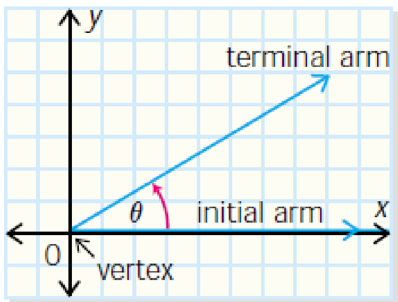
Angles and Angle Measure

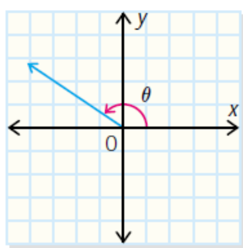
Standard Form



An angle θ is in **standard position** if the vertex of the angle is at the origin and the initial arm lies along the positive x-axis. The terminal arm can lie anywhere along the arc of rotation.

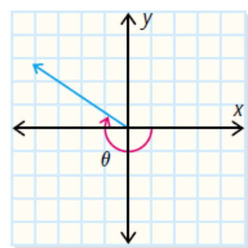
The initial arm of an angle rotates to its terminal position, either in a positive, counterclockwise direction or a negative, clockwise direction.

A **positive angle** is formed by the counterclockwise rotation of the terminal arm.



a positive angle

A **negative angle** is formed by the clockwise rotation of the terminal arm.



a negative angle

DEGREES AND RADIANS

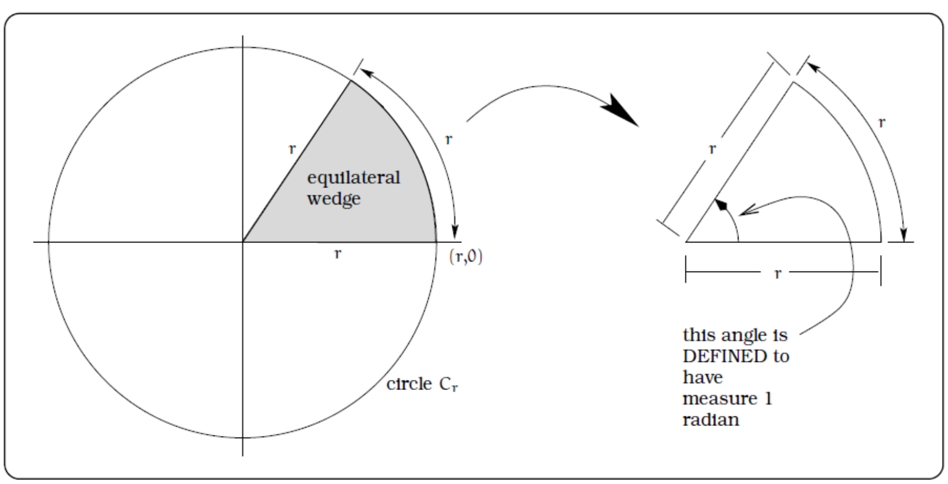
The measure of an angle is determined by the amount of rotation from the initial arm to the terminal arm. Angles can be measured in ***degrees*** or ***radians***.

One degree is a unit of angle measure that is equivalent to the rotation in $\frac{1}{360}$ th of a circle.

One radian is the measure of a central angle subtended in a circle by an arc equal in length to the radius of the circle.

The circumference of a circle is given by the formula $C = 2\pi r$, so there would be 2π radians in one complete rotation of the terminal arm in a circle.

So, **360° = 2π radians**



You can use this information to translate rotations into radian measure.

1 full rotation _____° = _____ radians	$\frac{1}{6}$ rotation _____° = _____ radians
$\frac{1}{2}$ rotation _____° = _____ radians	$\frac{1}{8}$ rotation _____° = _____ radians
$\frac{1}{4}$ rotation _____° = _____ radians	$\frac{1}{12}$ rotation _____° = _____ radians

To convert between degree and radian measure:

If 360° = 2π radians, then 1° = __________ = __________

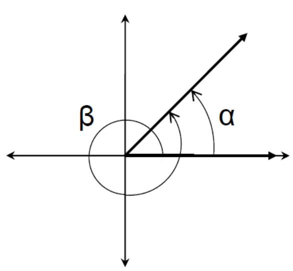
If 2π radians = 360°, then 1 radian = __________ = __________

To convert degrees to radians, multiply by __________

To convert radians to degrees, multiply by __________

COTERMINAL ANGLES

Coterminal angles are angles that have the same terminal arm. In the diagram shown, α and β are coterminal angles.



