Combining Transformations

Multiple transformations can be applied to a function using the general transformation model:

$y-k=af(b(x-h))$ *or* $y=af(b(x-h))+k$

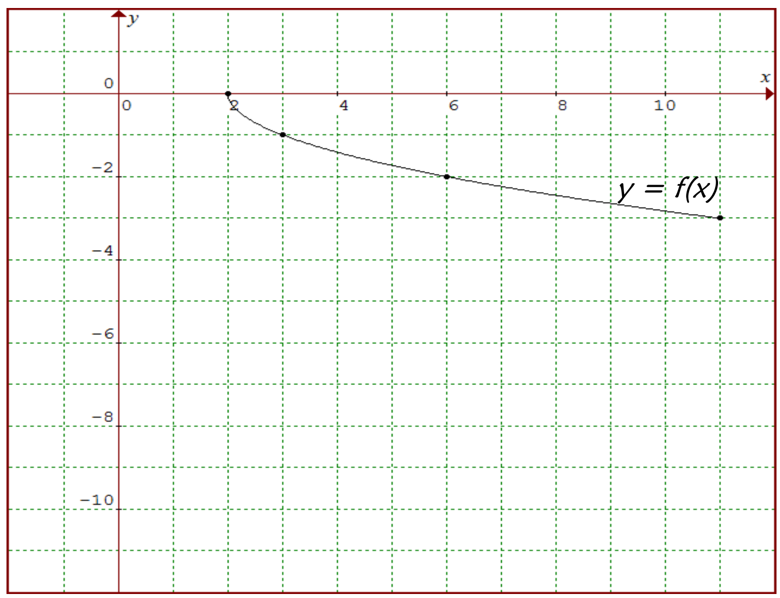
To sketch the graph of a function of this form, the stretches and reflections (values of a and b) occur ***before*** *the translations (values of h and k).*

Example 1: Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y=f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a. $y=3f(2x)$

b. $y=f(2x+4)$



Solution:

a. Compare the function $y=3f(2x)$ to

$$y=af(b(x-h))+k$$

a = _____ b= _____

h = _____ k = _____

The graph of $y=f(x)$ is vertically stretched by a factor of _____ and horizontally stretched by a factor of _____.

First, apply the vertical stretch by multiplying the y-values by _____.

(2, 0) → (2,)
(3, –1) → (3,)
(6, –2) → (6,)
(11, –3) → (11,) Plot the points and graph $y=3f(x)$.

Then, using the new image points from above, apply the horizontal stretch by multiplying the x-values by _____

(2, 0) → (, 0)
(3, –3) → (, –3)
(6, –6) → (, –6)
(11, –9) → (, –9) Plot the points and graph $y=3f(2x)$.

Would performing the stretches in reverse order change the final result? __________

Mapping Rule: ______________________________

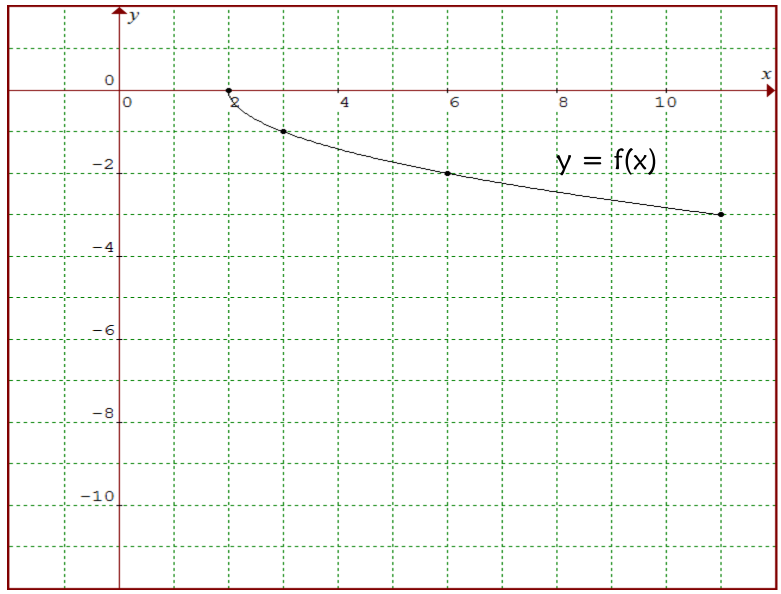
b. Rewrite the function $y = f(2x+4)$ in the form

