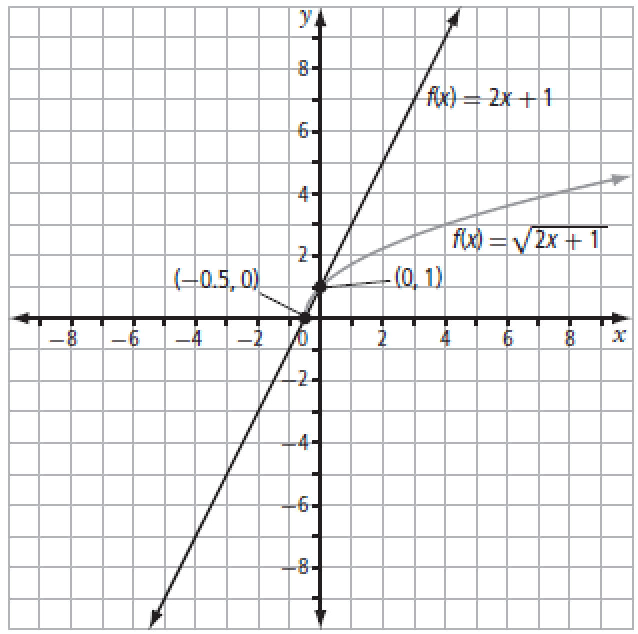
Square Root of a Function

Graphing $y = f(x)$ and $y = \sqrt{f(x)}$



- To graph $y = \sqrt{f(x)}$, you can set up a table of values for the graph of y = f(x). Then, take the square root of the elements in the range, while keeping the elements in the domain the same. The mapping of this transformation would be $(x, y) \rightarrow (x, \sqrt{y})$.

- Graphing $y = \sqrt{f(x)}$ from the graph of $y = f(x)$ is only defined when $f(x) \geq 0$ since you cannot take the square root of negative numbers. When $f(x) < 0$, the graph of $y = \sqrt{f(x)}$ does not exist.

- When graphing $y = \sqrt{f(x)}$, pay attention to the invariant points, which are points that are the same for $y = f(x)$ as they are for $y = \sqrt{f(x)}$. The invariant points are (x, 0) and (x, 1) because if $f(x) = 0$ then $\sqrt{f(x)} = 0$, and if $f(x) = 1$ then $\sqrt{f(x)} = 1$.

Domain and Range of $y = \sqrt{f(x)}$

- The domain of $y = \sqrt{f(x)}$ is any value of x for which $f(x) \geq 0$.
  (Since you cannot take the square root of a negative number.)

- The range is the square root of any positive or zero y-value in $y = f(x)$.

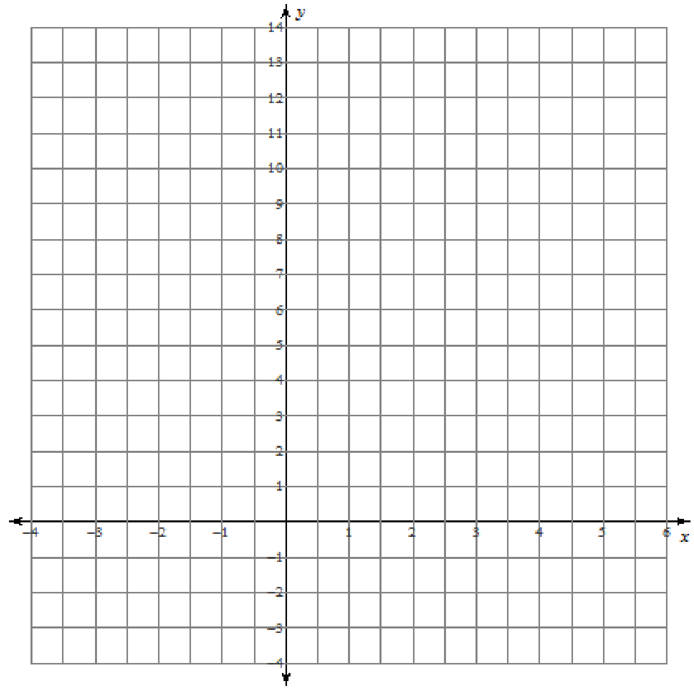
The Graph of $y = \sqrt{f(x)}$

Value of f(x)	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Location of $y = \sqrt{f(x)}$ relative to $y = f(x)$	The graph of $y = \sqrt{f(x)}$ is ***undefined***.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ ***intersect*** on the x-axis.	The graph of $y = \sqrt{f(x)}$ is ***above*** the graph of $y = f(x)$.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ ***intersect***.	The graph of $y = \sqrt{f(x)}$ is ***below*** the graph of $y = f(x)$.

Consider the graphs of $f(x) = 2x + 1$ and $f(x) = \sqrt{2x+1}$ shown above. Note that the graph of $f(x) = \sqrt{2x+1}$ is *undefined* for _________. The graphs of $f(x) = 2x + 1$ and $f(x) = \sqrt{2x+1}$ intersect at _________ and at _________. The graph of $f(x) = \sqrt{2x+1}$ is *above* the graph of $f(x) = 2x + 1$ for _______________, and *below* the graph of $f(x) = 2x + 1$ for _________.

Example 1: Compare Graphs of a Linear Function and the Square Root of the Function

a. Given $f(x) = 4x - 3$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
b. Compare the graphs.



Solution:

a.

x	$f(x) = 4x - 3$	$f(x) = \sqrt{4x - 3}$
0		
0.75		
0.77		
0.8		
1		
2		
3		
4		

b. **Comparison:**

The graphs of $f(x) = 4x - 3$ and $f(x) = \sqrt{4x - 3}$

intersect at __________ and at __________.

These are referred to as __________________ points.

The x-intercept of the graph of $f(x) = 4x - 3$ is also the x-intercept *and* the ______________ point of the graph of the $f(x) = \sqrt{4x - 3}$.

The graph of $f(x) = \sqrt{4x - 3}$ is *above* the graph of $f(x) = 4x - 3$ for ____________________.

The graph of $f(x) = \sqrt{4x - 3}$ is *below* the graph of $f(x) = 4x - 3$ for _______________.

Domain of $f(x) = 4x - 3$: ___________________

Range of $f(x) = 4x - 3$: _____________________

Domain of $f(x) = \sqrt{4x - 3}$: ____________________

Range of $f(x) = \sqrt{4x - 3}$: ____________________

Example 2: Graph the Square Root of a Function from the Graph of the Function and Explore the Domains and Ranges

For each of the following functions, y = f(x), sketch the graph of $y=\sqrt{f(x)}$. Determine the intercepts, domain and range of y=f(x) and $y=\sqrt{f(x)}$.

a. $y=-x-2$ and $y=\sqrt{-x-2}$

b. $y=x^2-6x+13$ and $y=\sqrt{x^2-6x+13}$

c. $y=(x+3)^2-4$ and $y=\sqrt{(x+3)^2-4}$

d. $y=9-x^2$ and $y=\sqrt{9-x^2}$

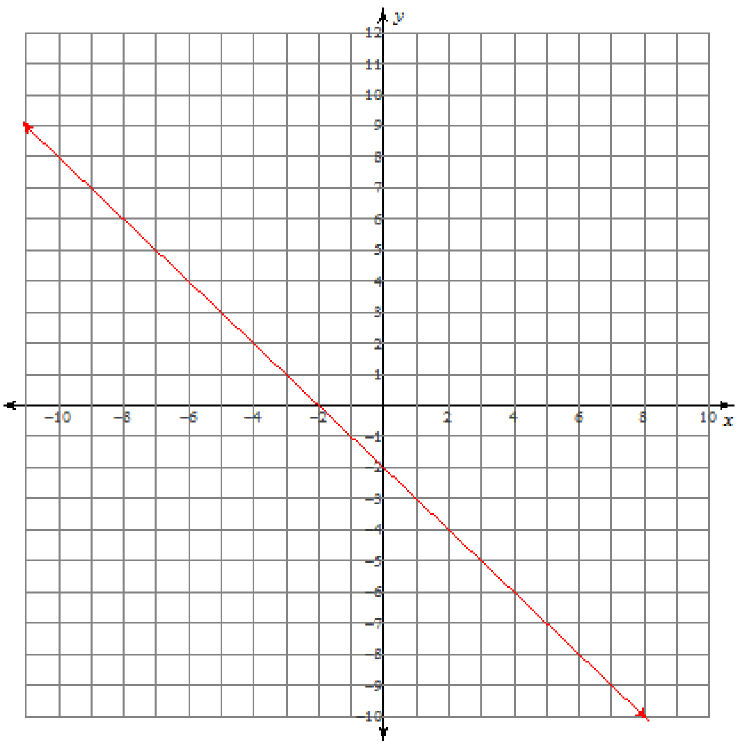
e. $y=-x^2-1$ and $y=\sqrt{-x^2-1}$

Solution:

On the same grid as the graph of y = f(x):

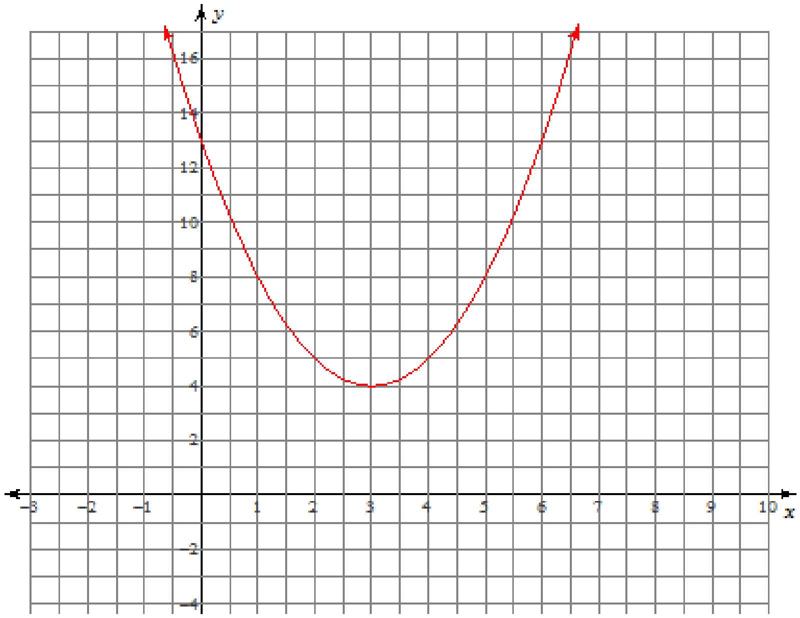
- Plot the invariant points.
- Draw a smooth curve between the invariant points, and *above* the graph of y = f(x).
- Plot a few other points. If possible, choose values of f(x) which have simple square roots.

a. $y=-x-2$ and $y=\sqrt{-x-2}$



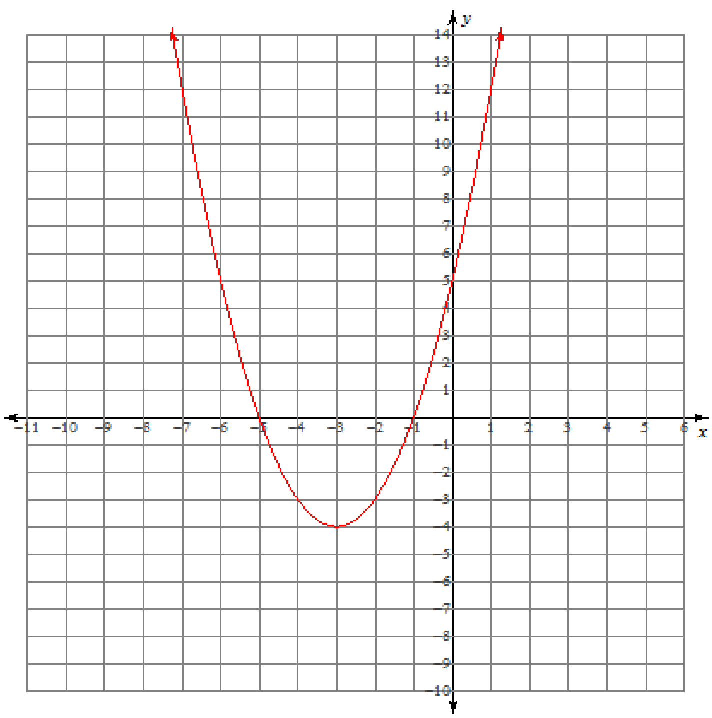
Function	$y=-x-2$	$y=\sqrt{-x-2}$
x-intercept		
y-intercept		
domain		
range		

b. $y = x^2 - 6x + 13$ and $y = \sqrt{x^2 - 6x + 13}$



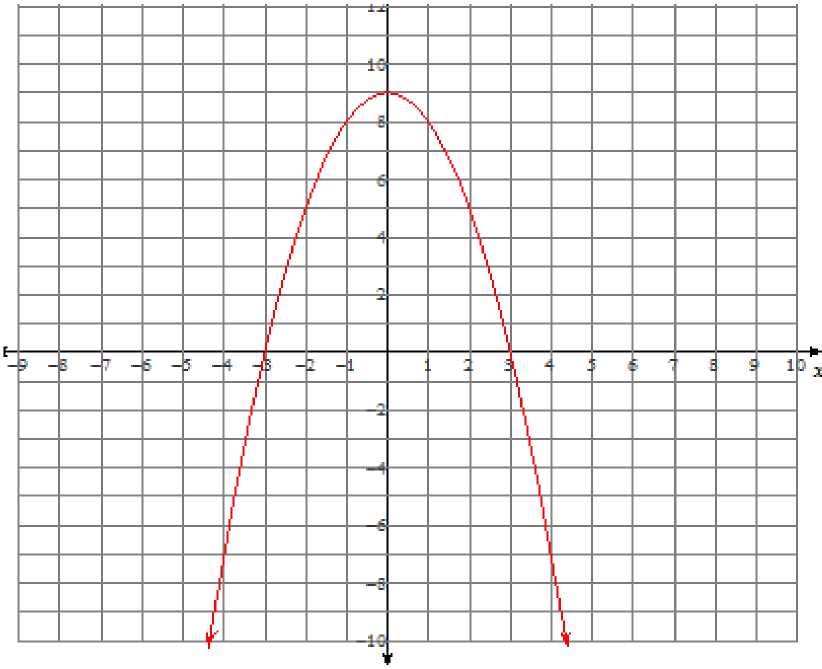
Function	$y = x^2 - 6x + 13$	$y = \sqrt{x^2 - 6x + 13}$
x-intercept		
y-intercept		
domain		
range		

c. $y = (x+3)^2 - 4$ and $y = \sqrt{(x+3)^2 - 4}$



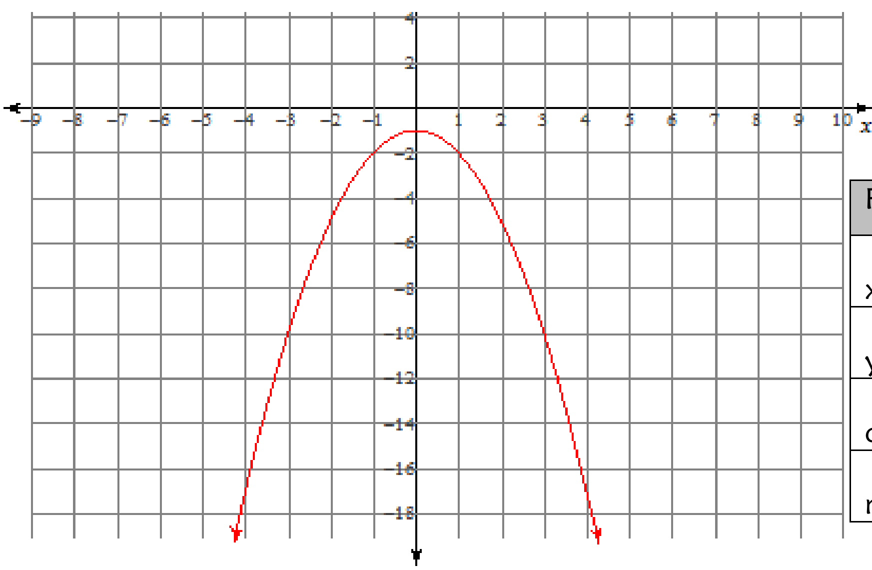
Function	$y = (x+3)^2 - 4$	$y = \sqrt{(x+3)^2 - 4}$
x-intercept		
y-intercept		
domain		
range		

d. $y = 9 - x^2$ and $y = \sqrt{9 - x^2}$



Function	$y = 9 - x^2$	$y = \sqrt{9 - x^2}$
x-intercept		
y-intercept		
domain		
range		

e. $y = -x^2 - 1$ and $y = \sqrt{-x^2 - 1}$



Function	$y = -x^2 - 1$	$y = \sqrt{-x^2 - 1}$
x-intercept		
y-intercept		
domain		
range		

Example 3: Determine Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

For the functions in each pair :

- Determine the coordinates of the invariant points.
- Sketch the graph of $y = f(x)$ using key points (invariant points, vertex, etc.) and use this graph to help you sketch the graph of $y = \sqrt{f(x)}$.
- Determine the domain and range of each function.

a. $y = 4x - 2$, $y = \sqrt{4x - 2}$ b. $y = 12 - 3x^2$, $y = \sqrt{12 - 3x^2}$ c. $y = 0.5x^2 - 5$, $y = \sqrt{0.5x^2 - 5}$

Solution:

a. $y = 4x - 2$ and $y = \sqrt{4x - 2}$

Invariant points:
Sketches:
Domain and Range: Domain of $y = 4x - 2$: ______________ Range of $y = 4x - 2$: ______________ Domain of $y = \sqrt{4x - 2}$: ______________ Range of $y = \sqrt{4x - 2}$: ______________

b. $y = 12 - 3x^2$ and $y = \sqrt{12 - 3x^2}$

Invariant points:

Sketches:

Domain and Range:

Domain of $y = 12 - 3x^2$: ______________________ Range of $y = 12 - 3x^2$: ______________________

Domain of $y = \sqrt{12 - 3x^2}$: ______________________ Range of $y = \sqrt{12 - 3x^2}$: ______________________

c. $y = 0.5x^2 - 5$ and $y = \sqrt{0.5x^2 - 5}$

Invariant points:

Sketches:

Domain and Range:

Domain of $y = 0.5x^2 - 5$: ______________________ Range of $y = 0.5x^2 - 5$: ______________________

Domain of $y = \sqrt{0.5x^2 - 5}$: ______________________ Range of $y = \sqrt{0.5x^2 - 5}$: ______________________