By adding or subtracting multiples of 360° or 2π radians (one full rotation), you can write an infinite number of angles that are coterminal with any given angle.

For example, some angles coterminal with 70° are:

$70° + (360°)(1) = 430°$ $\quad$ $70° - (360°)(1) = -290°$

$70° + (360°)(2) = 790°$ $\quad$ $70° - (360°)(2) = -650°$

In general the angles coterminal with 70° are $70° \pm (360°)n$, where n is any natural number.

Some angles coterminal with $\frac{5\pi}{6}$ are:

$\frac{5\pi}{6} + 2\pi(1) = \frac{17\pi}{6}$ $\quad$ $\frac{5\pi}{6} - 2\pi(1) = \frac{-7\pi}{6}$

$\frac{5\pi}{6} + 2\pi(2) = \frac{29\pi}{6}$ $\quad$ $\frac{5\pi}{6} - 2\pi(2) = \frac{-19\pi}{6}$

The angles coterminal with $\frac{5\pi}{6}$ are $\frac{5\pi}{6} \pm 2\pi n$, where n is any natural number.

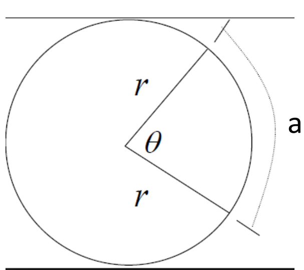
GENERAL FORM:

Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation from the terminal arm you arrive back at the same terminal arm. Angles coterminal with any angle θ can be described using the expression:

$$\boxed{\theta \pm (360°)n \textbf{ OR } \theta \pm 2\pi n, \textbf{ where n is any natural number}}$$

This way of expressing an infinite number of angles is called ***general form***.

ARC LENGTH OF A CIRCLE



An arc of a circle refers to a portion of the circumference of a circle. ***Arc length*** refers to the length of that arc.

Arcs that subtend the same size central angle do not necessarily have the same arc length. The arc length depends on the radius of the circle.

For a central angle, θ, in radians, the arc length, a, is given by the formula:

a = θr, where

where a = arc length*
θ = size of central angle, measured in radians
r = radius* of circle

* Note that the arc length and the radius must be measured in the same units.

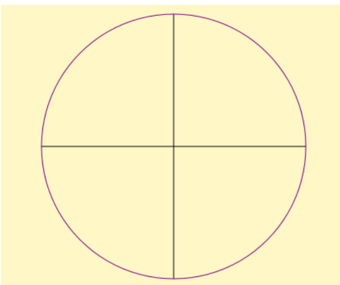
Example 1: Convert Between Degree and Radian Measure

Draw each angle in standard position. Convert each degree measure to radian measure and each radian measure to degree measure. Give answers as both exact and approximate measures to the nearest hundredth of a unit.

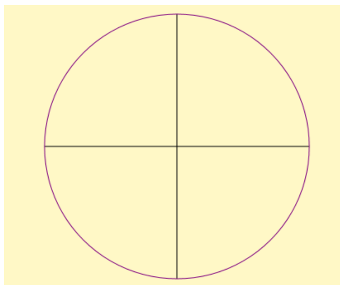
a. $135°$ b. $\frac{5\pi}{6}$ c. 4 d. $-120°$

Solution:

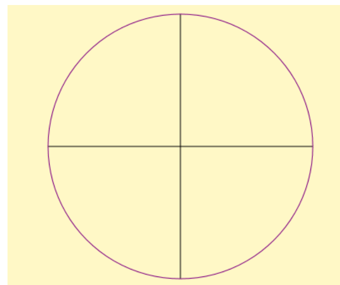
a. $135°$



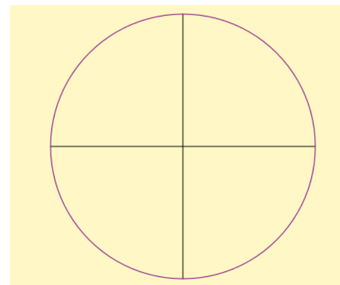
b. $\frac{5\pi}{6}$



c. 4



d. $-120°$



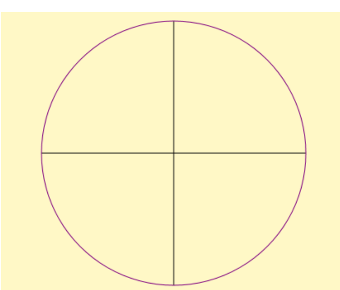
Example 2: Identify Coterminal Angles

Determine one positive and one negative angle measure that is coterminal with each angle. In which quadrant does the terminal arm lie?

