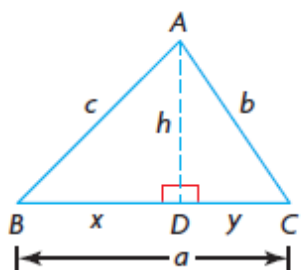


Proving and Applying the Cosine Law

The Sine Law cannot always help you determine unknown angle measures or side lengths. Another relationship known as the **Cosine Law** may be needed. Consider the following derivation of the Cosine Law:



Step 1

I drew an acute triangle ABC . Then I drew the height from A to BC and labelled the intersection point as point D . I labelled this line segment h . I labelled BD as x and DC as y .

$$h^2 = c^2 - x^2$$

$$h^2 = b^2 - y^2$$

Step 2

I wrote two different expressions for h^2 .

$$c^2 - x^2 = b^2 - y^2$$

$$c^2 = x^2 + b^2 - y^2$$

Step 3

I set the two expressions equal to each other and solved for c^2 .

$$x = a - y, \text{ so}$$

$$c^2 = (a - y)^2 + b^2 - y^2$$

$$c^2 = a^2 - 2ay + y^2 + b^2 - y^2$$

$$c^2 = a^2 + b^2 - 2ay$$

Step 4

I wrote an equivalent equation that only used the variable y and simplified.

$$\cos C = \frac{y}{b}, \text{ so}$$

$$b \cos C = y$$

Step 5

I determined an equivalent expression for y .

$$c^2 = a^2 + b^2 - 2ay$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Step 6

I substituted the expression $b \cos C$ for y in my equation.

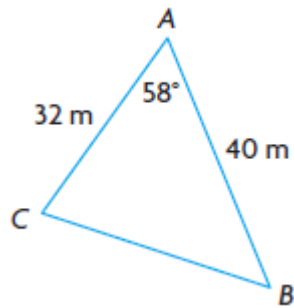
COSINE LAW: $c^2 = a^2 + b^2 - 2ab \cos C$

OR

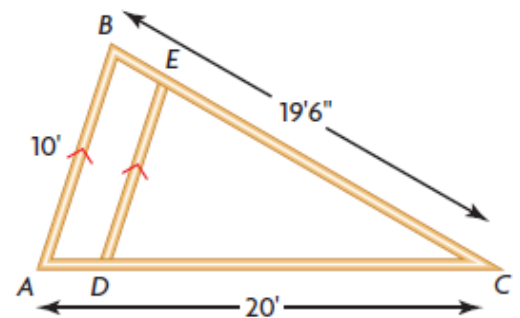
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 1: Use the Cosine Law to Determine a Side Length

Determine the length of CB to the nearest metre.

**Example 2: Use the Cosine Law to Determine an Angle Measure**

The diagram at the right shows the plan for a roof, with support beam DE parallel to AB . The local building code requires the angle formed at the peak of a roof to fall within a range of 70° to 80° so that snow and ice will not build up. Will this plan pass the local building code?



Example 3: Solving a Problem Using the Cosine Law

A pilot delivers supplies to a remote camp by flying 255 km in the direction $N52^\circ E$. While at the camp, the pilot receives a radio message to pick up a passenger at a village before returning home. The village is 85 km $S21^\circ E$ from the camp. What is the total distance, to the nearest kilometer, that the pilot will have flown by the time she returns to her starting point?