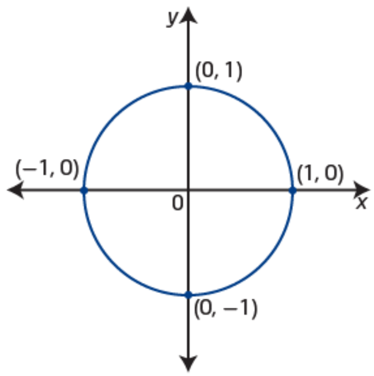
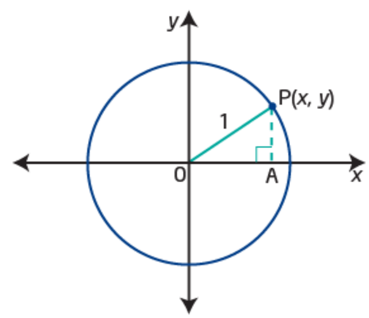
The Unit Circle



Unit Circle: *The unit circle has radius 1 unit and is centred at the origin on the Cartesian plane.*

THE EQUATION OF THE UNIT CIRCLE



- Consider any point P on the unit circle with coordinates (x, y).
- Drop a perpendicular from point P to point A on the x-axis.
- State the lengths of the sides of the $90° \Delta POA$

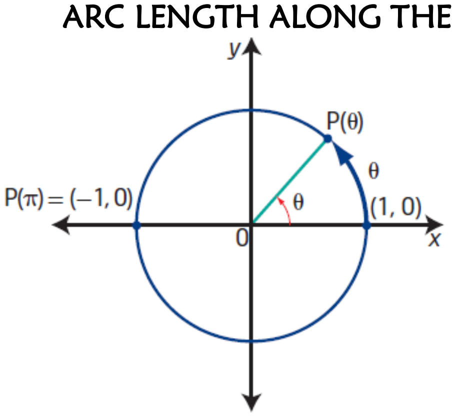
  OP = ________ OA = ________ PA = ________

- Pythagorean Theorem applies:

  $(OP)^2 = (OA)^2 + (PA)^2$

- Does the point (0.6, 0.8) lie on the circumference of the unit circle? Justify your answer.

ARC LENGTH ALONG THE UNIT CIRCLE



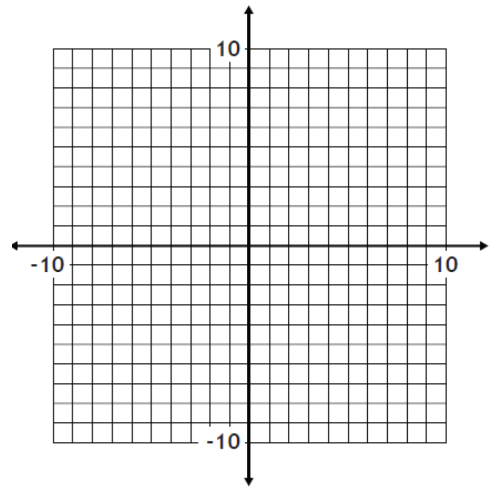
The formula $a = \theta r$ (where a is the arc length, θ, is the angle measure in radians, and r is the radius) applies to any circle. Remember that *r* and *a* must be measured in the same units. When considering the unit circle, this equation simplifies to ______________, since r = 1. Thus the subtended arc and the central angle have the same *numerical* value.

The function P takes real-number values for the central angle or the arc length on the unit circle and matches them with specific points. For example, if $\theta = \pi$, then the point on the unit circle is (–1, 0). Thus, you can write P(π)=(–1, 0).

Example 1: Equation of any Circle Centred at the Origin

Determine the equation of the circle with centre at the origin and radius 7.
Does the point (0, 7) lie on the circle? The point (5, 4.9)? Justify your response.

Solution:



Choose a point, P, on the unit circle with coordinates (x, y). Drop a perpendicular from P to the x-axis. Pythagorean Theorem applies:

Since this is true for every point P on the circle, then the equation of the circle is ______________________.

The point (0, 7) _______________ lie on the circle since ___________________.

The point (5, 4.9) _______________ lie on the circle since ___________________.

