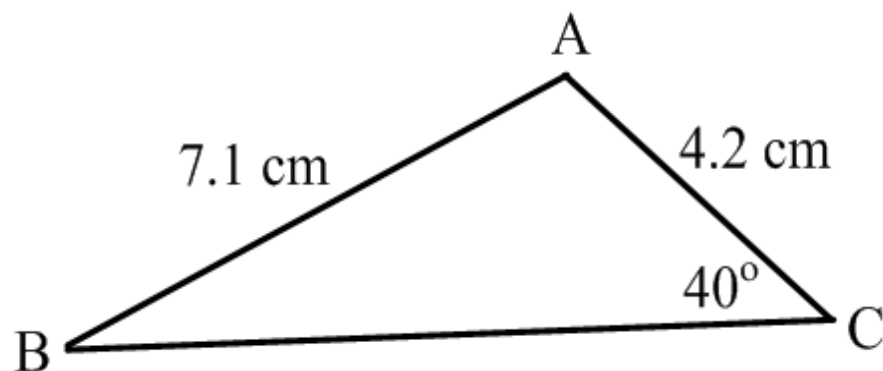


Proving and Applying the Sine Law

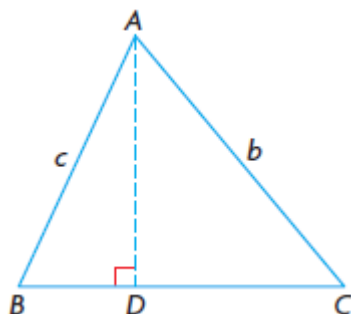
Suppose that you are asked to calculate the measure of $\angle B$ in the *oblique* triangle shown. A *primary trigonometric ratio* cannot be used to determine the measure of $\angle B$ *directly*. Why?



Show how you could determine the measure of $\angle B$ *indirectly* by first adding a line segment from vertex A representing the height of the triangle. Calculate this height by using a primary trigonometric ratio. Use the two *right* triangles you created to determine the measure of $\angle B$.

PROVING AND APPLYING THE SINE LAW

There is a side-angle relationship in oblique triangles known as the **Sine Law** that can be used to determine measures of unknown angles and sides *directly*, rather than having to create right angle triangles and solve for these values *indirectly*. Consider the following derivation of the Sine Law:



In $\triangle ABD$,

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AD}{c}$$

$$c \sin B = AD$$

In $\triangle ACD$,

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

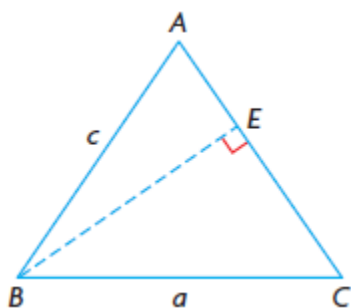
$$\sin C = \frac{AD}{b}$$

$$b \sin C = AD$$

$$c \sin B = b \sin C$$

$$\frac{c \sin B}{\sin C} = b$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$



In $\triangle ABE$,

$$\sin A = \frac{BE}{c}$$

$$c \sin A = BE$$

In $\triangle CBE$,

$$\sin C = \frac{BE}{a}$$

$$a \sin C = BE$$

$$c \sin A = a \sin C$$

$$c = \frac{a \sin C}{\sin A}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

SINE LAW: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

OR

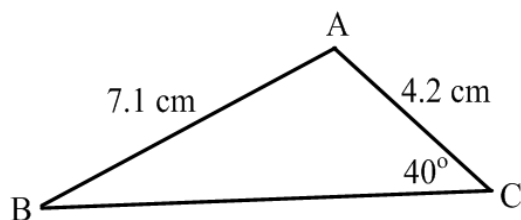
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1: Use the Sine Law to Determine a Side Length

In $\triangle ABC$, $a = 15.0$ m, $\angle B = 65^\circ$, and $\angle C = 35^\circ$. Use the Sine Law to determine the length of *side c* to the nearest tenth of a metre.

Example 2: Use the Sine Law to Determine an Angle Measure

Use the Sine Law to calculate the measure of $\angle B$ in the oblique triangle we worked with before.

**Example 3: Solving a Problem Using the Sine Law**

Maggie uses a winch and chains attached to the ceiling, as shown in the diagram, to lift engines in her garage. Determine the angle that each chain makes with the ceiling.

