

Proofs That Are Not Valid

Invalid Proof – A proof that contains an error in reasoning or that contains invalid assumptions.

Circular Reasoning – An argument that is incorrect because it makes use of the conclusion that is meant to be proved.

Example 1: Use Reasoning to Determine the Validity of a Proof

Jack claims that he can prove that $4 = 3$ using the following argument:

- Suppose that we have values a , b , and c such that $a + b = c$
- Since we can write $a = 4a - 3a$, $b = 4b - 3b$, and $c = 4c - 3c$, then we can write:

$$(4a - 3a) + (4b - 3b) = (4c - 3c)$$

- This can be rearranged as follows:

$$4a + 4b - 4c = 3a + 3b - 3c$$

- Each side can be written in factored form as follows:

$$4(a + b - c) = 3(a + b - c)$$

- Divide both sides by $(a + b - c)$ to obtain:

$$4 = 3$$

Where is the error in Jack's reasoning?

Example 2: Use Reasoning to Determine the Validity of a Proof

Kate has been told that the following number trick always results in a value of 5:

Choose any number. Add 3. Double. Add 4. Divide by 2. Subtract the number that you started with.

Each time Kate tries the trick she ends up with 5, however, her deductive proof does not give the same result:

Steps	Deductive Reasoning
Choose any number.	n
Add 3.	$n + 3$
Double.	$2n + 6$
Add 4.	$2n + 10$
Divide by 2.	$2n + 5$
Subtract the number that you started with.	$n + 5$

Find the error in Kate's proof and correct it.

Example 3: Use Reasoning to Determine the Validity of a Proof

Grant claims that he can prove that $x - 1 = 1 - x$ for all values of x using the following argument:

	Left Side	Right Side
	$x - 1$	$1 - x$
Square both sides	$(x - 1)^2$	$(1 - x)^2$
	$=$	$=$

Since $x^2 - 2x + 1 = 1 - 2x + x^2$, then it follows that $x - 1 = 1 - x$.

Where was the error in Grant's reasoning?

Remember:

A single error in reasoning can destroy a deductive proof. Common errors include:

- A false assumption or generalization
- An error in reasoning, like division by zero
- A calculation error
- Faulty logic

When you are writing and/or checking a mathematical proof, be sure that it is clear and correct!