> **The equation of the *unit* circle is:** $x^2 + y^2 = 1$
> **In *general*, the equation of a circle centred at the origin with radius *r* is:** $x^2 + y^2 = r^2$

Example 2: Determine Coordinates for a Point on the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the given conditions. Draw a diagram for each case and label the coordinates.

a. the x-coordinate is $\frac{1}{2}$ b. the y-coordinate is $\frac{-2}{3}$

Solution:

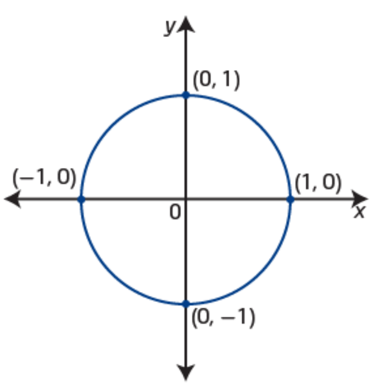
a. Remember that all coordinates on the unit circle satisfy the equation $x^2 + y^2 = 1$.

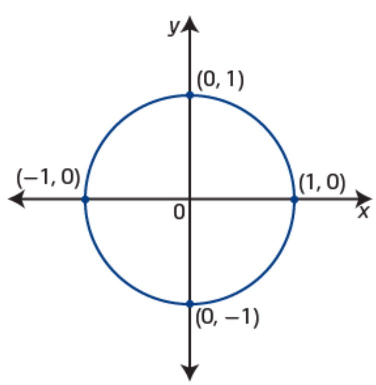
The x-coordinate is $\frac{1}{2}$.

Since the x-coordinate is positive, the point could be in quadrant _____ or _____.

$x^2 + y^2 = r^2$

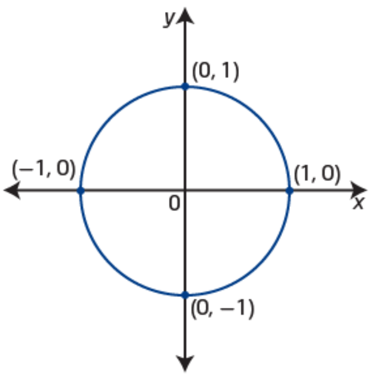
$\left(\frac{1}{2}\right)^2 + y^2 = 1^2$

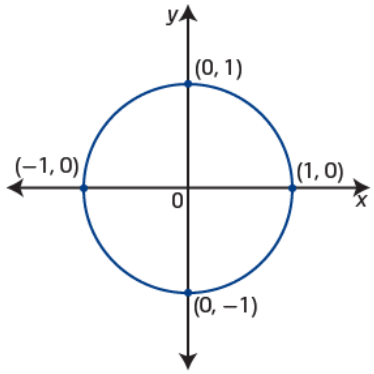




b. The y-coordinate is $\frac{-2}{3}$.

Since the y coordinate is negative, the point could be in quadrant _____ or _____.





