Specialized Factoring Techniques

Some polynomials, when written in a specific form, can be factored easily. It is important to be able to recognize these specialized factoring techniques.

The Sum or Difference of Cubes

These cubic expressions are binomials in the form ax^3+d^3 or ax^3-d^3. The factors are found as follows:
$(ax)^3+d^3=(ax+d)(a^2x^2-adx+d^2)$
$(ax)^3-d^3=(ax-d)(a^2x^2+adx+d^2)$

Example 1: Factoring a Sum or Difference of Cubes

Factor completely:

x^3+27	x^3-8	$64x^3+125$	$27x^3-1$	$3x^4-192x$

Quartic Expressions Factored as Quadratics

Quartic expressions in the form ax^4+bx^2+c can be expressed in quadratic form as am^2+bm+c, where $m=x^2$. If the quadratic expression can be factored, then the quartic expression can be factored in the same way.

Example 2: Factoring Quartic Expressions as Quadratics

Factor completely:

x^4-5x^2+4	$4x^4-37x^2+9$

Grouping to Find a Common Factor

A common factor can sometimes be found for specific groups of terms in a polynomial expression. The expression is written in the necessary order and each group of terms is then factored, leaving a common factor in brackets, which in turn is factored out.

Example 3: Grouping to Find a Common Factor

Factor completely:

$x^3 - 2x^2 - 16x + 32$	$x^5 - 5x^4 - 10x^3 + 50x^2 + 9x - 45$	$8x^5 - 40x^4 + 32x^3 - x^2 + 5x - 4$

Grouping to Get a Difference of Squares

If a polynomial expression can be grouped in the form $(x+m)^2 - n^2$, then it can be factored as a difference of squares.

Example 4: Grouping to Get a Difference of Squares

Factor completely:

$x^4 + 5x^2 + 9$	$x^4 - 6x^2 + 1$	$x^4 - 19x^2 + 9$

Factor completely:

a. $x^3 - 64$
b. $5x^4 - 135x$
c. $x^3 - 5x^2 - 9x + 45$
d. $x^4 - 13x^2 + 36$
e. $x^4 + 8x^3 - 2x^2 - 16x$
f. $2x^6 - 20x^4 - 16x^3 + 160x$
g. $x^4 - 16x^2 + 36$
h. $4x^3 - 8x^2 - 25x + 50$
i. $36x^2 - x^4 - 100$
j. $x^4 - x^3 - x + 1$
k. $x^5 - 2x^4 - x^3 + 2x^2 - 12x + 24$
l. $5x^6 + 15x^5 - 50x^4 - 80x^2 - 240x + 800$
m. $x^4 - 6x^2 + 25$
n. $250x^4 - 16x$
o. $x^4 - 11x^2 + 49$
p. $4x^4 + 15x^2 - 4$

Answers:

$a.\ (x-4)(x^2+4x+16)$

$b.\ (5x)(x-3)(x^2+3x+9)$

$c.\ (x-5)(x-3)(x+3)$

$d.\ (x-3)(x+3)(x-2)(x+2)$

$e.\ (x)(x^2-2)(x+8)$

$f.\ (2x)(x-2)(x^2+2x+4)(x^2-10)$

$g.\ (x^2+2x-6)(x^2-2x-6)$

$h.\ (x-2)(2x-5)(2x+5)$

$i.\ -(x^2+4x-10)(x^2-4x-10)$

$j.\ (x-1)(x-1)(x^2+x+1)$

$k.\ (x-2)(x-2)(x+2)(x^2+3)$

$l.\ 5(x-2)(x-2)(x+2)(x^2+4)(x+5)$

$m.\ (x^2-4x+5)(x^2+4x+5)$

$n.\ 2x(5x-2)(25x^2+10x+4)$

$o.\ (x^2+5x+7)(x^2-5x+7)$

$p.\ (2x-1)(2x+1)(x^2+4)$