a. $160°$ b. $-450°$ c. $\frac{7\pi}{3}$ d. $\frac{-9\pi}{4}$

Solution:

a. $160°$

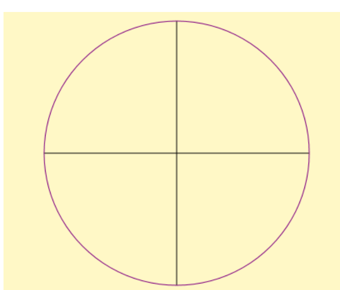


Terminal arm is in quadrant _____

Angles coterminal with 160°:

____

b. $-450°$

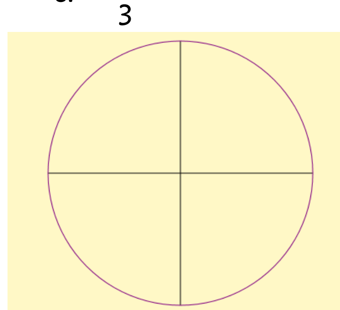


This is referred to as a ________________________ angle

Angles coterminal with –450°:

____

c. $\frac{7\pi}{3}$

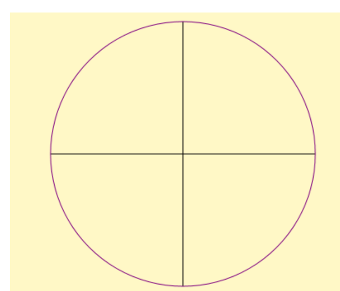


Terminal arm is in quadrant _____

Angles coterminal with $7\pi/3$:

____

d. $\frac{-9\pi}{4}$



Terminal arm is in quadrant _____

Angles coterminal with $-9\pi/4$:

____

Example 3: Express Coterminal Angles in General Form

a. Express the angles coterminal with $150°$ in general form. Identify the angles coterminal with $150°$ that satisfy the domain $-720° \le \theta \le 720°$.

b. Express the angles coterminal with $\frac{2\pi}{3}$ in general form. Identify the angles coterminal with $\frac{2\pi}{3}$ in the domain $-4\pi \le \theta \le 4\pi$.

Solution:

a. General form for angles coterminal with $150°$: ______________________________

n	1	2	3
$150° - (360°)n$			
$150° + (360°)n$			

Thus the angles that satisfy the domain $-720° \le \theta \le 720°$ are: ____________________________________

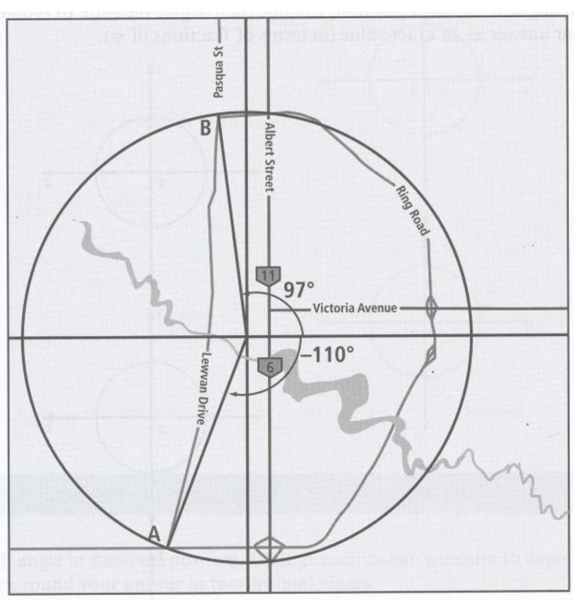
b. General form for angles coterminal with $\frac{2\pi}{3}$: ______________________________

n	1	2	3
$\frac{2\pi}{3} - 2\pi n$			
$\frac{2\pi}{3} + 2\pi n$			

Thus the angles that satisfy the domain $-4\pi \le \theta \le 4\pi$ are: ____

Example 4: Determine Arc Length in a Circle

The ring road around the eastern part of the city of Regina is almost a semicircle. Estimate the length of the ring road (from A to B) if the radius of the circle is 4.9 km.



Solution: