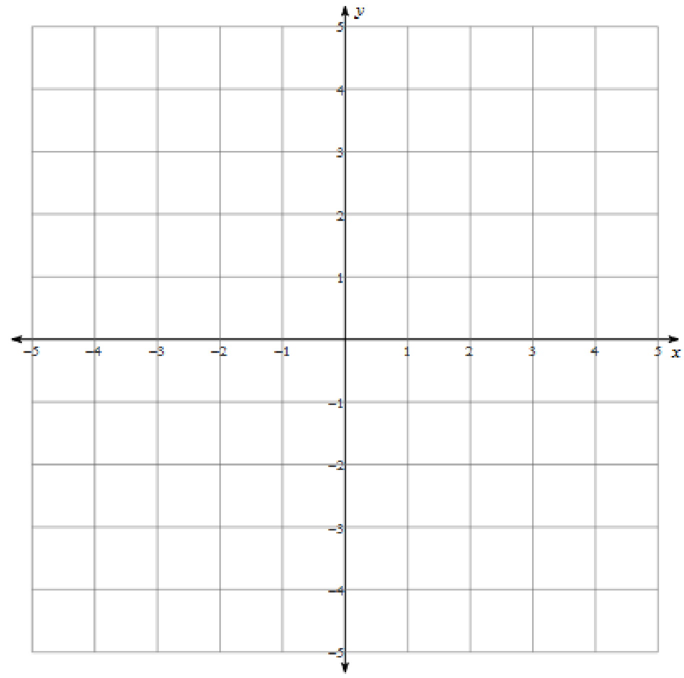
d.



The half-plane *above* the boundary line is shaded and the line is *dashed*; _______________.

Example 3:
Graph each linear inequality.

a. $y > 4x - 3$



STEP 1: Graph the boundary line _____________.
The inequality symbol is $>$,
so this line will be ________________.

STEP 2: Shade the correct half-plane.
The inequality symbol is $>$,
so shade _____________ the line.

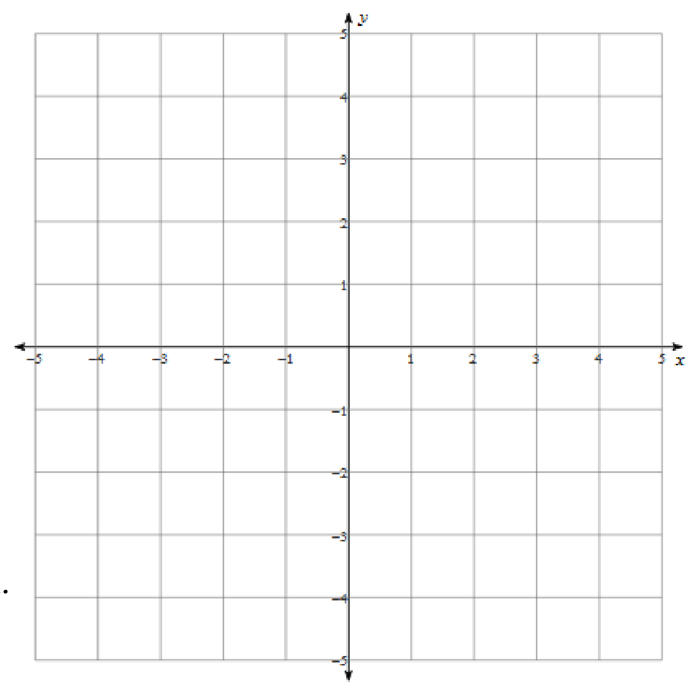
STEP 3: Substitute a point from the shaded region into the inequality to see if the correct region is shaded.

Substitute ________:

$y > 4x - 3$

The ordered pair ________ is a solution. The correct region is shaded.

b. $-2y - 4 \geq x$



STEP 1: Rearrange the inequality for y.

$-2y - 4 \geq x$

STEP 2: Graph the boundary line _______________.
The inequality symbol is $\leq$,
so this line will be _______________.

STEP 3: Shade the correct half-plane.
The inequality symbol is $\leq$,
so shade ____________ the line.

STEP 4: Substitute a point from the shaded region into the *original* inequality to see if the correct region is shaded.

Substitute ________:

$-2y - 4 \geq x$

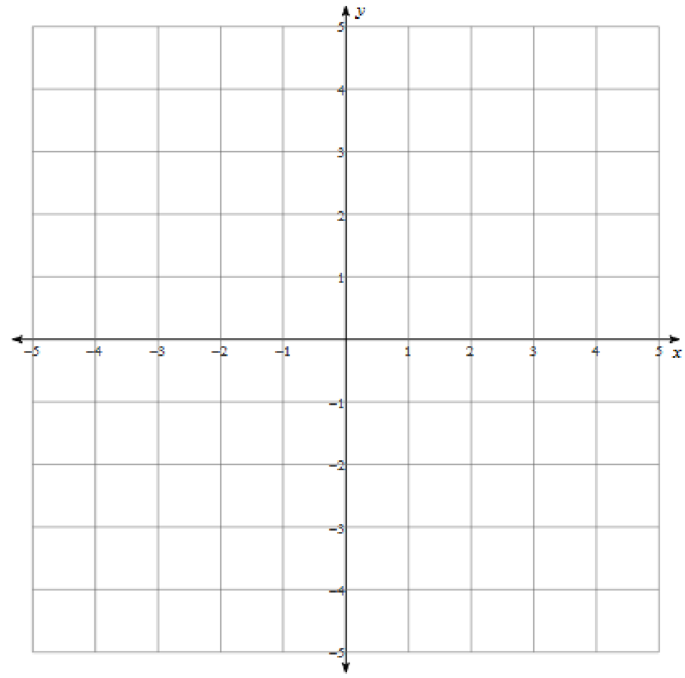
The ordered pair ________ is a solution. The correct region is shaded.

Vertical and Horizontal Boundary Lines

Example 4:

Graph each linear inequality.

a. $y < 2$



STEP 1: Graph the ________________ boundary line ________.
The inequality symbol is <,
so this line will be _______________.

STEP 2: Shade the correct half-plane.
The inequality symbol is <,
so shade ____________ the line.

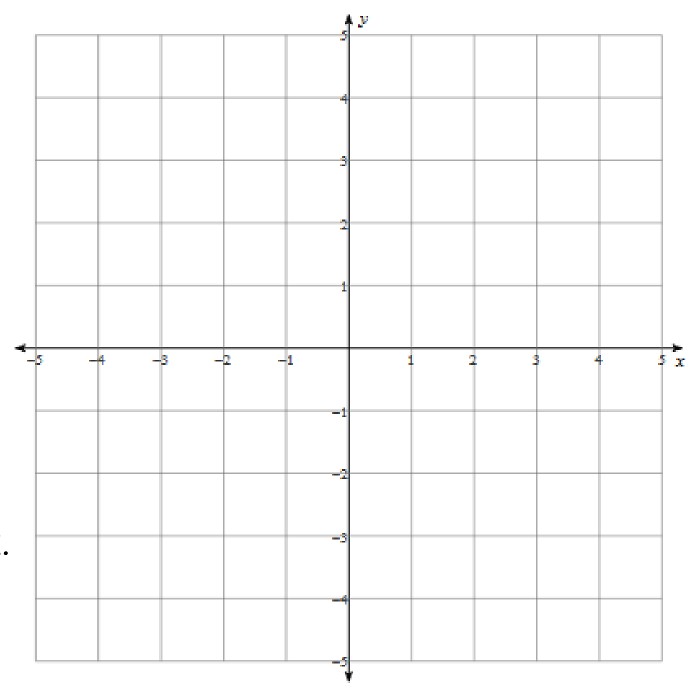
STEP 3: Substitute a point from the shaded region into the inequality to see if the correct region is shaded.

Substitute ________:

$y < 2$

The ordered pair ________ is a solution. The correct region is shaded.

b. $x + 1 \geq 0$



STEP 1: Rearrange the inequality for x.

$x + 1 \geq 0$

STEP 2: Graph the _______________ boundary line ________.
The inequality symbol is $\geq$,
so this line will be _______________.

STEP 3: Shade the correct half-plane.
The inequality symbol is $\geq$,
so shade ___________________ the line.

STEP 4: Substitute a point from the shaded region into the *original* inequality to see if the correct region is shaded.

Substitute ________:

$x + 1 \geq 0$

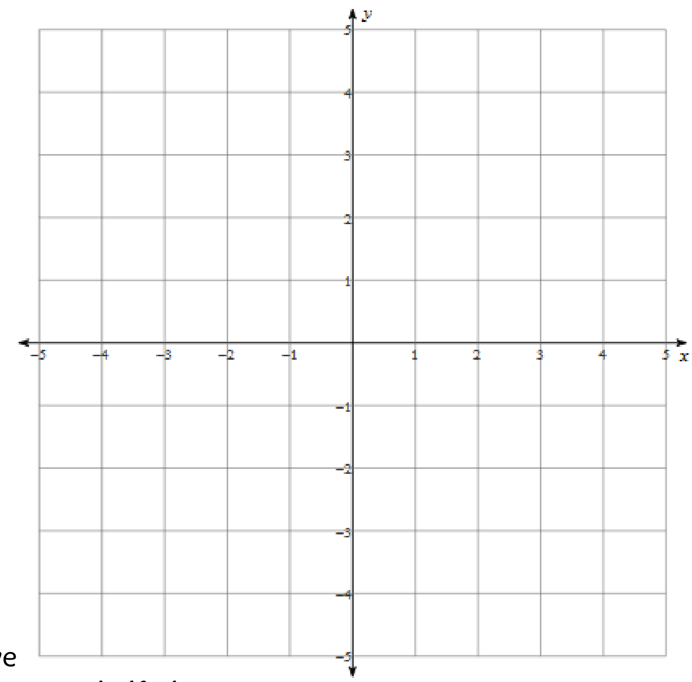
The ordered pair ________ is a solution. The correct region is shaded.

Graphing discrete solution sets

Example 5:

Graph each inequality.

a. $\{(x,y)\,|\,2x-5y\geq 10,\ x\in I, y\in I\}$.



STEP 1: Rearrange the inequality for y.

$$2x-5y\geq 10$$

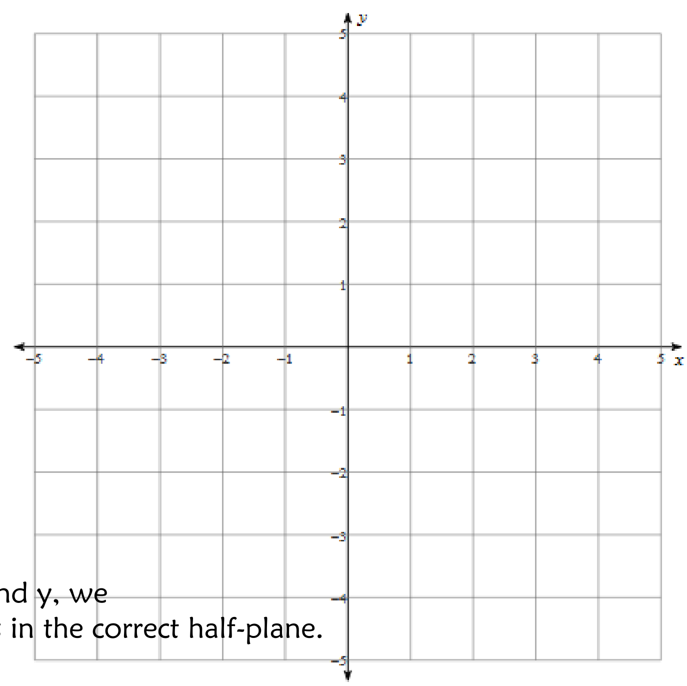
STEP 2: Graph the boundary line ____________.
The inequality symbol is $\leq$,
so this line will be ______________.

STEP 3: Since we are dealing with *integer* values for x and y, we will *stipple* only points with integer coordinates in the correct half-plane.
The inequality symbol is $\leq$,
so stipple ___________ the line.

STEP 4: Substitute a point from the solution region into the *original* inequality to see if the correct region is stippled.

Substitute _______: $2x-5y\geq 10$

b. $\{(x,y)\,|\,2x<3y,\ x\in N, y\in N\}$.



STEP 1: Rearrange the inequality for y.

$$2x<3y$$

STEP 2: Graph the boundary line ____________.
The inequality symbol is >,
so this line will be ______________.

STEP 3: Since we are dealing with *natural number* values for x and y, we will *stipple* only points with natural number coordinates in the correct half-plane.
The inequality symbol is >,
so stipple ___________ the line.

STEP 4: Substitute a point from the solution region into the *original* inequality to see if the correct region is stippled.

Substitute _______: $2x<3y$

Graphing Linear Inequalities to Solve Problems

Example 6:

One part of an exam has a combination of 2-point questions and 3-point questions. The total number of points earned for that part of the exam can be no more than 90.

a. Write an inequality for the situation.

STEP 1: Define the variables in this situation and determine any restrictions.

STEP 2: Write a linear inequality to represent the problem.

b. Graph the inequality.

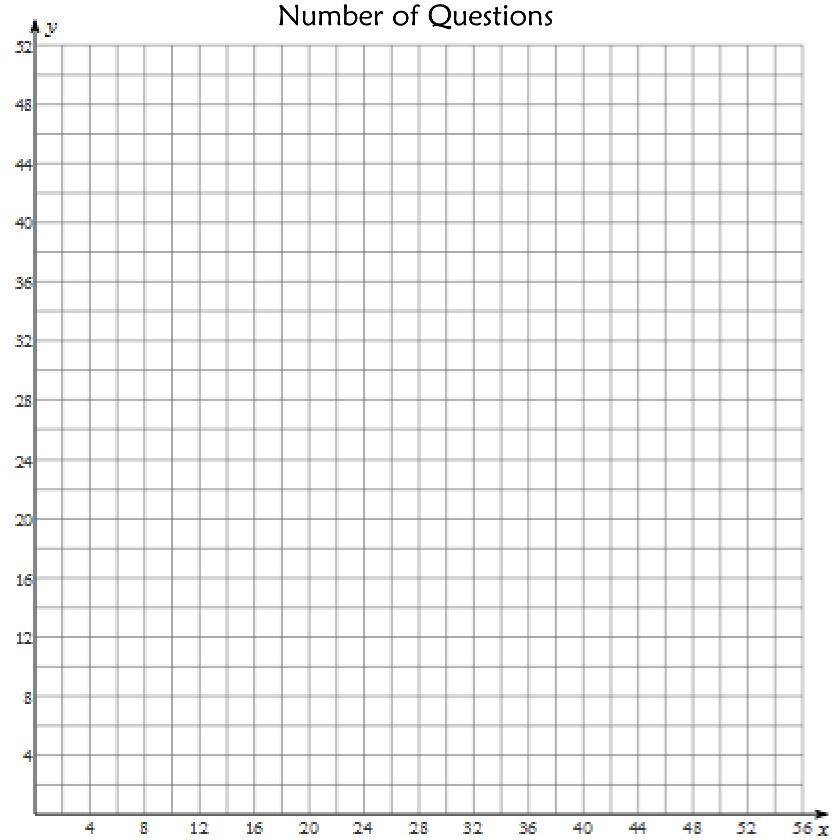
STEP 1: Label the axes.

STEP 2: Rearrange the inequality for y.

STEP 3: Graph the boundary line ______________.
The inequality symbol is $\leq$,
so this line will be _______________.

STEP 4: *Stipple* the correct half-plane.
The inequality symbol is $\leq$,
so stipple ____________ the line.

Number of Questions



c. Describe two combinations of questions that could be used on the exam.

Example 7:

A sports store has a net revenue of \$100 on every pair of downhill skis sold and \$120 on every snowboard sold. The manager's goal is to have a net revenue of more than \$600 dollars a day from the sales of these two items.

a. Write an inequality for the situation.

STEP 1: Define the variables in this situation and determine any restrictions.

STEP 2: Write a linear inequality to represent the problem.

b. Graph the inequality.

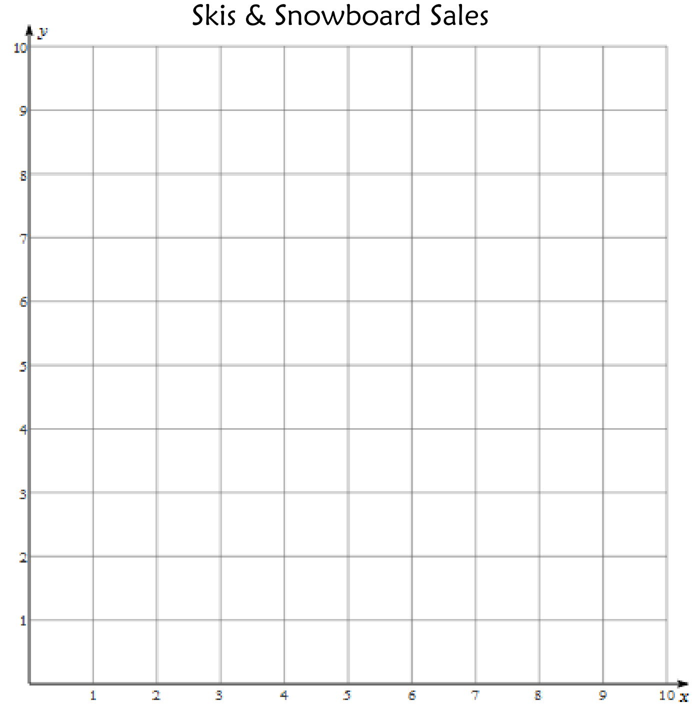
STEP 1: Label the axes.

STEP 2: Rearrange the inequality for y.

STEP 3: Graph the boundary line __________________.
The inequality symbol is >,
so this line will be _______________.

STEP 4: *Stipple* the correct half-plane.
The inequality symbol is >,
so stipple ____________ the line.

Skis & Snowboard Sales



c. Describe one combination of downhill skis and snowboard sales that make sense and meet the manager's daily sales goal.

d. Describe one combination of downhill skis and snowboard sales that will not meet the manager's daily sales goal.

IN SUMMARY:

When a *linear inequality* in two variables is represented *graphically*, its *boundary* divides the Cartesian plane into two half planes. *One* of these half planes represents the solution set of the linear inequality, which may or may not include points on the boundary itself.

Follow these steps when asked to graph a *linear inequality*:

1) Rearrange the inequality for y.

2) Write the *equation* of the ***boundary line*** and graph this line.

   - If the inequality ***includes*** the possibility of equality ($\leq$ or $\geq$), draw a ***solid*** line.

   - If the inequality ***excludes*** the possibility of equality ($<$ or $>$), draw a ***dashed*** line.

3) Shade (for *continuous* data) or stipple (for *discrete* data) the solution region.

   - Shade or stipple ***above*** the boundary if the inequality is $>$ or $\geq$. Shade or stipple ***below*** the boundary if the inequality is $<$ or $\leq$.

   - Substitute the coordinates of a ***test point*** from the solution region into the *original* linear inequality to verify that you have shaded or stippled the correct region.