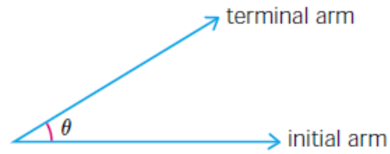
Angles in Standard Position

A **ray** is part of a line. It has one endpoint and continues without end in the other direction.

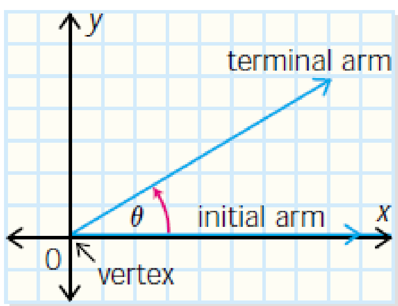
An **angle** is formed by the union of two rays with a common endpoint. It is formed by rotating a given ray about its endpoint to a terminal position.

The original ray is the **initial arm** of the angle and the second ray is the **terminal arm** of the angle. The common endpoint is the **vertex** of the angle.



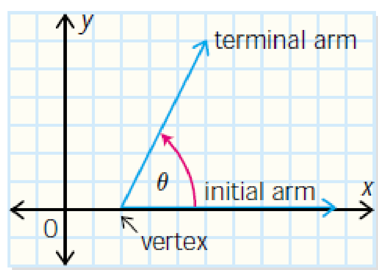
Angles are often named with Greek letters: θ (theta), α (alpha), β (beta)

Standard Form

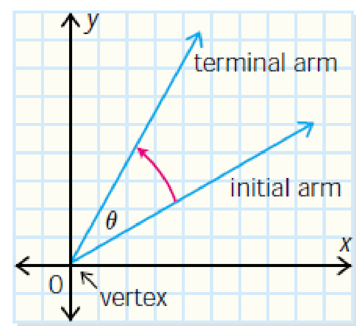


An angle θ is in **standard position** if the vertex of the angle is at the origin and the initial arm lies along the positive x-axis. The terminal arm can lie anywhere along the arc of rotation.

Not Standard Form

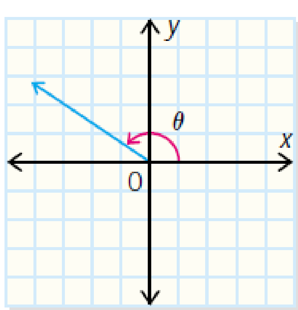


Not Standard Form



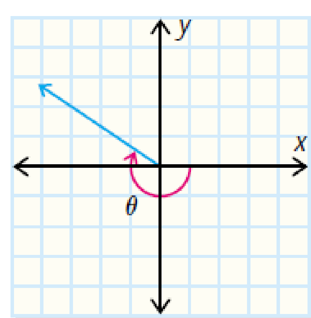
The initial arm of an angle rotates to its terminal position either in a positive, counterclockwise direction or a negative, clockwise direction.

A **positive angle** is formed by the counterclockwise rotation of the terminal arm.



a positive angle

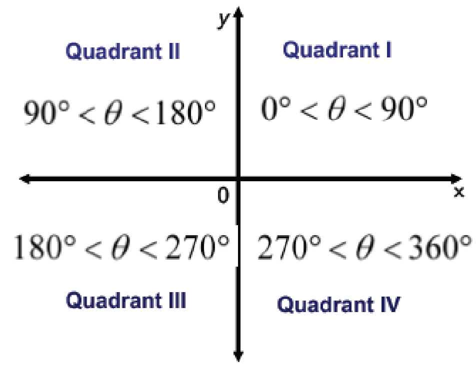
A **negative angle** is formed by the clockwise rotation of the terminal arm.



a negative angle

The Cartesian plane is divided into four quadrants by the x-and y-axes. If θ is a positive angle, then the terminal arm lies in

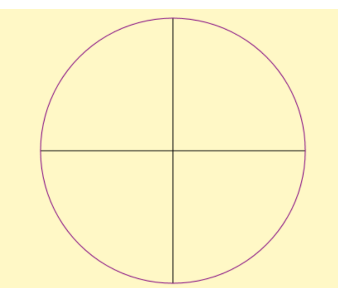
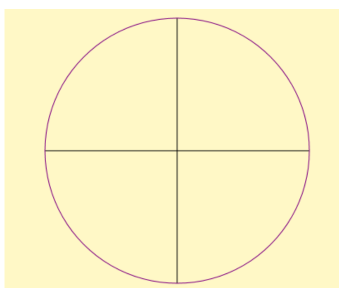
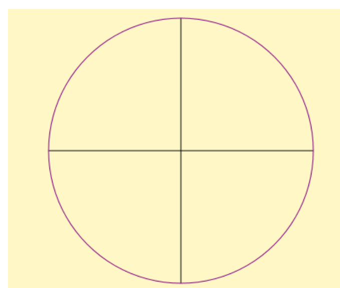
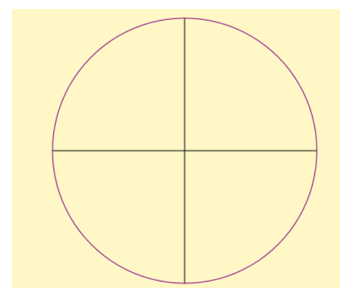
- Quadrant I when $0^\circ < \theta < 90^\circ$
- Quadrant II when $90^\circ < \theta < 180^\circ$
- Quadrant III when $180^\circ < \theta < 270^\circ$
- Quadrant IV when $270^\circ < \theta < 360^\circ$



Example 1: Sketch an Angle in Standard Position, $0^\circ \le \theta \le 360^\circ$

Sketch each angle in standard position. State the quadrant in which the terminal arm lies.

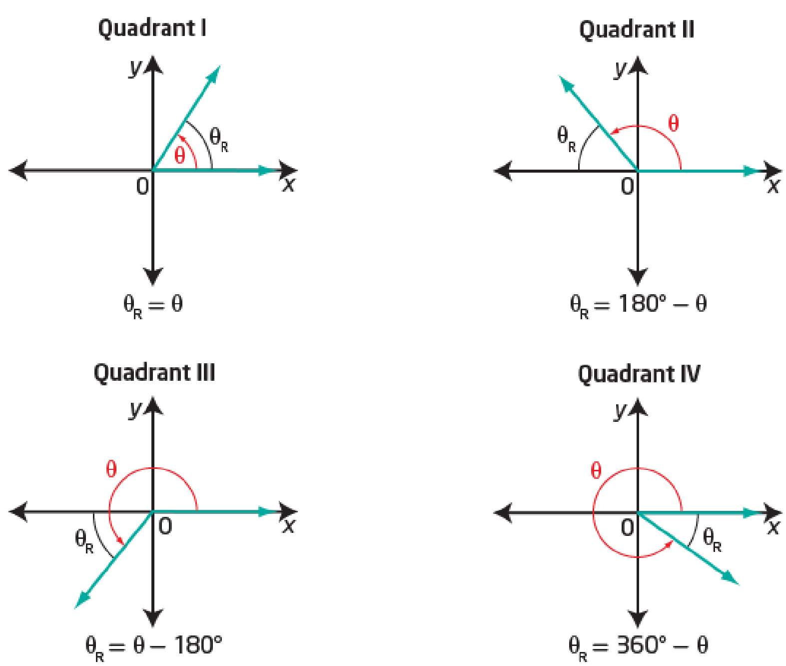
a) 55° b) 322° c) 250° d) 171°

Reference Angles

For each angle in standard position, there is a corresponding acute angle called the **reference angle.** The reference angle is the acute angle formed between the terminal arm and the x-axis. **The reference angle is always positive and between** 0° **and** 90°.

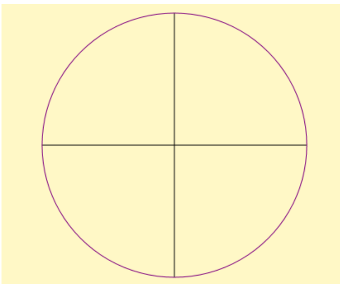
θ_R, the reference angle, is illustrated for angles, θ, in standard position.



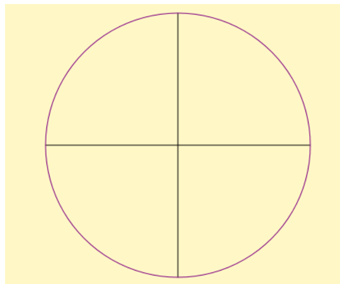
Example 2: Determine a Reference Angle

Determine the reference angle θ_R for each angle θ in standard position. Sketch and label θ and θ_R.

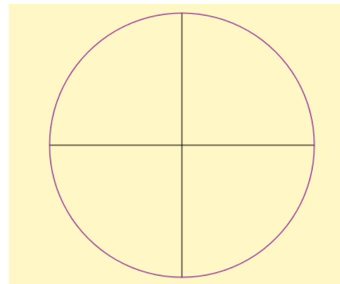
a) $\theta = 315^o$



b) $\theta = 152^o$



c) $\theta = 230^o$



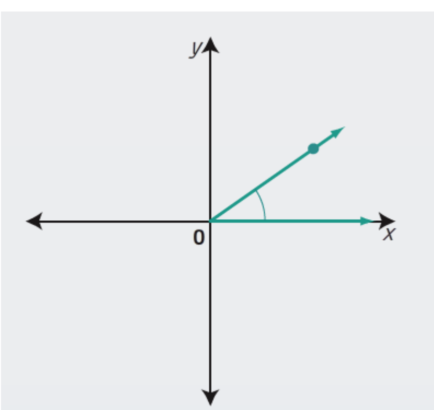
Example 3: Determine the Angle in Standard Position

Determine the measure of the angle in standard position when an angle of $35°$ is reflected in the

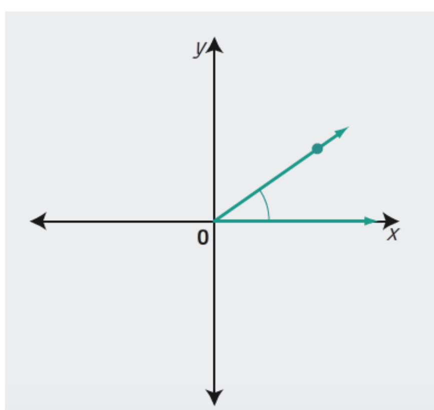
a) y-axis
b) x-axis
c) y-axis and then in the x-axis

Solution:

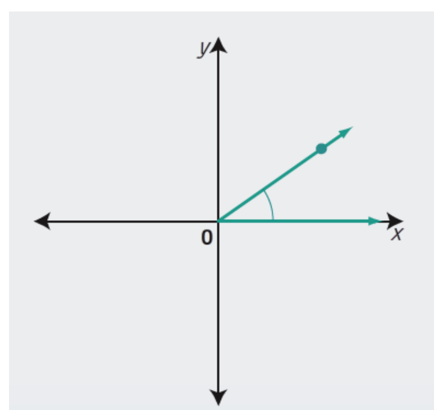
a) Reflect the angle of $35°$ in the y-axis.



b) Reflect the angle of $35°$ in the x-axis.



c) Reflect the angle of $35°$ in the y-axis and then in the x-axis.



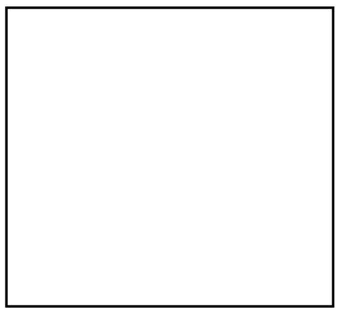
Special Right Triangles

There are two special right triangles for which you can determine the *exact* values of the primary trigonometric ratios.

i) $45° - 45° - 90°$ triangle

Drawing the diagonal of a square with a side length of 1 unit gives a $45° - 45° - 90°$ triangle.

Determine the length of the hypotenuse.



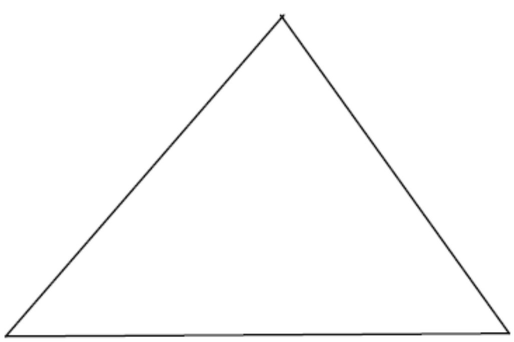
Using your diagram above, complete the following:

$\sin 45° =$ $\cos 45° =$ $\tan 45° =$

ii) $30° - 60° - 90°$ triangle

Drawing an altitude in an equilateral triangle with a side length of 2 units gives a $30° - 60° - 90°$ triangle.

Determine the lengths of the missing sides.



Using your diagram above, complete the following:

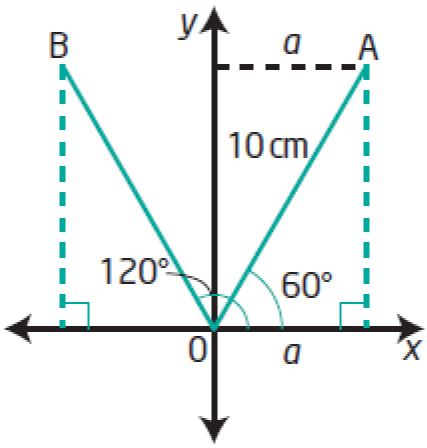
$\sin 60° =$ $\cos 60° =$ $\tan 60° =$

$\sin 30° =$ $\cos 30° =$ $\tan 30° =$

Example 4: Find an Exact Distance

A piano teacher uses a metronome to help piano students keep time. The pendulum arm of the metronome is 10 cm long. For one particular tempo, the setting results in the arm moving back and forth from a start position of $60°$ to $120°$. What horizontal distance does the tip of the arm move in one beat?

Solution:



OA and OB represent the start and end positions, respectively, of the metronome arm for one beat. The tip of the arm moves a horizontal distance equal to a to reach the vertical position from A.

Find the horizontal distance a :

Since the reference angle for 120° is ________, the tip of the metronome arm moves this *same* horizontal distance past the vertical position to reach B.

Therefore, the horizontal distance traveled by the tip of the metronome arm in one beat is ________ cm.