

Proving Conjectures: Deductive Reasoning

Proof – A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

Deductive Reasoning – Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

Example 1: Connecting Conjectures with Reasoning

Use *inductive* reasoning to make a conjecture about the connection between the *sum* of 5 consecutive integers and the *median* of these numbers.

Use *deductive* reasoning to *prove* your conjecture.

Solution:

Inductive reasoning:

$$1. \quad \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{median} = \underline{\hspace{1cm}}$$

$$2. \quad \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{median} = \underline{\hspace{1cm}}$$

$$3. \quad \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{median} = \underline{\hspace{1cm}}$$

$$4. \quad \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{median} = \underline{\hspace{1cm}}$$

Conjecture: Whenever you add 5 consecutive integers, the sum is always _____ the median of these numbers.

Deductive reasoning:

Let *any* five consecutive integers be represented by: x , $x + 1$, $x + 2$, $x + 3$, and $x + 4$

Then, the *sum* of these integers = _____ = _____

The *median* of these integers = _____

5 times the median = _____

Since, in general, the sum of any five consecutive integers is equal to 5 times the median of these numbers, then our conjecture is true.

Example 2: Use Deductive Reasoning to Validate a Conjecture

Use deductive reasoning to prove the conjecture that the sum of any two *odd* integers will always be *even*.

Solution:

Deductive reasoning:

Let *any* two odd integers be represented by _____ and _____.

The *sum* of these integers = _____

= _____

= _____

Since any even number is divisible by 2 and $2(x + y + 1)$ is divisible by 2, then $2(x + y + 1)$ will always be even. The conjecture that the sum of any two odd integers will always be even is true.

Example 3: Use Deductive Reasoning to Validate a Conjecture

Use deductive reasoning to prove the conjecture that the difference between two consecutive perfect squares will always be odd.

Solution:

Deductive reasoning:

Let *any* two consecutive perfect squares be represented by _____ and _____.

The *difference* of these integers = _____

= _____

= _____

= _____

Since $2x + 1$ will always be odd, then the conjecture that “the difference between two consecutive perfect squares will always be odd” is true.

Example 4: Use Deductive Reasoning to Validate a Conjecture

Jared discovered a number trick in a book he was reading:

Choose any number. Double it. Add 6. Double again. Subtract 4. Divide by 4. Subtract 2.

Use inductive reasoning to make a conjecture about the relationship between the number chosen and the final result.

Use deductive reasoning to prove your conjecture.

Solution:

Steps	Inductive Reasoning		
Choose any number			
Double it			
Add 6			
Double again			
Subtract 4			
Divide by 4			
Subtract 2			

Conjecture: _____

Steps	Deductive Reasoning
Choose any number	
Double it	
Add 6	
Double again	
Subtract 4	
Divide by 4	
Subtract 2	

Therefore, the conjecture is true.