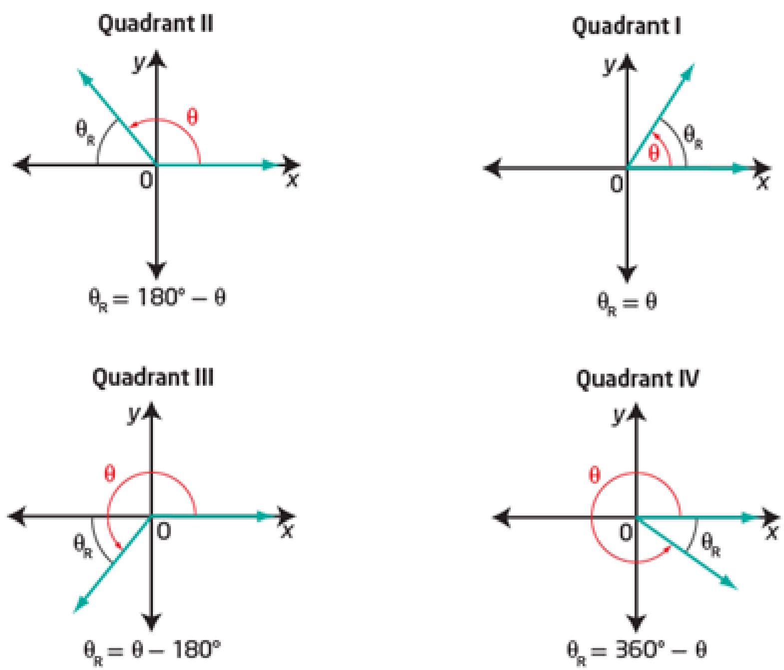
$$x^2 + y^2 = r^2$$

$$x^2 + \left(\frac{-2}{3}\right)^2 = 1^2$$

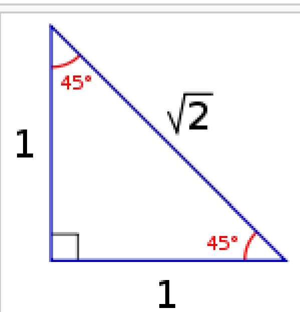
REFERENCE ANGLES

For each angle, θ, in standard position, there is a corresponding acute angle called the **reference angle.** The reference angle, θ_R, is the acute angle formed between the terminal arm of θ and the *x-axis.* **The reference angle is always positive** and $0° < \theta_R < 90°$.

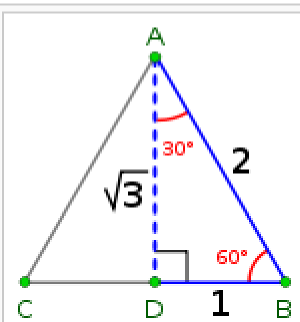
Examples of angles in standard position and their corresponding reference angles are illustrated in the various quadrants.



REVIEW OF SPECIAL TRIANGLES



The side lengths of a 45–45–90 triangle

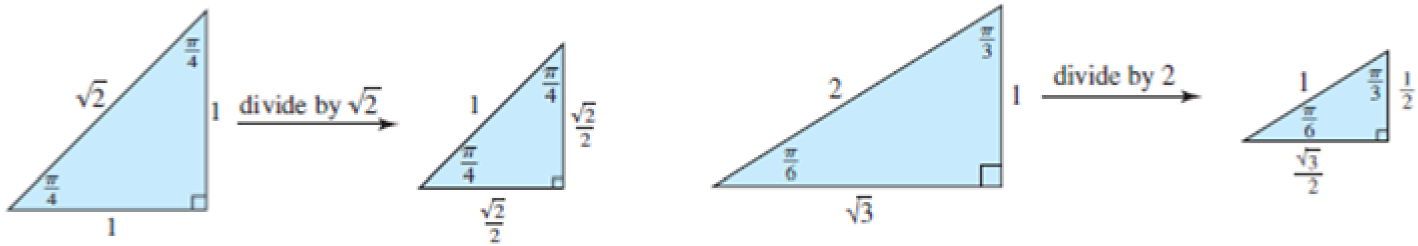


The side lengths of a 30–60–90 triangle

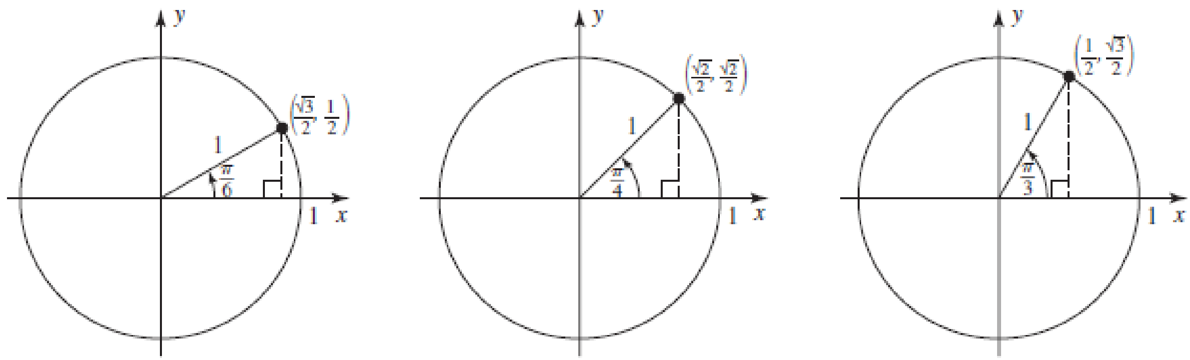
There are two "special" right triangles with which you need to be familiar; the $45° - 45° - 90°$ triangle and the $30° - 60° - 90°$ triangle. The "special" nature of these triangles is their ability to yield *exact* values instead of decimal approximations when dealing with trigonometric functions.

