Characteristics of Polynomial Functions

What is a polynomial function?

A polynomial function has the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$, where

- n is a whole number
- x is a variable
- the coefficients a_n to a_0 are real numbers
- the degree of the polynomial is n, the exponent of the greatest power of x
- a_n is the leading coefficient; the coefficient of the greatest power of x
- the constant term is a_0

degree of the polynomial

$$\Downarrow$$

$$f(x) = -4x^3 - 5x^2 + 6x + 7$$

$$\Uparrow \qquad \Uparrow$$

leading coefficient constant term

Example 1: Identify Polynomial Functions

Which of the following functions are polynomials? Justify your answer. State the degree, the leading coefficient, and the constant term of each *polynomial* function.

a) $g(x) = 4^x$

b) $h(x) = 6 - 5x^3 + 4x^2$

c) $n(x) = \sqrt[3]{x} - 2$

d) $f(x) = x^{-2} + 3$

e) $m(x) = 3x^4 + 2x^3 - 6x^2 + 5x - 1$

Solution:

	Type of Function	Degree	Leading Coefficient	Constant Term
$g(x) = 4^x$				
$h(x) = 6 - 5x^3 + 4x^2$				
$n(x) = \sqrt[3]{x} - 2$				
$f(x) = x^{-2} + 3$				
$m(x) = 3x^4 + 2x^3 - 6x^2 + 5x - 1$				

Investigating Characteristics of Polynomial Functions

Graph each function given below on a different set of coordinate axes using graphing technology. Sketch the results on the coordinate plane provided and complete the table associated with each set of functions.

End Behaviour: The behaviour of the y-values of a function as $|x|$ becomes very large

CONSTANT FUNCTION

	Example 1	Example 2
Equation of Function	y = 4	y = –3
Degree 0	Even degree	
Sketch of Graph		
Number of x-intercepts (for $y \neq 0$)		
End Behaviour		

LINEAR FUNCTION

	Example 1	Example 2	Example 3
Equation of Function	y = x	y = x + 2	y = –3x
Degree 1	Odd degree		
Sketch of Graph			
Number of x-intercepts			
End Behaviour			
Leading Coefficient			
Constant Term			

QUADRATIC FUNCTION

	Example 1	Example 2	Example 3	Example 4
Equation of Function	$y = x^2$	$y = x^2 - 2x + 6$	$y = x^2 + 7x + 10$	$y = -x^2 + 6x - 9$
Degree 2	Even degree			
Sketch of Graph				
Number of x-intercepts				
End Behaviour				
Leading Coefficient				
Constant Term				

CUBIC FUNCTION

	Example 1	Example 2	Example 3	Example 4
Equation of Function	$y = x^3$	$y = x^3 - 4x^2 + x + 6$	$y = x^3 - x^2 - 2x - 12$	$y = -x^3 - x^2 + 5x - 3$
Degree 3	Odd degree			
Sketch of Graph				
Number of x-intercepts				
End Behaviour				
Leading Coefficient				
Constant Term				

QUARTIC FUNCTION

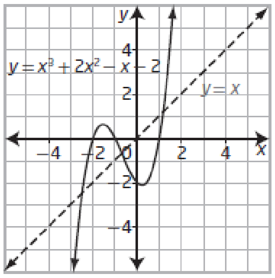
	Example 1	Example 2	Example 3	Example 4
Equation of Function	$y = x^4$	$y = x^4 - x^3 - 7x^2 + x + 6$	$y = x^4 + 5x^3 + 6x^2 - 4x - 8$	$y = -x^4 + 8x^3 - 15x^2 - 8x + 16$
Degree 4	Even degree			
Sketch of Graph				
Number of x-intercepts				
End Behaviour				
Leading Coefficient				
Constant Term				

QUINTIC FUNCTION

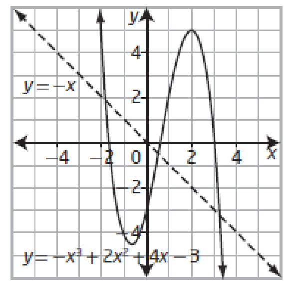
	Example 1	Example 2	Example 3	Example 4
Equation of Function	$y = x^5$	$y = x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4$	$y = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$	$y = -x^5 + 4x^4 - x^3 - 10x^2 + 4x + 8$
Degree 5	Odd degree			
Sketch of Graph				
Number of x-intercepts				
End Behaviour				
Leading Coefficient				
Constant Term				

Conclusions:

Graphs of Odd Degree Polynomial Functions



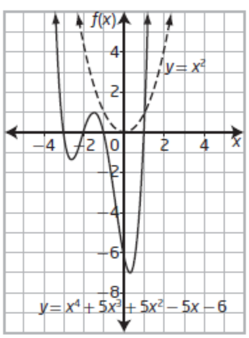
- Extend from quadrant ______ to quadrant ______ when the leading coefficient is ____________________.



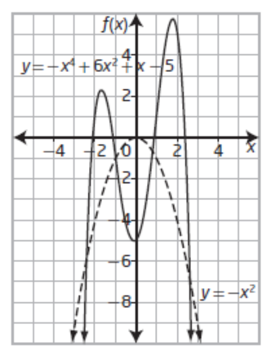
- Extend from quadrant ______ to quadrant ______ when the leading coefficient is ____________________.

- Have a least ______ x-intercept to a maximum of ______ x-intercepts, where n is the degree of the function.
- Have y-intercept a_o, the _______________ term of the function.
- Have __________ maximum or minimum values.
- Have domain _______________ and range _______________.

Graphs of Even Degree Polynomial Functions



- Extend from quadrant ______ to quadrant ______ when the leading coefficient is ____________________. (opens ____________________)



- Extend from quadrant ______ to quadrant ______ when the leading coefficient is ____________________. (opens ____________________)

- Have from ______ to a maximum of ______ x-intercepts, where n is the degree of the function .
- Have y-intercept a_o, the _______________ term of the function.
- Has a __________ or __________ value.
- Have domain _______________; the range depends on the __________ or __________ value of the function.

Example 2: Match a Polynomial Function With Its Graph

For each polynomial function, identify the following characteristics:

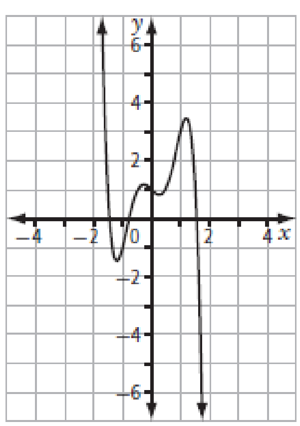
- the type of function and whether it is of even or odd degree
- the end behavior of the graph of the function
- the possible number of x-intercepts
- whether the graph has a maximum or minimum value
- the y-intercept

Then, match each function to its corresponding graph.

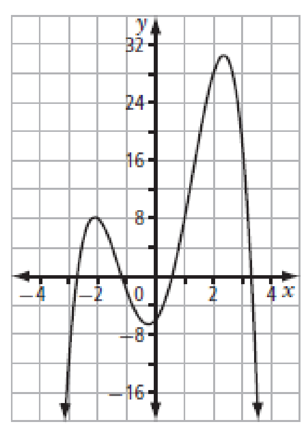
a. $f(x) = 2x^3 - 4x^2 + x + 2$
b. $g(x) = -x^4 + 10x^2 + 5x - 6$
c. $h(x) = -2x^5 + 5x^3 - x + 1$
d. $p(x) = x^4 - 5x^3 + 16$

Function	Type / Degree	End Behaviour	Possible # of x-intercepts	Max or Min Value	y-intercept	Matching Graph
a						
b						
c						
d						

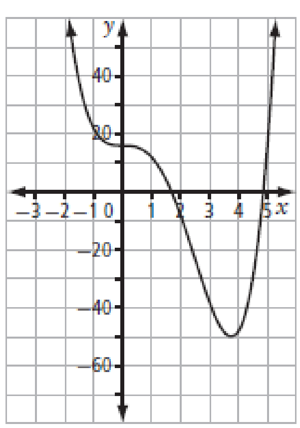
A



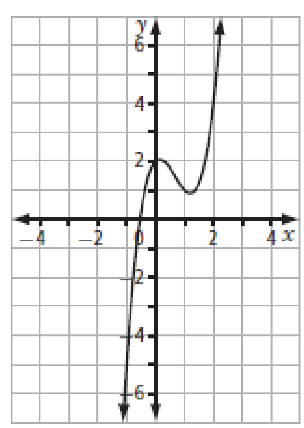
B



C



D



Example 3: Application of a Polynomial Function

An antibacterial spray is tested on a bacterial culture. The population, P, of bacteria t minutes after the spray is applied is modeled by the function $P(t) = -2t^3 - 2t^2 + 3t + 800$.

a. What is the population of the bacteria 3 minutes after the spray is applied?

$P(t) = -2t^3 - 2t^2 + 3t + 800$

b. How many bacteria were in the culture just before the spray was applied?

$P(t) = -2t^3 - 2t^2 + 3t + 800$

c. What is the population of the bacteria 8 minutes after the spray is applied? Why is this not realistic for this situation?

$P(t) = -2t^3 - 2t^2 + 3t + 800$