Systems of Linear Inequalities

Solutions of Systems of Inequalities

A system of linear inequalities is a set of two or more linear inequalities with two or more variables. The solution of a system of linear inequalities is the set of ordered pairs that make every linear inequality in the system true.

Example 1:

Tell whether or not the ordered pair is a member of the solution set for the system of inequalities.

a. (0, 4) $\begin{cases} y < -x + 7 \\ y > x - 5 \end{cases}$

Substitute the ordered pair into each inequality.

$y < -x + 7$ $y > x - 5$

The ordered pair (0, 4) makes _____, so it _____ a member of the solution set.

b. (3, 4) $\begin{cases} x + 2y \le 12 \\ 3x - 2y \le -1 \end{cases}$

Substitute the ordered pair into each inequality.

$x + 2y \le 12$ $3x - 2y \le -1$

The ordered pair (3, 4) makes _____, so it _____ a member of the solution set.

c. (–1, 2) $\begin{cases} y \ge -3x \\ -5x + 17y > 45 \end{cases}$

Substitute the ordered pair into each inequality.

$y \ge -3x$ $-5x + 17y > 45$

The ordered pair (–1, 2) makes _____, so it _____ a member of the solution set. There is no need to check the second inequality.

Graphing Solutions of Systems of Linear Inequalities

To graph a system of two linear inequalities, use arrows to indicate which side of the boundary to shade/stipple for *each* inequality. The region representing *both* inequalities is the solution region and this is the *only* region that needs to be shaded/stippled.

Example 2:

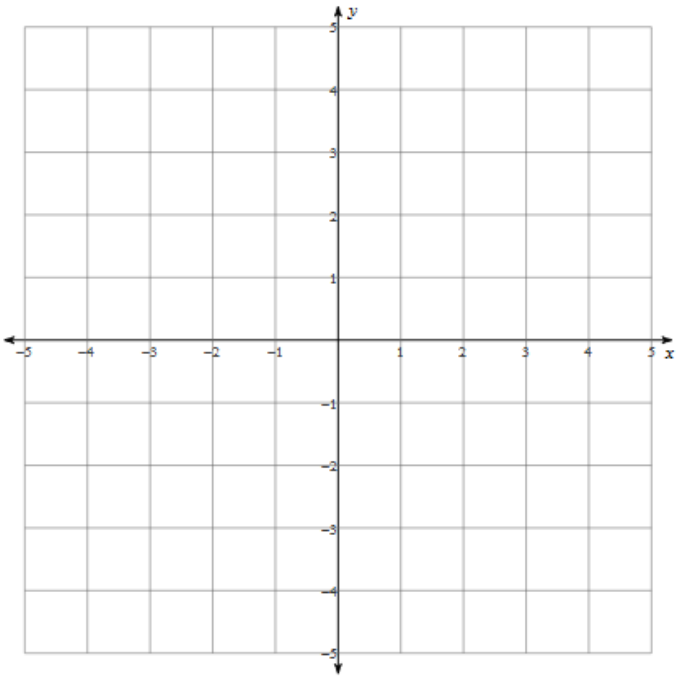
a. Graph the system $\{(x,y) \mid x+y<3,\ x\in R,\ y\in R\}$
$\{(x,y) \mid y\geq x+1,\ x\in R,\ y\in R\}$

STEP 1: Write the first inequality in slope-intercept form.

$x+y<3 \ \rightarrow$

$y\geq x+1$

STEP 2: Graph each inequality in the system.



b. Determine *graphically* whether or not each of the following ordered pairs are members of the solution set of the given system: (–4, –1), (0, 1), (0, 3), (2, 3)

The following points *are* solutions to the system:

The following points are *not* solutions to the system:

Example 3:

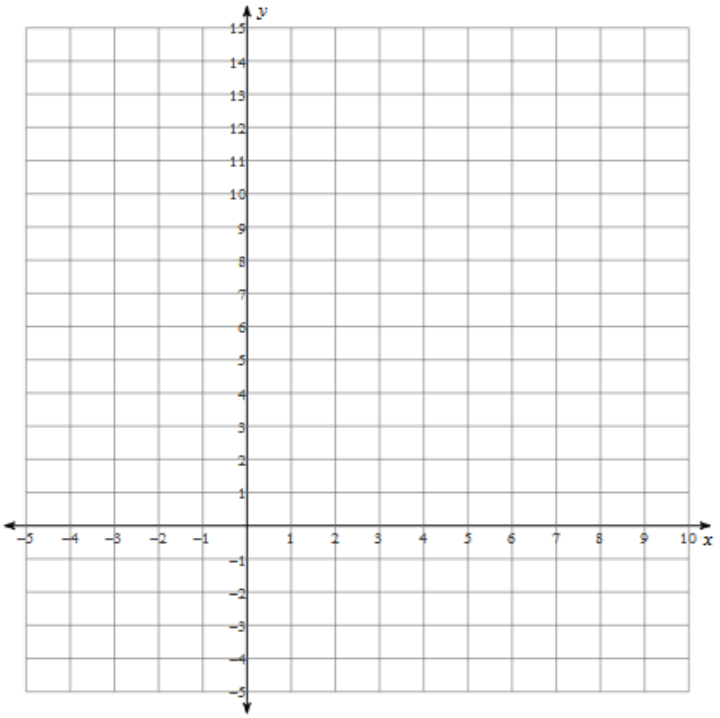
a. Graph the system $\{(x,y) \mid 3\leq x,\ x\in I, y\in I\}$
$\{(x,y) \mid y<2x,\ x\in I, y\in I\}$

STEP 1: Reverse the first inequality so that the variable is on the left.

$3\leq x \ \rightarrow$

$y<2x$

STEP 2: Graph each inequality in the system.



b. Choose a test point in the stippled region and verify that it is a solution to the system of inequalities.

More Than Two Inequalities

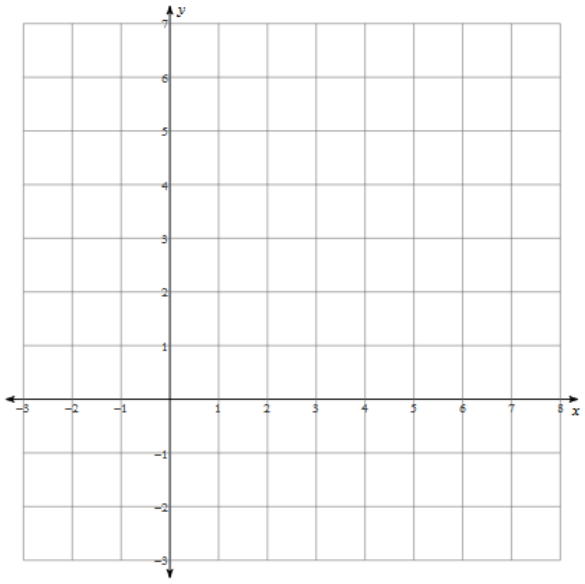
Example 4:

a. Graph the following system of linear inequalities:

$y \geq x+1$

$y \leq 3x$

$y \leq 4$



b. Choose a test point in the shaded region and verify that it is a solution to the system of inequalities.

Graphing Systems of Linear Inequalities to Solve Problems

The linear inequalities that restrict the solution to a problem are called ***constraints***. The graph of the solution of the system of linear inequalities that form the constraints is called the ***feasible region***. Every point in the feasible region satisfies all of the constraints.

Example 5:

The football team is selling T-shirts and sweatshirts to raise funds. For each T-shirt sold, the team makes $5. For each sweatshirt sold, the team makes $8. The team wants to make at least $1000 from sales. There are only 220 T-shirts and 80 sweatshirts available to sell.

a. Write the constraints for the situation, then graph these constraints.

STEP 1: Define the variables.

STEP 2: Write each constraint.

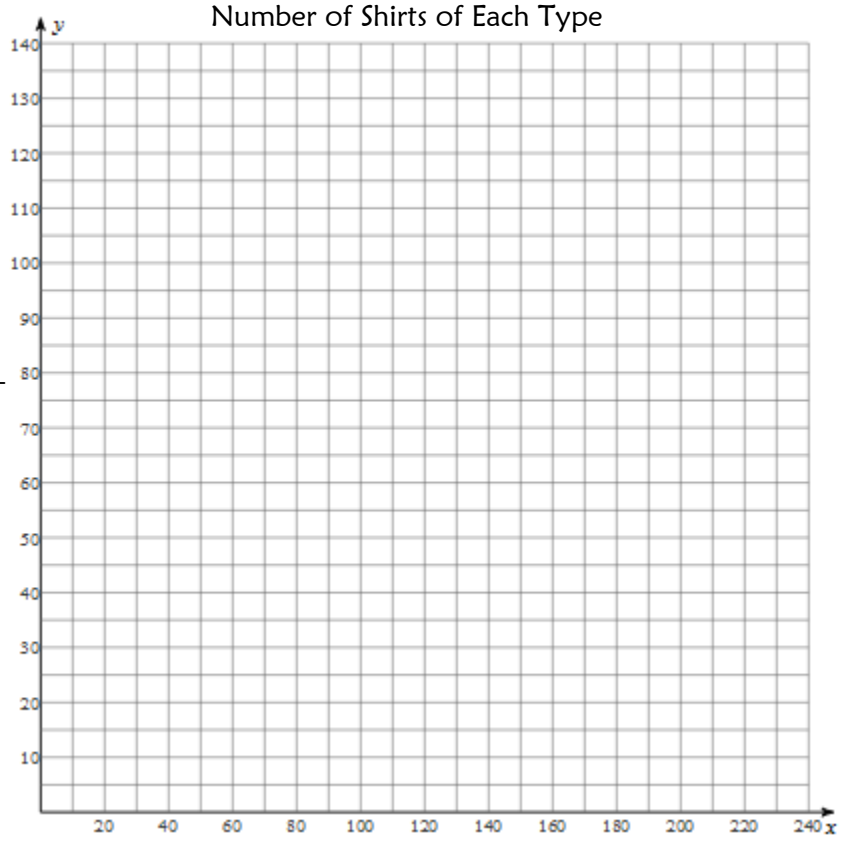
Amount of money from sales: _____

# of T-shirts team can sell: _____

# of sweatshirts team can sell: _____

STEP 3: Graph the system of inequalities.

Number of Shirts of Each Type



b. State a possible combination that satisfies the conditions of the problem.

Example 6:

Two scientists, Madison and Caleb, require the use of a lab. Madison needs the lab for at least as long as Caleb needs it, but, at most, for 12 hours. The lab is available for no more than 20 hours.

a. Write the constraints for the situation, then graph these constraints.

STEP 1: Define the variables.

STEP 2: Write each constraint.

Madison needs the lab for at least as long as Caleb: _____

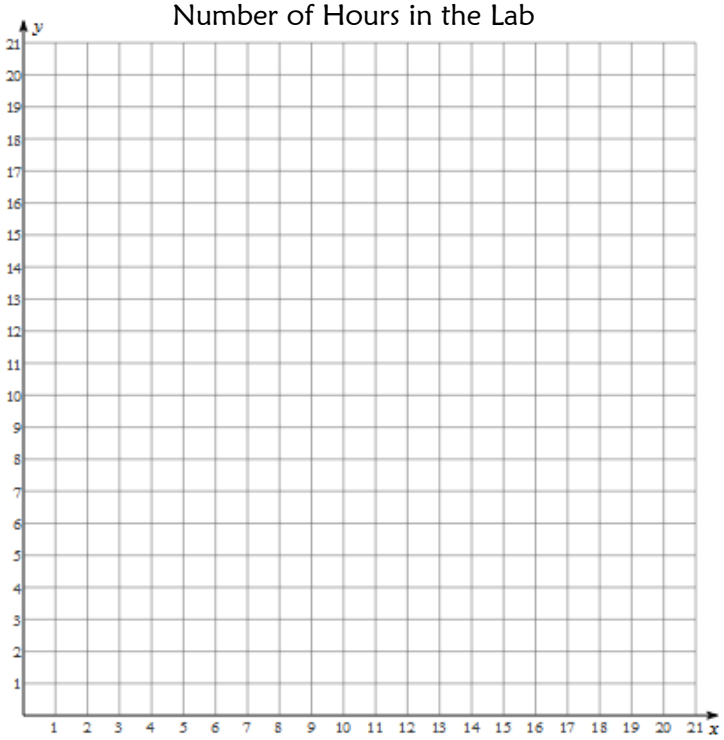
The lab is available for no more than 20 h: _____

The number of hours Madison uses the lab: _____

The number of hours Caleb uses the lab: _____

STEP 3: Graph the system of inequalities.

Number of Hours in the Lab



b. State a possible combination that satisfies the conditions of the problem.

Example 7:

A gardening company is buying bags of fertilizer with nitrogen and potash to use in its own mixture. The company needs at least 300 pounds of nitrogen and 80 pounds of potash. The company can buy a combination of Brand X bags, which cost $15 each and contain 8 pounds of nitrogen and 3 pounds of potash each, and Brand Y bags which cost $18 each and contain 12 pounds of nitrogen and 2 pounds of potash each. The company wants to spend no more than $750.

a. Write the constraints for the situation, then graph these constraints.

STEP 1: Organize the information in a table.

STEP 2: Define the variables.

	Brand X	Brand Y	Total (max or min)
nitrogen (lbs)			
potash (lbs)			
cost ($)			

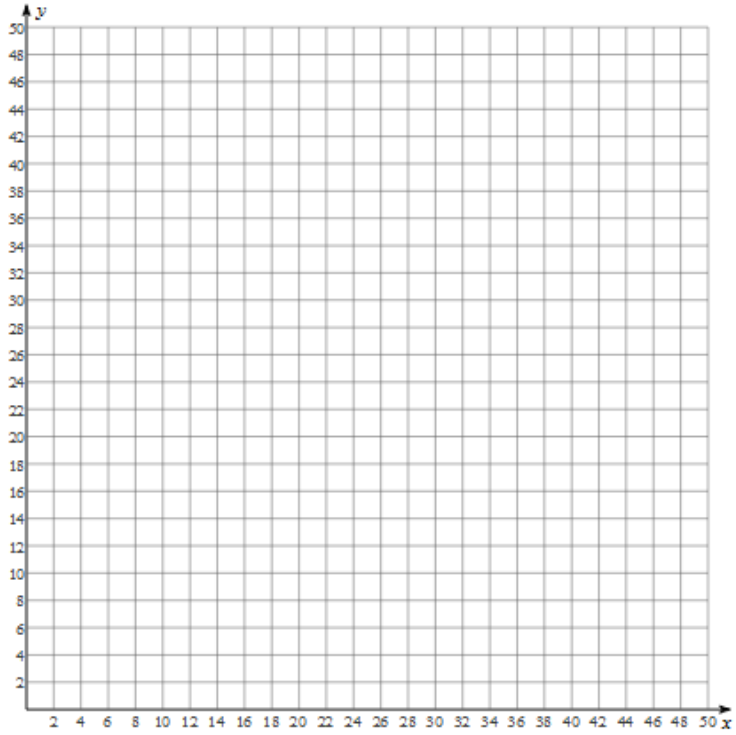
STEP 3: Write each constraint.

Amount of nitrogen is at least 300 pounds: _____

Amount of potash is at least 80 pounds: _____

The cost is, at most, $750: _____

STEP 4: Graph the system of inequalities.



b. State a possible combination that satisfies the conditions of the problem.