45° = ______ rad 30° = ______ rad 60° = ______ rad

These special triangles can be scaled to fit within the unit circle:



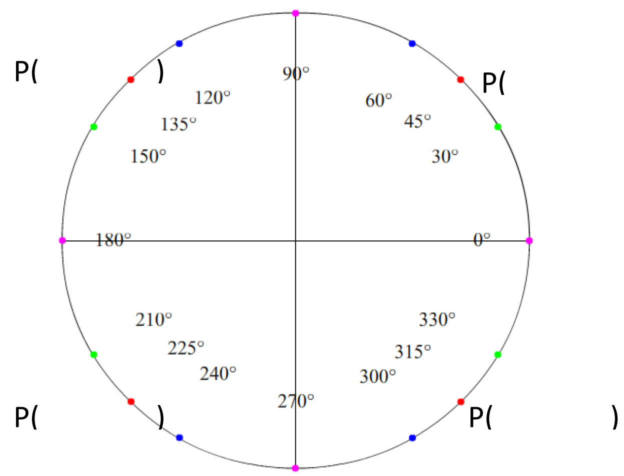
Determine the lengths of x and y for each special triangle contained within the unit circle:



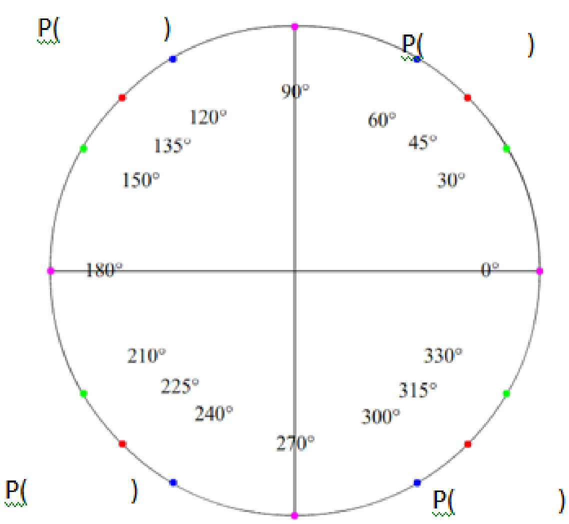
Example 3: Reference Angles of $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{6}$ on the Unit Circle

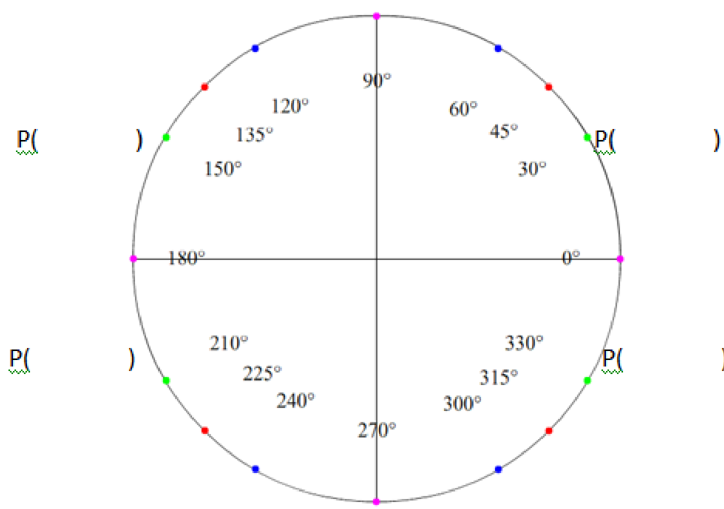
a. On the diagrams of the unit circle, show the angles that have reference angles of $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{6}$, respectively, in the interval $0 \le \theta \le 2\pi$.

b. Label the coordinates for each point P in part (a).



Solution:

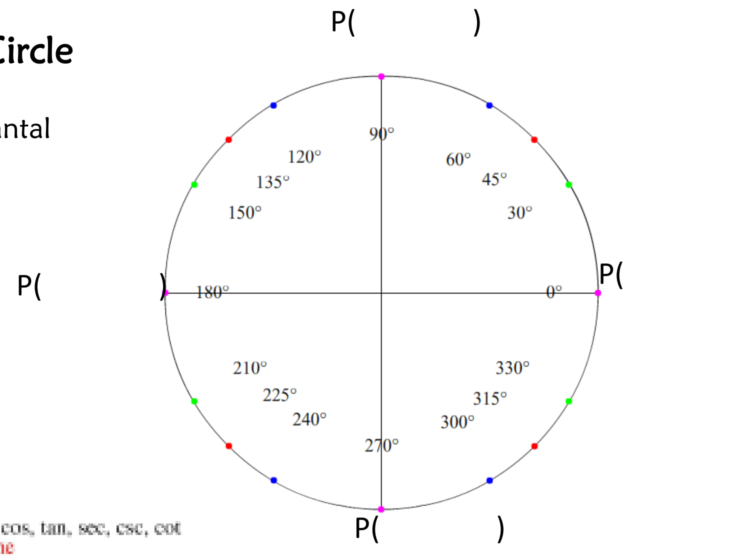




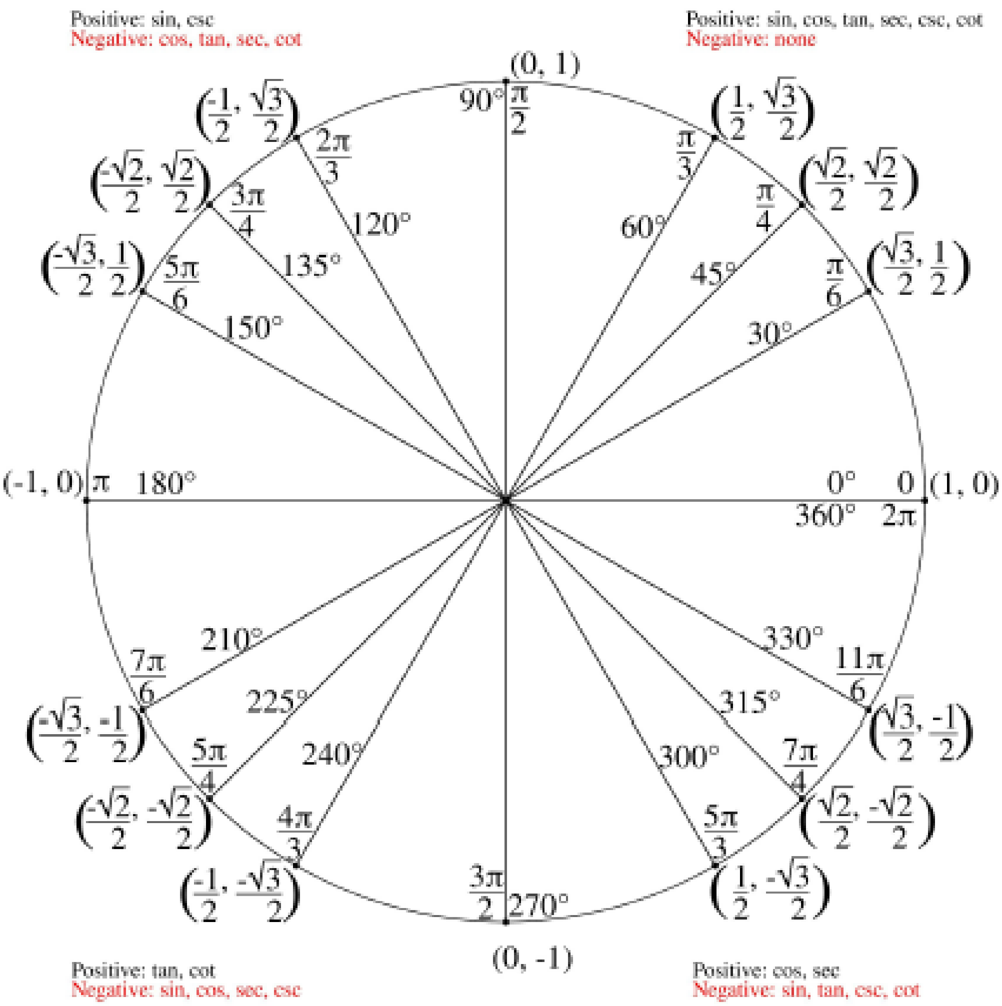
Example 4: Quandrantal Angles on the Unit Circle

a. On the diagram of the unit circle, show the quadrantal angles in the interval $0 \le \theta \le 2\pi$.

b. Label the coordinates for each point P in part (a).



THE UNIT CIRCLE



Coordinates on the Unit Circle

$P(\theta) = (x,\ y) = (\cos\theta,\ \sin\theta)$ holds true for any point $P(\theta)$ which is at the intersection of the terminal arm of angle θ and the unit circle.