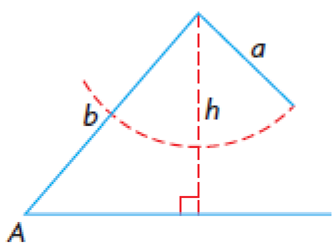
The Ambiguous Case of the Sine Law

NUMBER OF POSSIBLE TRIANGLES GIVEN SIDE-SIDE-ANGLE (SSA)

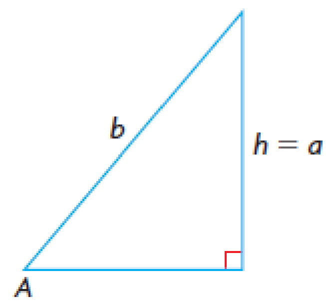
Suppose we are given side a, side b, and ∠A in △ABC. Let h represent the height of the "triangle". Note that $\sin A = \frac{h}{b} \rightarrow h = b\sin A$.

If ∠A is *acute*, then there are four possible cases to consider:

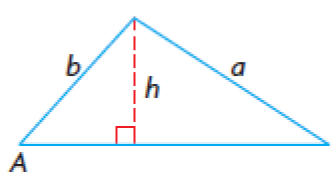
i) If ∠A is acute and $a < h$, then **no such triangle** exists.



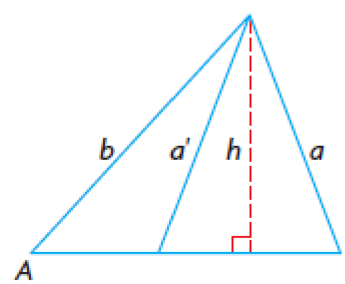
ii) If ∠A is acute and $a = h$, then **one *right* triangle** exists.



iii) If ∠A is acute and $a \geq b$, then **one triangle** exists.

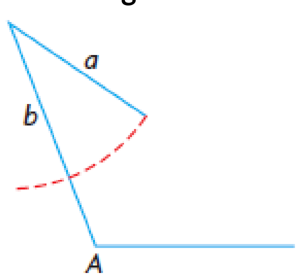


iv) If ∠A is acute and $h < a < b$, then **two possible triangles** exist.

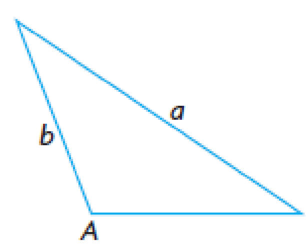


If ∠A is *obtuse*, then there are two possible cases to consider:

i) If ∠A is obtuse and $a \leq b$, then **no such triangle** exists.



ii) If ∠A is obtuse and $a > b$, then **one triangle** exists.



Example 1: Connecting the SSA situation to the number of possible triangles.

Given each SSA situation, determine how many triangles are possible.

a. In Δ ABC, $\angle A = 30^o$, $a = 6cm$, $b = 12cm$

b. In Δ DEF, $\angle D = 40^o$, $\mathrm{d} = 6cm$, $\mathrm{e} = 10cm$

c. In Δ KAT, $\angle K = 50^o$, $\mathrm{k} = 8cm$, $\mathrm{t} = 5cm$

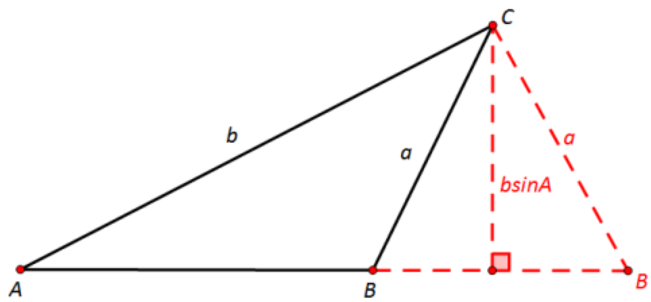
d. In Δ LMN, $\angle M = 35^o$, $\mathrm{m} = 5cm$, $\mathrm{n} = 6cm$

e. In Δ JKL, $\angle J = 150^o$, $\mathrm{j} = 8cm$, $\mathrm{k} = 10cm$

f. In Δ PQR, $\angle R = 150^o$, $\mathrm{r} = 9cm$, $\mathrm{p} = 6cm$

Given the measures of two side lengths and a non-contained angle (SSA), there will be *zero*, *one*, or *two* possible triangles. The ***ambiguous case of the sine law*** is the case where *two* possible triangles can be drawn given the available information.

Two possibilities exist if we are given measurements for acute angle A, side a, and side b in a triangle such that $h < a < b$. One possibility is that $\angle$B is *acute* and the other possibility is that $\angle$B is *obtuse*.



Example 2: The Ambiguous Case of the Sine Law

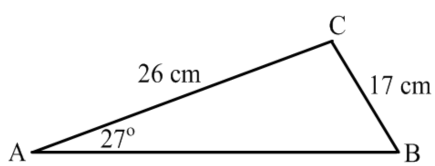
In ΔABC, $\angle$A = 27°, a = 17 cm and b = 26 cm. Calculate the measure of $\angle$B.

Solution:

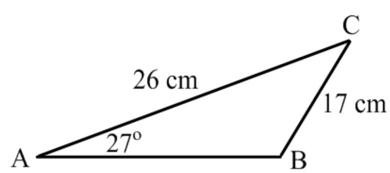
For ΔABC, h = bsinA =

Note that $h < a < b$, so there are two possible cases:

Case 1:



Case 2:



It is important to note that for one sine value, there is more than one possible angle. *Supplementary* angles will have the same sine value, that is, sin θ = sin (180° − θ). If we know the sine of an angle and we use the calculator to determine the measure of that angle, we will get the *acute* angle possibility. If we know that the angle is *obtuse*, then we can calculate its measure by finding the *supplement* of the acute angle.

In the diagrams above, using the sine law to solve for $\angle$B, we get:

So in **Case 1**, $\angle$B = __________ and in **Case 2**, $\angle$B = _________________ = __________.

Example 3: Solving a Problem Using the Sine Law

Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of the cloud. Albert's rope is 7.8 m long and makes an angle of 36° with the ground. Belle's rope is 5.9 m long. Assume that Albert and Belle form a triangle in a vertical plane with the weather balloon.

a) Is it necessary to consider the ambiguous case? Explain.
b) Sketch the possible diagrams for this situation.
c) Determine the possible the distances between Albert and Belle to the nearest tenth of a metre.

Solution:

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b) Sketch the possible diagrams for this situation.

c) Determine the possible the distances between Albert and Belle to the nearest tenth of a metre.