

Solving Radical Equations Algebraically & Graphically

Strategies for Solving Graphically

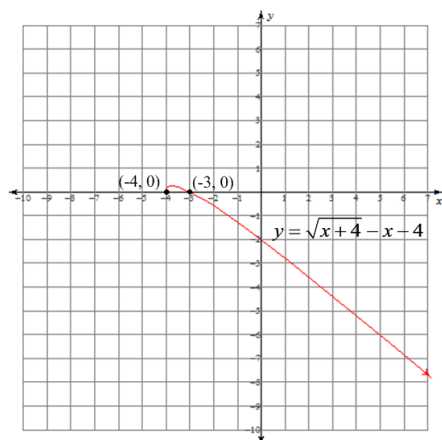
Method 1: Use a Single Function

Rearrange the radical equation so that one side is equal to zero.
Graph the corresponding function and find the x-intercepts of the graph.

Example: Solve $2 + \sqrt{x+4} = x+6$

Rearrange: $\sqrt{x+4} - x - 4 = 0$

Graph: $y = \sqrt{x+4} - x - 4$



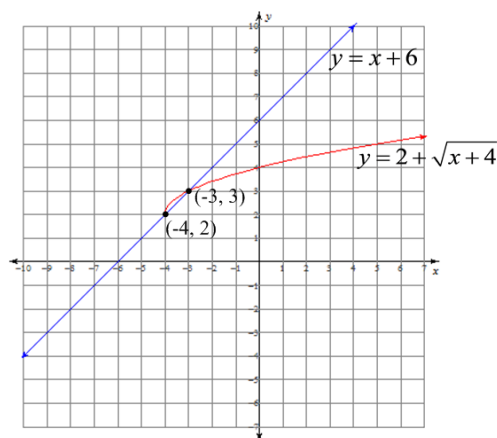
Solutions: $x = -3$ or $x = -4$

Method 2: Use a System of Two Functions

Express each side of the equation as a function. Graph these functions and determine the value of x at the point(s) of intersection.

Example: Solve $2 + \sqrt{x+4} = x+6$

Graph: $y = 2 + \sqrt{x+4}$ and $y = x+6$



Solutions: $x = -3$ or $x = -4$

Strategy for Solving Algebraically

Step 1: Isolate the radical.

Step 2: Square both sides of the equation to eliminate the radical.

Step 3: Continue to solve for x .

Step 4: Check for extraneous solution(s).

Example: Solve $2 + \sqrt{x+4} = x+6$

$$2 + \sqrt{x+4} = x+6$$

$$\sqrt{x+4} = x+4$$

$$x+4 = x^2 + 8x + 16$$

$$0 = x^2 + 7x + 12$$

$$0 = (x+3)(x+4)$$

$$x = -3 \text{ or } x = -4$$

Check $x = -3$:

$$LHS = 2 + \sqrt{-3+4} = 2 + \sqrt{1} = 3$$

$$RHS = -3 + 6 = 3$$

$$LHS = RHS$$

Check $x = -4$:

$$LHS = 2 + \sqrt{-4+4} = 2 + 0 = 2$$

$$RHS = -4 + 6 = 2$$

$$LHS = RHS$$

Example 1: Relate Roots and x-Intercepts

For the radical equation $2\sqrt{x-4}-3=0$:

- Algebraically determine the root(s).
- Graph the corresponding function (method 1) and determine the x-intercepts.
- Describe the connection between the root(s) of the equation and the x-intercept(s) of the graph of the corresponding function.

Solution:

[illegible]

The root(s), or solution(s), of a radical equation are equal to the _____ of the graph of the corresponding function.

Example 2: Solve a Radical Equation Involving an Extraneous Root

For the equation $\sqrt{x+5} = x+3$:

- Algebraically determine the root(s).
- Graph the corresponding functions (method 2) and determine the point(s) of intersection.
- Describe the connection between the root(s) of the equation to the point(s) of intersection of the two functions.

Solution:

[illegible]

The two functions intersect at the point _____. The value of x at this point, _____, is the solution to the equation.