

# Reflections and Stretches

**Reflection:** a transformation where each point of the original graph has an image point resulting from a reflection in a line. A reflection may result in a change of orientation of a graph while preserving its shape.

**Stretch:** a transformation in which the distance of each  $x$ -coordinate or  $y$ -coordinate from the line of reflection is multiplied by some scale factor. Scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection. A stretch changes the shape of the graph but not its orientation.

**Invariant Point:** a point on a graph that remains unchanged after a transformation is applied to it. Any point on a line of reflection is an invariant point.

## Example 1: Compare the Graphs of $y = f(x)$ , $y = -f(x)$ , and $y = f(-x)$

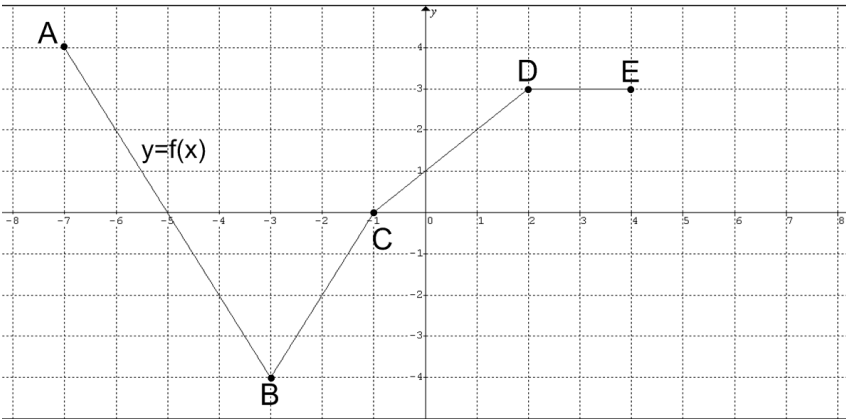
- a. Given the graph of  $y = f(x)$ , graph the function  $y = -f(x)$ . How is the graph of  $y = -f(x)$  related to the graph of  $y = f(x)$ ?

**Solution:**

Use key points on the graph of  $y = f(x)$  to create a table of values.

The image points on the graph of  $y = -f(x)$  have the same  $x$ -coordinates but different  $y$ -coordinates. Multiply the  $y$ -coordinates of points on the graph of  $y = f(x)$  by  $-1$ .

	x	$y = f(x)$
A	-7	4
B	-3	-4
C	-1	0
D	2	3
E	4	3
	x	$y = -f(x)$
A'	-7	
B'	-3	
C'	-1	
D'	2	
E'	4	



The graph of  $y = -f(x)$  is congruent to the graph of  $y = f(x)$ .

The points on the graph of  $y = f(x)$  relate to the points on the graph of  $y = -f(x)$  by the mapping \_\_\_\_\_.

The graph of  $y = -f(x)$  is a reflection of the graph of  $y = f(x)$  in the \_\_\_\_\_.

The points \_\_\_\_\_ and \_\_\_\_\_ are *invariant* points.

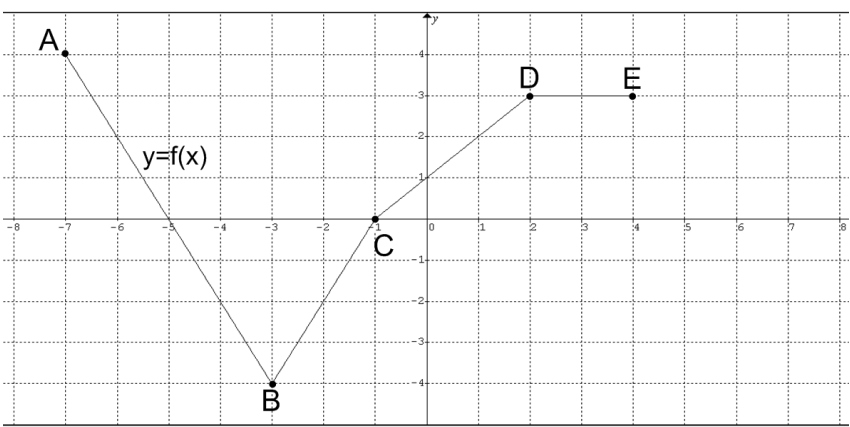
- b. Given the graph of  $y = f(x)$ , graph the function  $y = f(-x)$ . How is the graph of  $y = f(-x)$  related to the graph of  $y = f(x)$ ?

Solution:

Use key points on the graph of  $y = f(x)$  to create a table of values.

The image points on the graph of  $y = f(-x)$  have the same y-coordinates but different x-coordinates. Multiply the x-coordinates of points on the graph of  $y = f(x)$  by  $-1$ .

	x	$y = f(x)$
A	-7	4
B	-3	-4
C	-1	0
D	2	3
E	4	3
	x	$y = f(-x)$
A'		4
B'		-4
C'		0
D'		3
E'		3



The graph of  $y = f(-x)$  is congruent to the graph of  $y = f(x)$ .

The points on the graph of  $y = f(x)$  relate to the points on the graph of  $y = f(-x)$  by the mapping \_\_\_\_\_.

The graph of  $y = f(-x)$  is a reflection of the graph of  $y = f(x)$  in the \_\_\_\_\_.

The point \_\_\_\_\_ is an invariant point.

Example 2: Vertical Stretches

Given the graph of  $y = f(x)$ ,

- transform the graph of  $f(x)$  to sketch the graph of  $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

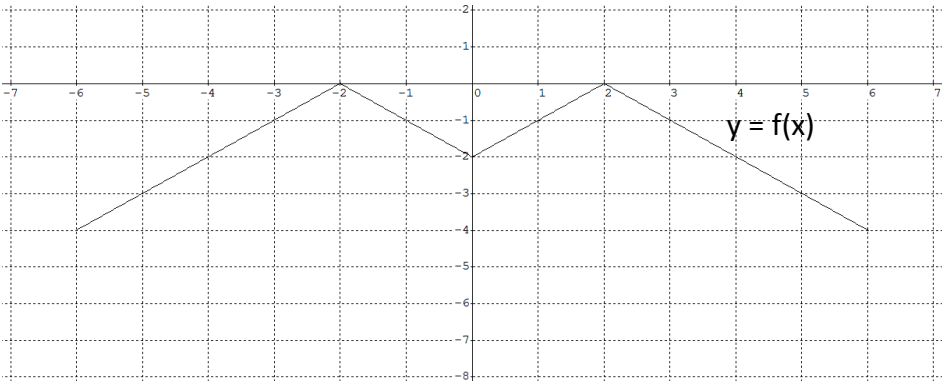
a.  $g(x) = 2f(x)$

b.  $g(x) = \frac{1}{2}f(x)$

Solution:

- a. Use the key points on the graph of  $y = f(x)$  to create a table of values. The image points on the graph of  $g(x) = 2f(x)$  have the same x-coordinates but different y-coordinates. Multiply the y-coordinates of points on the graph of  $y = f(x)$  by 2.

x	$y = f(x)$	$y = g(x) = 2f(x)$
-6		
-2		
0		
2		
6		



The points on the graph of  $f(x)$  relate to the points on the graph of  $g(x) = 2f(x)$  by the mapping \_\_\_\_\_.

The graph of  $g(x) = 2f(x)$  is a vertical stretch of the graph of  $f(x)$  about the \_\_\_\_\_ by a factor of 2.

The invariant points are \_\_\_\_\_ and \_\_\_\_\_.

The domain of  $f(x)$  is \_\_\_\_\_ or \_\_\_\_\_

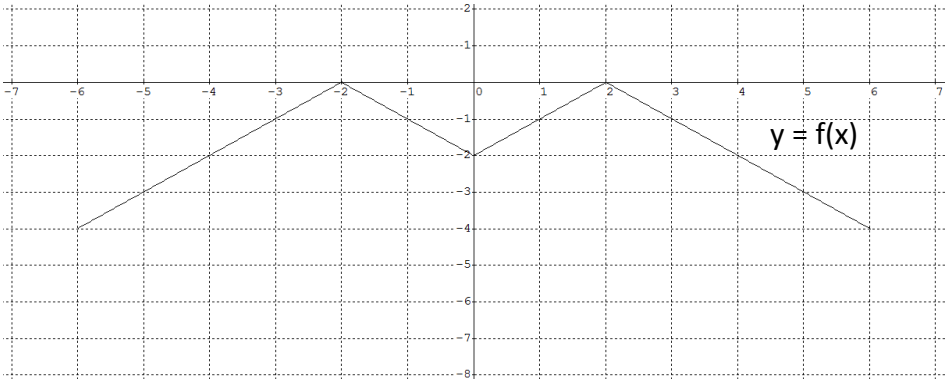
and the range is \_\_\_\_\_ or \_\_\_\_\_.

The domain of  $g(x)$  is \_\_\_\_\_ or \_\_\_\_\_

and the range is \_\_\_\_\_ or \_\_\_\_\_.

- b. Use the key points on the graph of  $y = f(x)$  to create a table of values. The image points on the graph of  $g(x) = \frac{1}{2}f(x)$  have the same x-coordinates but different y-coordinates. Multiply the y-coordinates of points on the graph of  $y = f(x)$  by  $\frac{1}{2}$ .

x	$y = f(x)$	$y = g(x) = \frac{1}{2}f(x)$
-6	-4	
-2	0	
0	-2	
2	0	
6	-4	



The points on the graph of  $f(x)$  relate to the points on the graph of  $g(x) = \frac{1}{2}f(x)$  by the mapping \_\_\_\_\_.

The graph of  $g(x) = \frac{1}{2}f(x)$  is a vertical stretch of the graph of  $f(x)$  about the \_\_\_\_\_ by a factor of  $\frac{1}{2}$ .

The invariant points are \_\_\_\_\_ and \_\_\_\_\_.

The domain of  $f(x)$  is \_\_\_\_\_ or \_\_\_\_\_  
and the range is \_\_\_\_\_ or \_\_\_\_\_.

The domain of  $g(x)$  is \_\_\_\_\_ or \_\_\_\_\_  
and the range is \_\_\_\_\_ or \_\_\_\_\_.

### Example 3: Horizontal Stretches

Given the graph of  $y = f(x)$ ,

- transform the graph of  $f(x)$  to sketch the graph of  $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

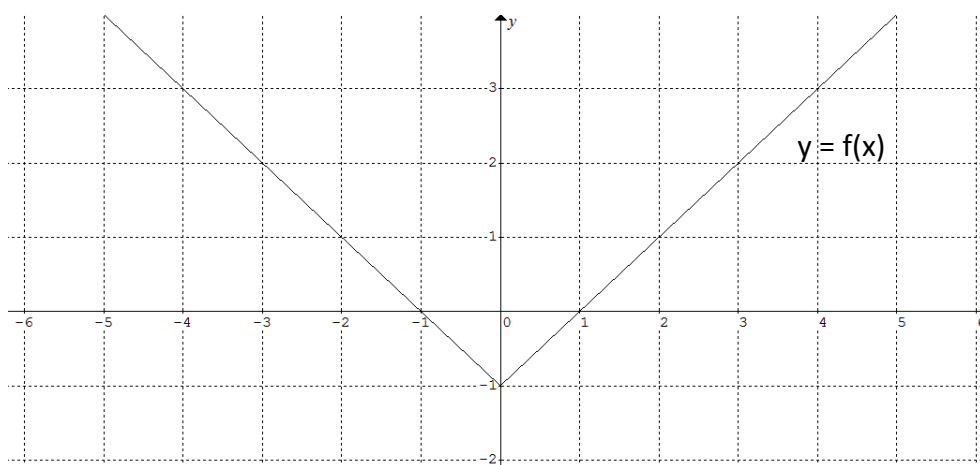
a.  $g(x) = f(2x)$

b.  $g(x) = f\left(\frac{1}{2}x\right)$

**Solution:**

- a. Use key points on the graph of  $y = f(x)$  to create a table of values. The image points on the graph of  $g(x) = f(2x)$  have the same y-coordinates but different x-coordinates. Multiply the x-coordinates of points on the graph of  $y = f(x)$  by  $\frac{1}{2}$ .

x	$y = f(x)$
	3
	1
	-1
	1
	3
x	$y = g(x) = f(2x)$
	3
	1
	-1
	1
	3



The points on the graph of  $f(x)$  relate to the points on the graph of  $g(x) = f(2x)$  by the mapping \_\_\_\_\_.

The graph of  $g(x) = f(2x)$  is a horizontal stretch of the graph of  $f(x)$  about the \_\_\_\_\_ by a factor of  $\frac{1}{2}$ .

The invariant point is \_\_\_\_\_.

The domain of  $f(x)$  is \_\_\_\_\_ or \_\_\_\_\_

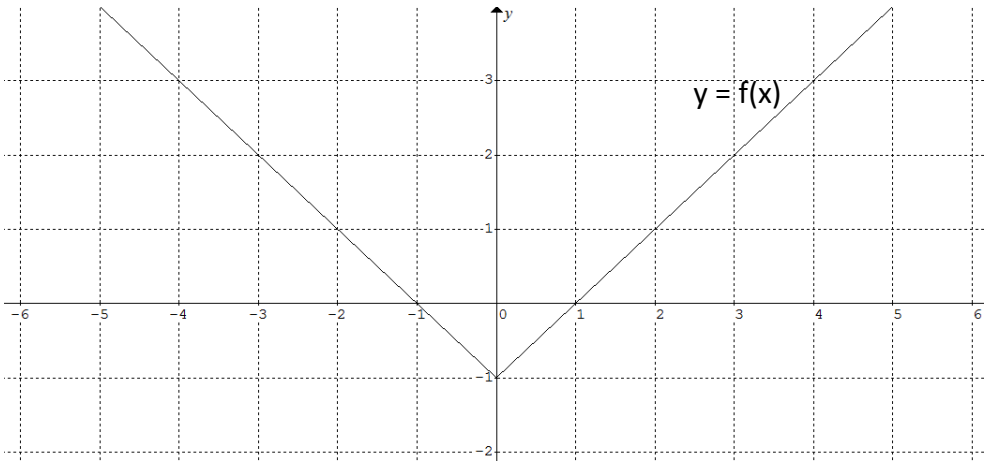
and the range is \_\_\_\_\_ or \_\_\_\_\_.

The domain of  $g(x)$  is \_\_\_\_\_ or \_\_\_\_\_

and the range is \_\_\_\_\_ or \_\_\_\_\_.

- b. Use key points on the graph of  $y = f(x)$  to create a table of values. The image points on the graph of  $g(x) = f(\frac{1}{2}x)$  have the same y-coordinates but different x-coordinates. Multiply the x-coordinates of points on the graph of  $y = f(x)$  by 2.

x	y = f(x)
	2
	1
	0
	-1
	0
	1
	2
x	y = g(x) = f(\frac{1}{2}x)
	2
	1
	0
	-1
	0
	1
	2



The points on the graph of  $f(x)$  relate to the points on the graph of  $g(x) = f(\frac{1}{2}x)$  by the mapping \_\_\_\_\_.

The graph of  $g(x) = f(\frac{1}{2}x)$  is a horizontal stretch of the graph of  $f(x)$  about the \_\_\_\_\_ by a factor of 2.

The invariant point is \_\_\_\_\_.

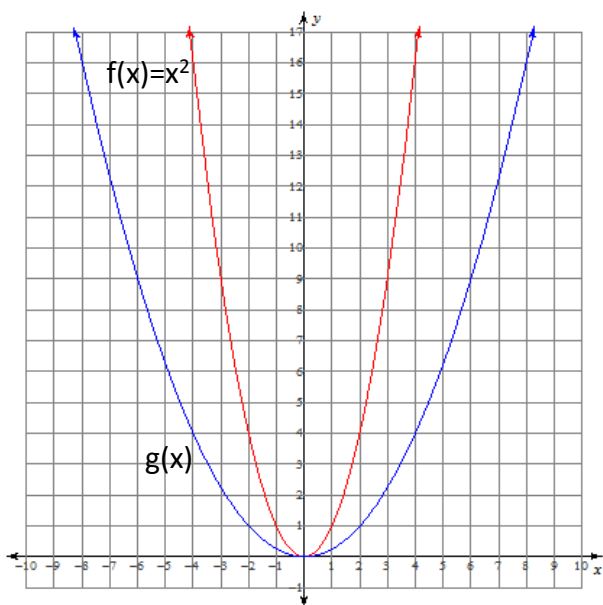
The domain of  $f(x)$  is \_\_\_\_\_ or \_\_\_\_\_  
and the range is \_\_\_\_\_ or \_\_\_\_\_.

The domain of  $g(x)$  is \_\_\_\_\_ or \_\_\_\_\_  
and the range is \_\_\_\_\_ or \_\_\_\_\_.

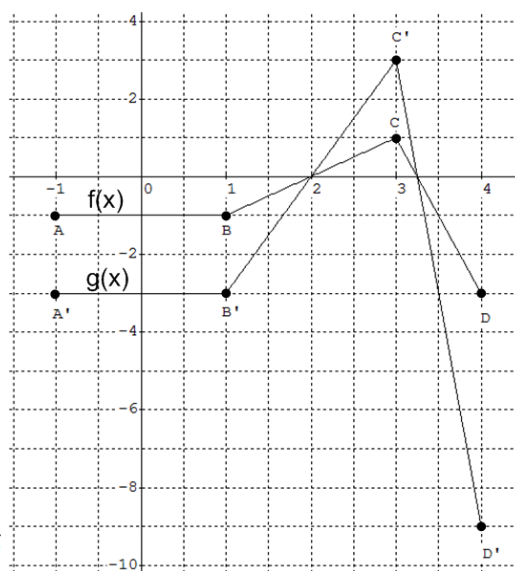
### Example 4: Write the Equation of a Transformed Function

The graph of the function  $y=f(x)$  has been transformed by either a stretch or a reflection. Write the equation of the transformed graph,  $g(x)$ .

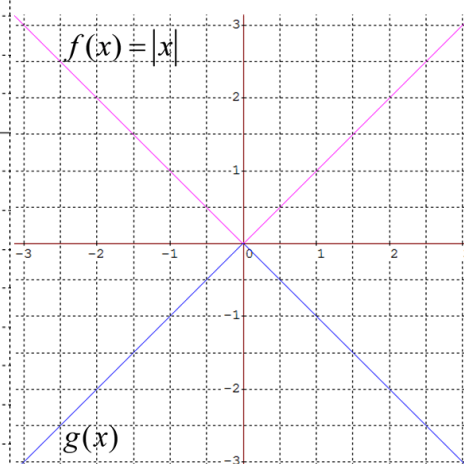
a.



b.



c.



#### Solution:

- a. The shape has changed so the graph has been transformed by a stretch. In this case, the stretch can be described in two ways.

**Case 1:** Check for a pattern in the y-coordinates.

x	y = f(x)	y = g(x)
-4		
-2		
0		
2		
4		

A vertical stretch results when the vertical distances of the transformed graph are a constant multiple of those of the original graph with respect to the x-axis.

The transformation can be described by the mapping \_\_\_\_\_.

The equation of the transformed function is  $g(x) = \underline{\hspace{2cm}}$  or  $g(x) = \underline{\hspace{2cm}}$ .

**Case 2:** Check for a pattern in the x-coordinates.

x	y = f(x)	x	y = g(x)
	16		16
	4		4
	1		1
	0		0
	1		1
	4		4
	16		16

A horizontal stretch results when the horizontal distances of the transformed graph are a constant multiple of those of the original graph with respect to the y-axis.

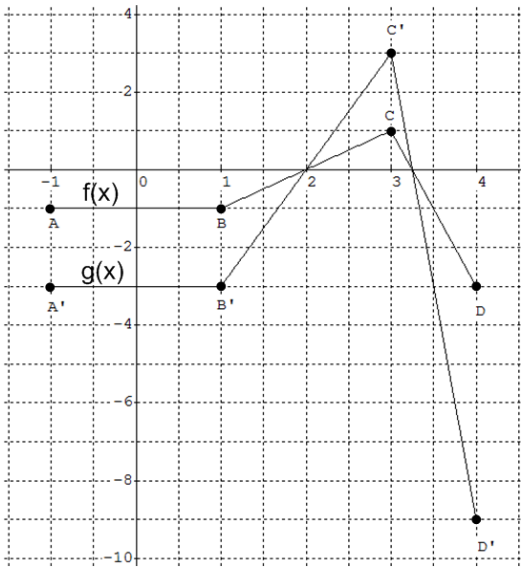
The transformation can be described by the mapping \_\_\_\_\_.

The equation of the transformed function is  $g(x) = \underline{\hspace{2cm}}$  or  $g(x) = \underline{\hspace{2cm}}$ .

- b. The shape has changed so the graph has been transformed by a stretch. In this case, the stretch can be described as a vertical stretch.

Check for a pattern in the y-coordinates.

x	y = f(x)	y = g(x)

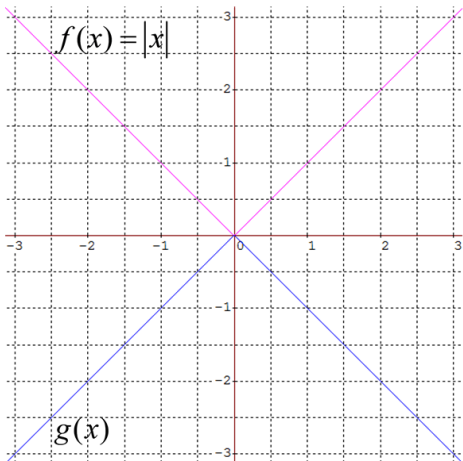


The transformation can be described by the mapping \_\_\_\_\_.

The equation of the transformed function is  $g(x) =$  \_\_\_\_\_.

- c. The shape of the graph has not changed. The graph has been transformed by a reflection in the \_\_\_\_\_.

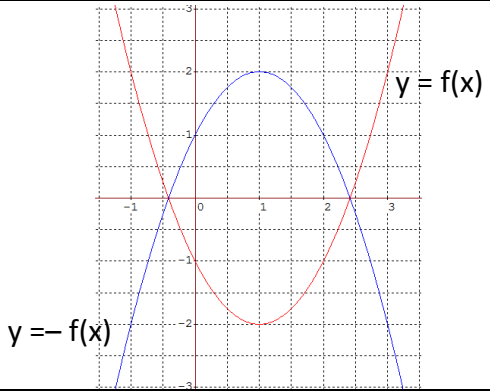
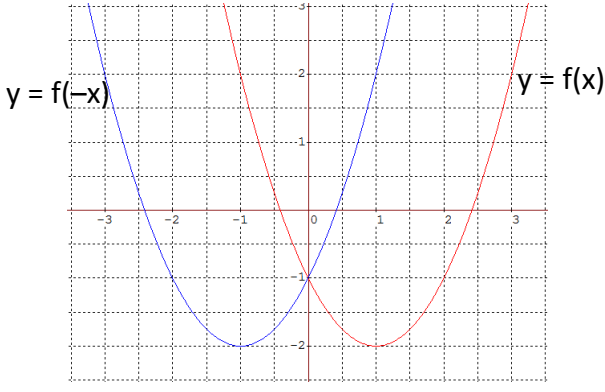
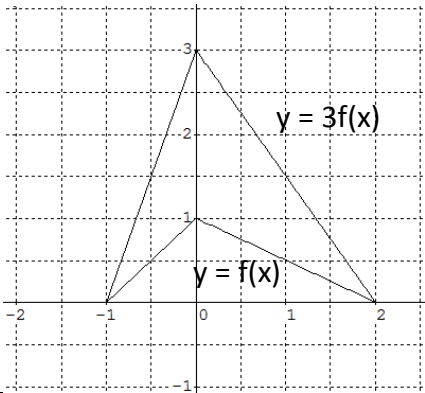
x	y = f(x)	y = g(x)



The transformation can be described by the mapping \_\_\_\_\_.

The equation of the transformed function is  $g(x) =$  \_\_\_\_\_ or  $g(x) =$  \_\_\_\_\_.

Reflections and Stretches of the function  $y = f(x)$ 

Function	Transformation from $y = f(x)$	Mapping	Example
$y = -f(x)$	<ul style="list-style-type: none"> <li>a reflection in the x-axis</li> </ul>	$(x, y) \rightarrow (x, -y)$	
$y = f(-x)$	<ul style="list-style-type: none"> <li>a reflection in the y-axis</li> </ul>	$(x, y) \rightarrow (-x, y)$	
$y = af(x)$	<ul style="list-style-type: none"> <li>A vertical stretch about the x-axis by a factor of <math> a </math>.</li> <li>If <math>a &lt; 0</math>, then the graph is also reflected in the x-axis</li> </ul>	$(x, y) \rightarrow (x, ay)$	
$y = f(bx)$	<ul style="list-style-type: none"> <li>A horizontal stretch about the y-axis by a factor of <math>\frac{1}{ b }</math>.</li> <li>If <math>b &lt; 0</math>, then the graph is also reflected in the y-axis</li> </ul>	$(x, y) \rightarrow (\frac{x}{b}, y)$	