Factored Form of a Quadratic Function

One method that we have used to graph quadratic functions that are given in the form $f(x) = ax^2 + bx + c$ is to create a table of values. Another method is to determine the direction of opening, x-intercepts, y-intercept and its corresponding "symmetrical" point, axis of symmetry, and the vertex, and use these characteristics to sketch the graph.

- To determine the x-intercepts, write the function in factored form, then set f(x) = 0 and solve for x. Note that it is not always possible to factor a quadratic expression.

- If it is not possible to determine the x-intercepts by factoring, we can determine the y-intercept (which is "c") and its corresponding "symmetrical" point. To do this, we set f(x) = c and solve for x. This method is sometimes referred to as *partial factoring.*

- In either case above, we can then determine the axis of symmetry and vertex coordinates.

Example 1:

Sketch the graph of the quadratic function $f(x) = 2x^2 + 14x + 12$. State the domain and range of the function.

Solution:

Direction of opening: ____________________

y-intercept: __________

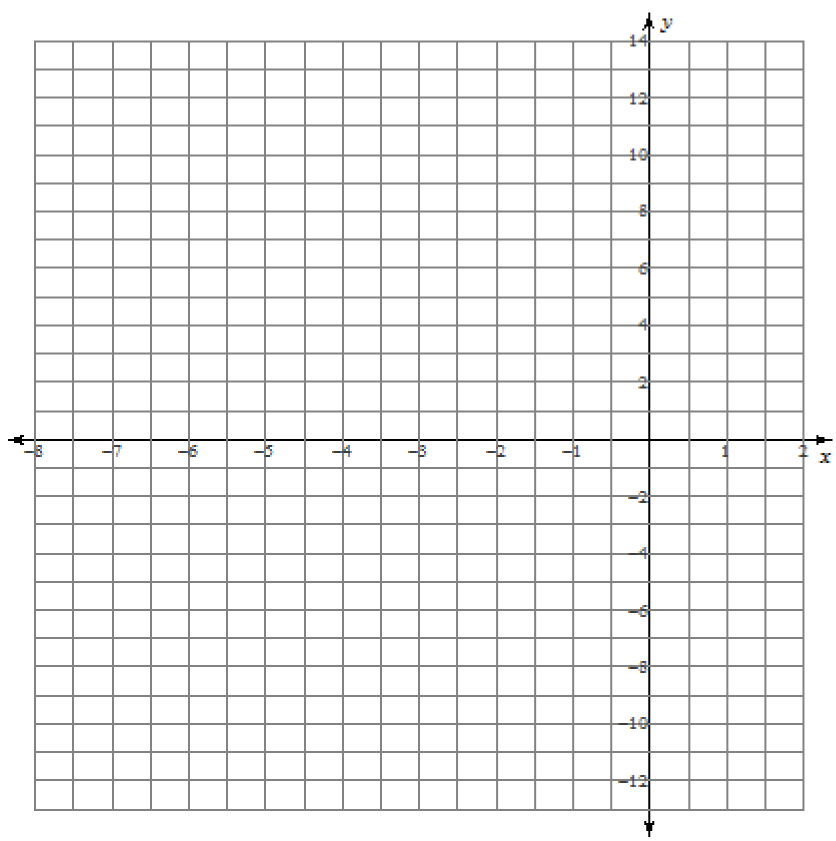
x-intercepts: _________________________

Axis of symmetry: __________

Vertex: __________

Domain: ____________________

Range: _________________________



Example 2:

Sketch the graph of the quadratic function $f(x) = -x^2 + 6x + 10$. State the domain and range of the function.

Solution:

Direction of opening: ____________________

We cannot use the factoring method to determine the x-intercepts of the function $f(x) = -x^2 + 6x + 10$ since $-x^2 + 6x + 10$ cannot be factored. Instead, we will use *partial factoring* to determine two *other* points that are the same distance from the axis of symmetry The y-intercept will be one of these two points. We can then use these two points to determine the x-coordinate of the vertex and then substitute this value into the equation to determine the y-coordinate of the vertex.

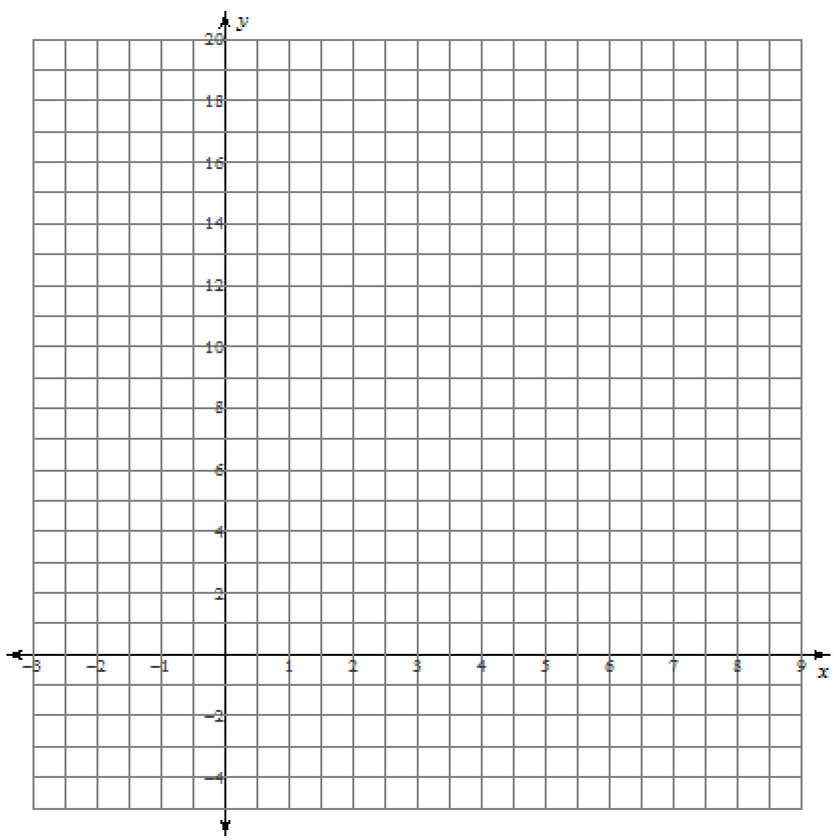
y-intercept and corresponding symmetrical point: _________________________

Axis of symmetry: __________

Vertex: __________

Domain: ____________________

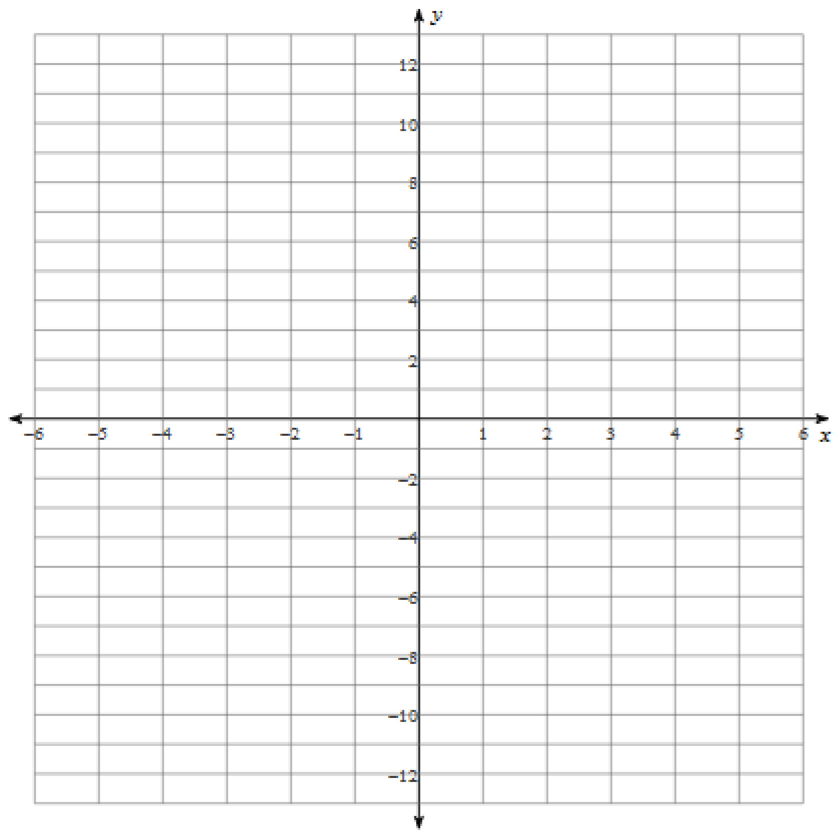
Range: _________________________



Example 3:

Sketch the graph of the quadratic function $y = -3x^2 + 6x + 9$ and state the domain and range.

Solution:



Direction of opening: _______________

Use partial factoring to determine two points that are equidistant from the axis of symmetry.

Let y = ______

Coordinates of the two points:
__________ & __________

Determine the x-intercepts.

Let y = ______

x-intercepts: ___________ & __________

Determine the coordinates of the vertex.

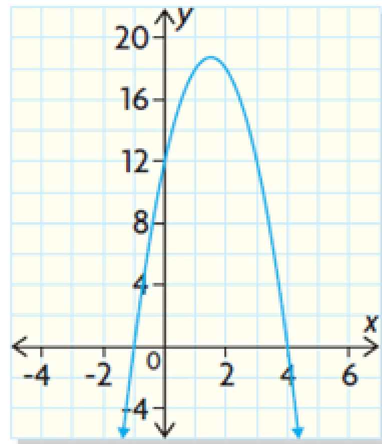
Vertex coordinates: __________

Sketch the graph.

Domain: ________________ Range: ________________________

Example 4:

Determine the function that defines this parabola.
Write the function in standard form.



Solution:

The *factored* form of a quadratic function can be written as y = a(x – r)(x – s), where r and s are the x-intercepts of the graph of the function.

The x-intercepts are ______ and ______, so substitution of these values into the function gives:

To determine the value of “a”, we can substitute another point (x, y) from the graph:

In factored form, the quadratic function is written:

In standard form, the quadratic function is written:

Example 5:

The members of a Ukrainian church hold a fundraiser every Friday night in the summer. They usually charge $6 for a plate of perogies. They know, from previous Fridays, that 120 plates of perogies can be sold at the $6 price but, for each $1 price increase, 10 fewer plates will be sold. What should the members charge if they want to raise as much money as they can for the church?

Solution:

Let x represent ____

Let R represent total revenue (in dollars)

R = ______________________

The graph of this ______________ function will be a ______________ opening

___________, so the function will have a ______________ R value.

We can determine this value, R, as well as the corresponding number of price increases, x, by determining the vertex coordinates:

Example 6:

Ataneq takes tourists on dogsled rides. He needs to build a kennel to separate some of his dogs from the other dogs in his team. He has budgeted for 40 m of fence. To save on materials, he plans to place the kennel against part of his home. What dimensions should Ataneq use to maximize the area of the kennel?

Solution:

Let x represent ____

Then __________ represents the length of the kennel

Let A represent the area of the kennel

A = _________________

The graph of this _______________ function will be a _______________ opening

___________, so the function will have a _______________ A value.

We can determine this value, A, as well as the corresponding width, x, by determining the vertex coordinates: