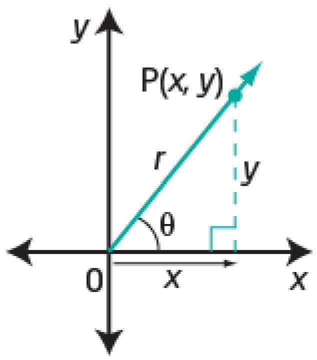
Trigonometric Ratios of Any Angle



Suppose θ is any angle in standard position and P (x,y) is any point on its terminal arm at a distance of r from the origin. The value of r can then be determined using the Pythagorean Theorem:

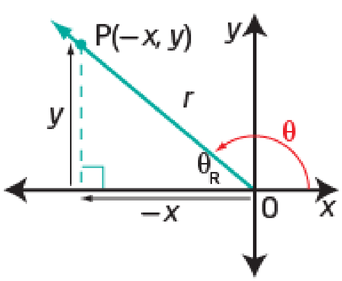
$$r = \sqrt{x^2 + y^2}$$

This reference triangle can be used to determine the primary trigonometric ratios for θ in terms of x, y, and r:

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \qquad \cos\theta = \frac{adj}{hyp} = \frac{x}{r} \qquad \tan\theta = \frac{opp}{adj} = \frac{y}{x}$$

Using the diagrams below, determine the signs of the primary trigonometric ratios in each quadrant.

QUADRANT II



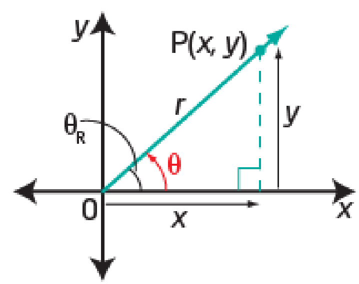
$\theta = 180° - \theta_R$

$\sin\theta = -$

$\cos\theta = -$

$\tan\theta = -$

QUADRANT I



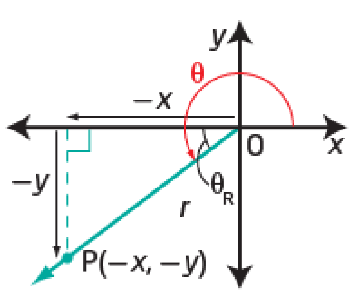
$\theta = \theta_R$

$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

$\cos\theta = -$

$\tan\theta = -$

QUADRANT III



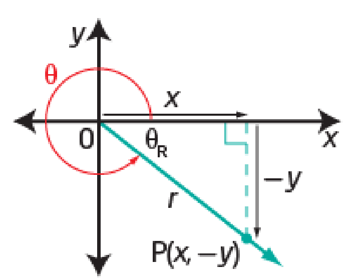
$\theta = 180° + \theta_R$

$\sin\theta = -$

$\cos\theta = -$

$\tan\theta = -$

QUADRANT IV



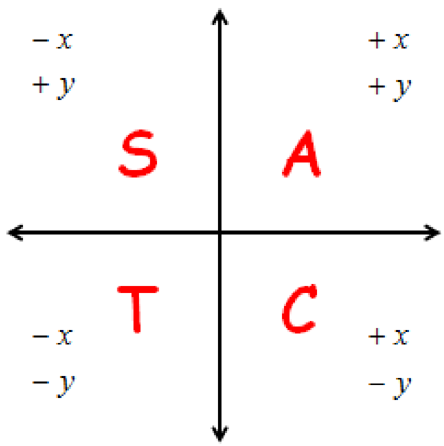
$\theta = 360° - \theta_R$

$\sin\theta = -$

$\cos\theta = -$

$\tan\theta = -$

This information can be summarized in the CAST rule. This acronym lets us know which primary trigonometric ratios are positive in each of the quadrants.



A - ______________________________

S - ______________________________

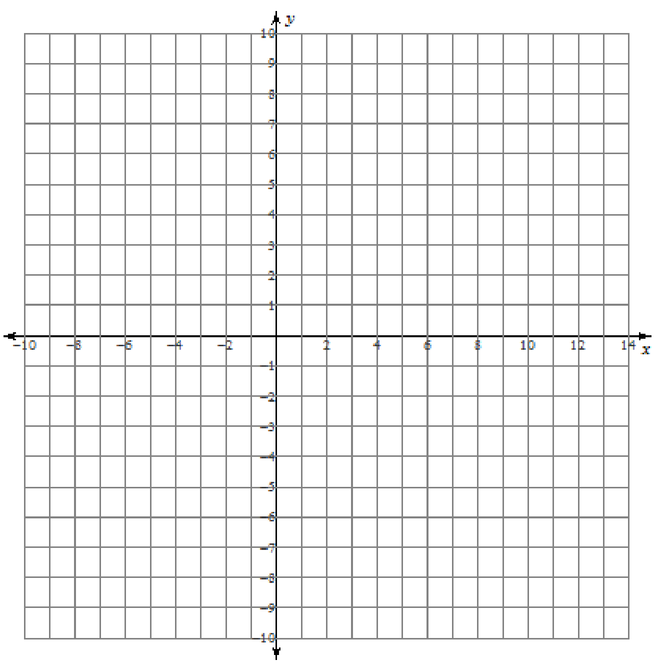
T - ______________________________

C - ______________________________

Example 1: Write Trigonometric Ratios for Angles in Any Quadrant

The point P (12, –9) lies on the terminal arm of a positive angle θ in standard position. Determine the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

Solution:



Plot the point P (12, –9) on the Cartesian plane.

Draw a line segment from the origin to point P to represent the terminal arm of the angle.

Label the angle θ and the reference angle θ_R

Calculate **r** using the Pythagorean Theorem.

Substitute the values of x, y and r into the formulas to calculate the three ratios.

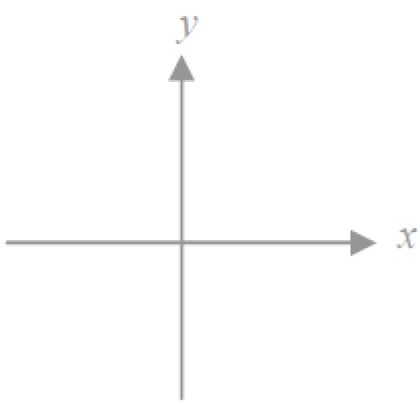
$\sin\theta = \frac{\quad}{\quad}$ $\cos\theta = \frac{\quad}{\quad}$ $\tan\theta = \frac{\quad}{\quad}$

Example 2: Determine the Exact Value of a Trigonometric Ratio

Determine the exact values of the sine, cosine and tangent ratios for $\theta = 120°$.

Solution:

Sketch the angle in standard position. Determine the measure of the reference angle, θ_R. Construct the appropriate special triangle and label the sides accordingly. Determine the trigonometric ratios.

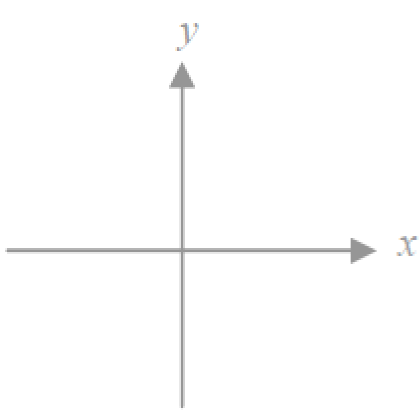


Example 3: Determine Trigonometric Ratios

Suppose θ is an angle in standard position with its terminal arm in quadrant III, and $\sin\theta = -\frac{2}{3}$. What are the exact values of $\cos\theta$ and $\tan\theta$?

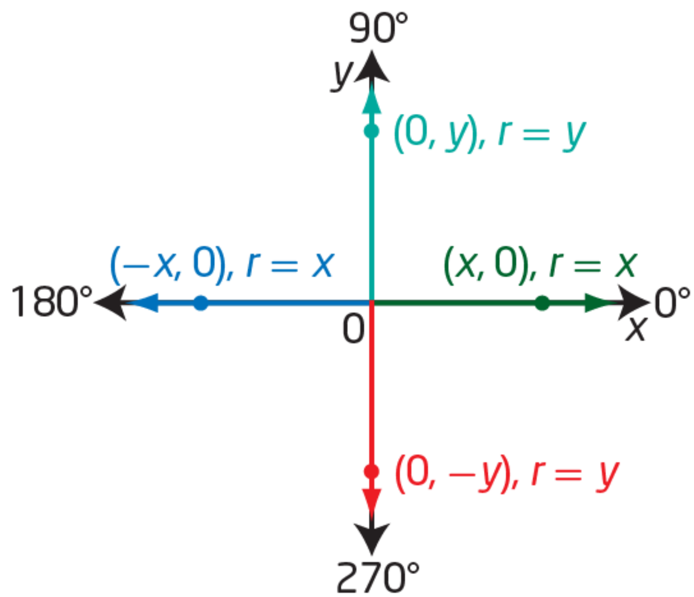
Solution:

Sketch a diagram. Use the definition of sine to find the values of y and r. Remember to consider the quadrant that the terminal arm lies in when determining the value of y. Note that r is always positive.



Use the Pythagorean Theorem to find x, then determine the exact values of $\cos\theta$ and $\tan\theta$.

Quadrantal Angles



A **quadrantal angle** is an angle in standard position whose terminal arm lies on one of the axes.

Examples of quadrantal angles include $0°,\ 90°,\ 180°,\ 270°,\ 360°$.

Example 4: Determine Trigonometric Ratios of Quadrantal Angles

Determine the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the quadrantal angles $0°,\ 90°,\ 180°,\ 270°$.

	$0°$	$90°$	$180°$	$270°$
$\sin\theta = \frac{y}{r}$				
$\cos\theta = \frac{x}{r}$				
$\tan\theta = \frac{y}{x}$				

Example 5: Solve for an Angle Given its Exact Sine, Cosine or Tangent Value

Solve for θ:

a. $\sin\theta = \frac{1}{\sqrt{2}},\ 0 \le \theta \le 360^o$ b. $\cos\theta = -\frac{1}{2},\ 0 \le \theta \le 360^o$

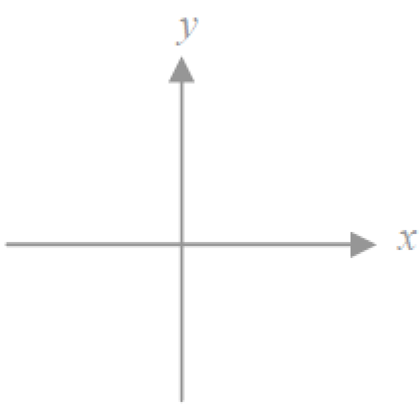
Solution:

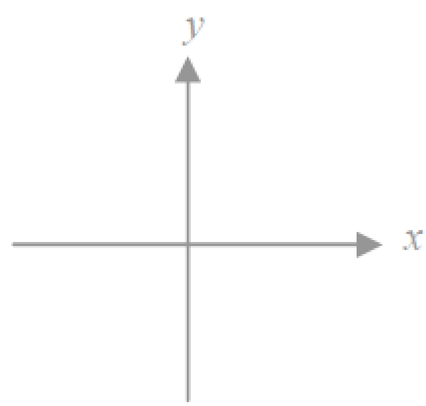
a. $\sin\theta = \frac{1}{\sqrt{2}},\ 0 \le \theta \le 360^o$

What reference angle has a sine value of $\frac{1}{\sqrt{2}}$? (Note: Think of your special triangles) ______________

Since the ratio for $\sin\theta$ is *positive*, the terminal arm lies in either quadrant ________ or ________.

Thus the two angles for which $\sin\theta = \frac{1}{\sqrt{2}},\ 0 \le \theta \le 360^o$ are __________ and __________.



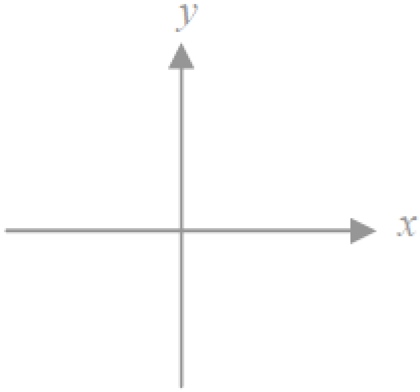


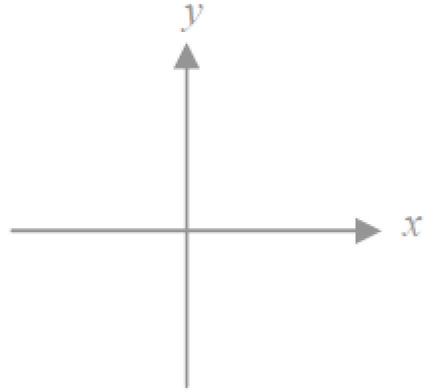
b. $\cos\theta = -\frac{1}{2},\ 0 \le \theta \le 360^o$

What reference angle has a cosine value of $\frac{1}{2}$? ____________

Since the ratio for $\cos\theta$ is *negative*, the terminal arm lies in either quadrant ________ or ________.

Thus the two angles for which $\cos\theta = -\frac{1}{2},\ 0 \le \theta \le 360^o$ are __________ and __________.





Example 6: Solving an Angle Given its Approximate Sine, Cosine or Tangent Value

Solve for θ to the nearest tenth of a degree:

$\tan\theta = -0.8891$, $0^o \le \theta \le 360^o$

Solution:

The tangent ratio is negative, so the angles in standard position lie in quadrant _______ and quadrant _______.

Use a calculator to determine the reference angle that has $\tan\theta_R = 0.8891$.

$\theta_R =$ ________

With a reference angle of __________, the measures of θ are as follows:

In quadrant II:

$\theta =$

$\theta =$

In quadrant IV:

$\theta =$

$\theta =$