

Exploring the Validity of Conjectures and Using Reasoning to Find a Counterexample to a Conjecture

A conjecture that you make as a result of inductive reasoning may or may not be true. You can prove a conjecture false by finding just one counterexample.

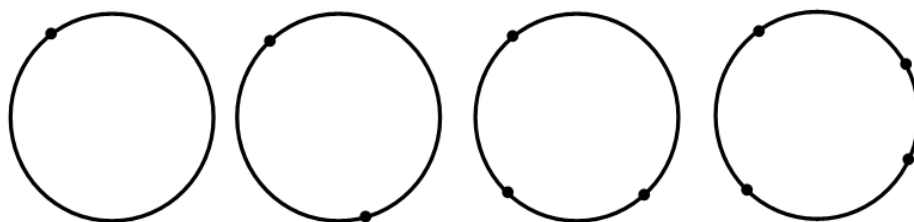
Counterexample – An example that invalidates a conjecture

Example 1: Testing a Conjecture

Make and test a conjecture regarding the number of regions a circle will be divided into if points on a circle are joined by chords.

Solution:

STEP 1: Find examples.



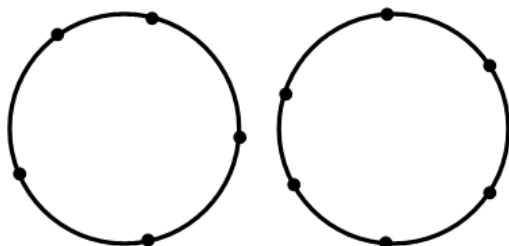
Join the points on each circle by chords and count the number of regions the circle is divided into. Record your results.

Number of Points	1	2	3	4
Number of Regions				

STEP 2: Look for a pattern and form a conjecture.

As the number of connected points on the circle increases by 1, the number of regions created within the circle _____.

STEP 3: Gather more evidence to test your conjecture.



Number of Points	1	2	3	4	5	6
Number of Regions						

Upon further testing, we found a counterexample to our conjecture. This counterexample *disproves* the conjecture.

Example 2: Testing and Revising a Conjecture

Make a conjecture about the difference between consecutive perfect squares. Develop evidence to test your conjecture.

Solution:

STEP 1: Find examples.

$$2^2 - 1^2 =$$

$$3^2 - 2^2 =$$

$$4^2 - 3^2 =$$

$$9^2 - 8^2 =$$

$$12^2 - 11^2 =$$

STEP 2: Look for a pattern and form a conjecture.

STEP 3: Gather more evidence to test your conjecture. (Can you find a counterexample to your conjecture? If so, what is it?)

STEP 4: Based on further evidence, revise your conjecture.

Notes:

- Gathering further evidence can help support a conjecture, but it does not prove the conjecture.
- A single counterexample is enough to disprove a conjecture.

YOUR TURN:

1. Give an example that *supports* each the following statements, then, show that each statement is false by finding a *counterexample*.

- a. **Every multiple of 5 ends in 5.**

Supporting example: _____

Counterexample: _____

- b. **The sum of two prime numbers is an even number.**

Supporting example: _____

Counterexample: _____

- c. **In any isosceles triangle, all three angles are acute.**

Supporting example: _____

Counterexample: _____

- d. **The sum of a multiple of 3 and a multiple of 6 must be odd.**

Supporting example: _____

Counterexample: _____

- e. **The difference between a multiple of 11 and a multiple of 7 is even or divisible by 3.**

Supporting example: _____

Counterexample: _____

2. Ryan made the following conjecture:

“The sum of two numbers is greater than either of the two addends.”

- a. Give a *counterexample* to this conjecture. _____

- b. *Revise* Ryan’s conjecture such that *no* counterexample exists.