$$y = af(b(x-h)) + k$$

$y = f(2x+4)$ becomes ____________________

a = _____ b= _____

h = _____ k = _____



The graph of $y = f(x)$ is stretched __________________________ by a factor of _____,

and then translated _____ units to the _______________.

First, apply the horizontal stretch by multiplying the x-values by _____.

(2, 0) → (, 0)

(3, –1) → (, –1)

(6, –2) → (, –2)

(11, –3) → (, –3) Plot the points and graph $y = f(2x)$.

Then, using the new image points from above, apply the horizontal translation by _________________________ each x-value.

(1, 0) → (, 0)

(1.5, –1) → (, –1)

(3, –2) → (, –2)

(5.5, –3) → (, –3) Plot the points and graph $y = f(2(x+2))$.

Note that the horizontal *stretch* must be performed *before* the horizontal *translation* in order to get the correct final result.

Mapping Rule: ________________________________

Example 2: Combination of Transformations

State the combination of transformations that must be applied to the graph of the function $y = f(x)$ in order to obtain the graph of the transformed function, $g(x) = -2f\left(\frac{1}{2}(x-1)\right) + 4$. Write the corresponding mapping rule, then apply the mapping rule to key points on f(x) to obtain the corresponding image points on g(x).

Sketch the graph of $g(x)$. Write the specific equation for $g(x)$.

Solution:

Compare $g(x) = -2f\left(\frac{1}{2}(x-1)\right) + 4$ to $y = af(b(x-h)) + k$ to obtain the following values:

a = ______, b = ______, h = ______, k = ______

To obtain the graph of g(x), the graph of f(x) must be reflected through the _______________, stretched _________________________ by a factor of _____ and _________________________ by a factor of _____.

The graph would then be translated _____ unit __________ and _____ units __________.

Mapping Rule: _____________________________________

Apply the mapping rule to key points on f(x) to obtain the corresponding image points on g(x), then sketch the graph of g(x).

$f(x) \rightarrow g(x)$

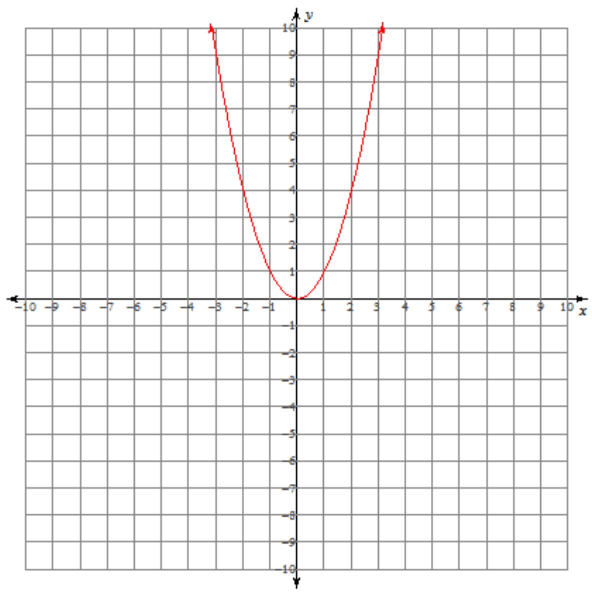
$(-2, 4) \rightarrow$ (___, ___)

$(-1, 1) \rightarrow$ (___, ___)

$(0, 0) \rightarrow$ (___, ___)

$(1, 1) \rightarrow$ (___, ___)

$(2, 4) \rightarrow$ (___, ___)



The equation of the transformed function is: ______________________________

Example 3: Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x-h))+k$. Explain your answer.

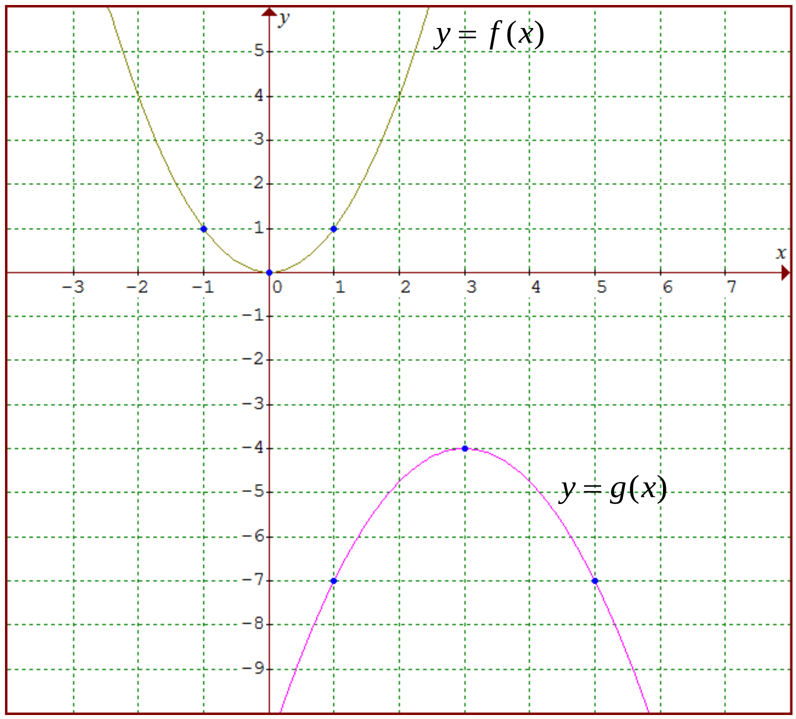
Solution:

Locate key points on f(x) and their image points on g(x):

(–1, 1) → (1, –7)

(0, 0) → (3, –4)

(1 , 1) → (5, –7)



Stretches and Reflections:

To determine horizontal and vertical stretch factors, compare distances between key points.

Horizontally on $f(x)$ key points are ______ units apart, and on $g(x)$ key points are ______ units apart.

So, horizontal stretch factor = ______

Vertically on $f(x)$ key points are ______ unit apart, and on $g(x)$ key points are ______ units apart.

So, vertical stretch factor = ______

Also, we can see that the graph has been reflected in the ______________, so ______ is negative.

So, a = ______ b = ______

Translations:

The point (0, 0) is not affected by stretches or reflections so we can use this to determine the horizontal and vertical translations.

So, this point has moved _____ units __________ and _____ units __________.

So, h = ______ k = ______

Substitute the values of a, b, h, and k into $y = af(b(x-h)+k$.

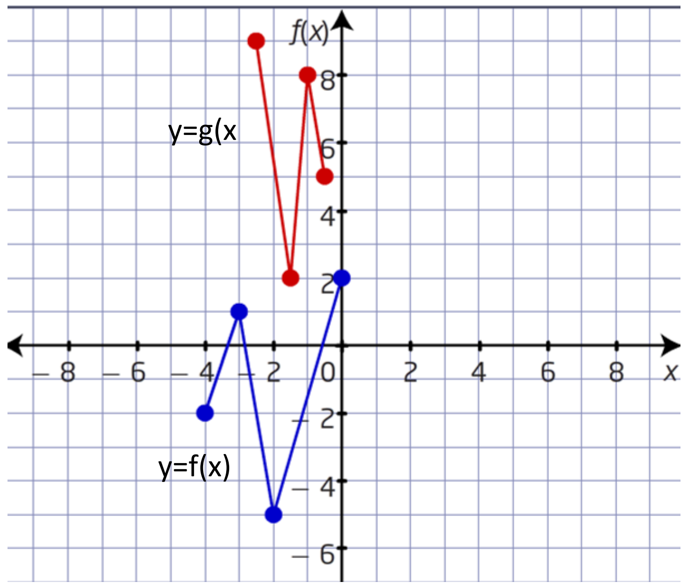
The equation of the transformed function is: ____

Example 4: Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x-h))+k$. Explain your answer.

Solution:

Compare the locations of the key points on the original graph, f(x), and the transformed graph, g(x), to determine whether or not there have been any reflections and/or stretches. It might be helpful to label which points on f(x) correspond with their image points on g(x).



Reflection in the x-axis: __________

Reflection in the y-axis: __________

Vertical stretch factor: __________

Horizontal stretch factor: __________

To determine whether or not there have been any vertical and/or horizontal translations, consider where the key points on f(x) will be located after the reflections and stretches listed above have been applied, then determine what translations will be necessary to obtain the final image points on g(x).

f(x)			h(x)			g(x)	
x	y		x	y		x	y
–4	–2						
–3	1	→			→		
–2	–5						
0	2						

Compare the intermediate function, h(x), to the final function, g(x), to determine the translations.

Horizontal translation: __________

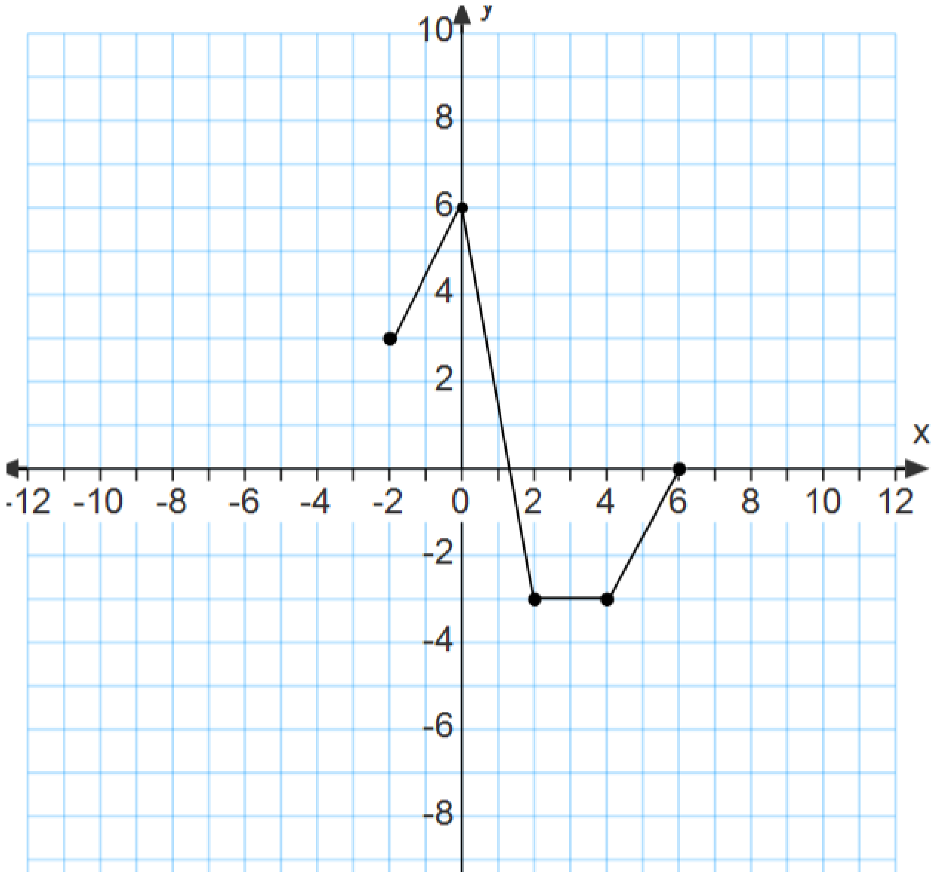
Vertical translation: __________

Final mapping rule: ____

The equation of the transformed function is: ____

Your turn:

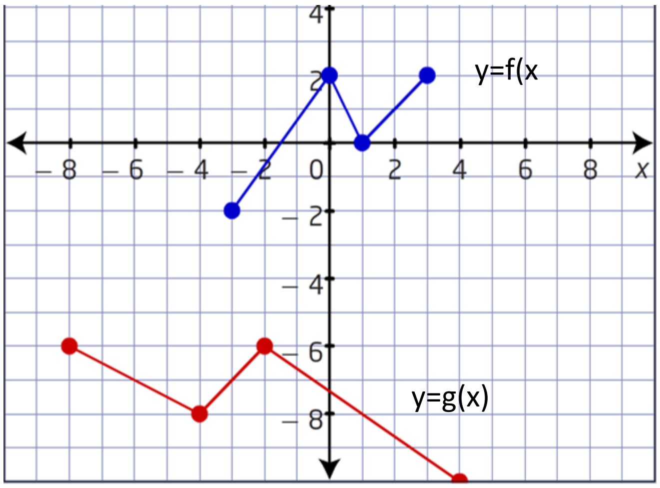
1. The graph of y = f(x) is given below. Sketch the graph of $y=\frac{1}{3}f\left(-\frac{1}{2}(x-4)\right)-3$.



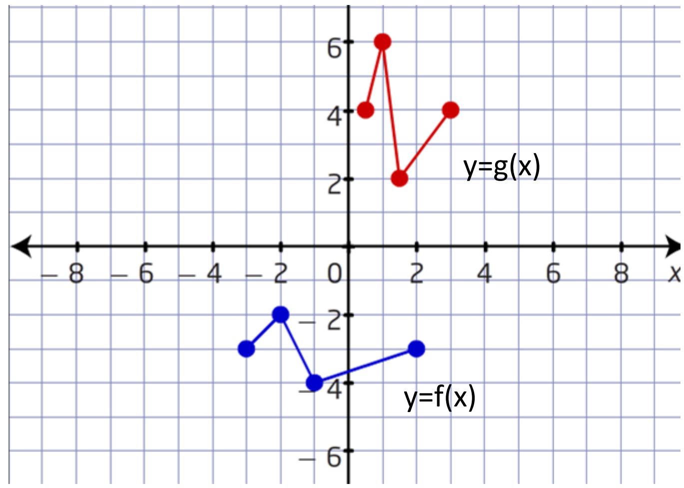
$y=f(x)$		**MAPPING RULE:**	$y=\frac{1}{3}f\left(-\frac{1}{2}(x-4)\right)-3$	
x	y		x	y
−2	3			
0	6			
2	−3			
4	−3			
6	0			

2. The graph of the function $y = g(x)$ represents a transformation of the graph $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x-h)) + k$.

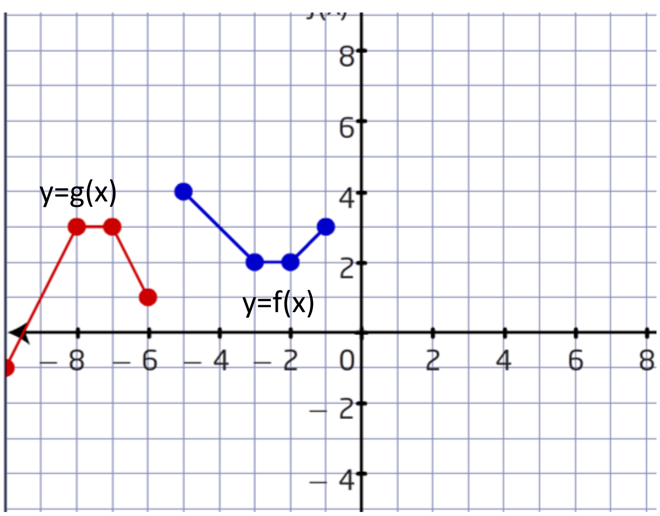
a.



b.



c.



d.