Introduction to Trigonometric Equations

Solving an equation means to determine the value(s) of a variable that make the equation true. For example, $\sin\theta = \frac{1}{2}$ is true when $\theta = 30^o$ and $\theta = 150^o$ and every angle coterminal with 30^o *or* 150^o. These angles are solutions to the trigonometric equation $\sin\theta = \frac{1}{2}$. Note that the variable θ is often used to represent the unknown angle, but any other variable is allowed.

Steps to Solving First-Degree Trigonometric Equations

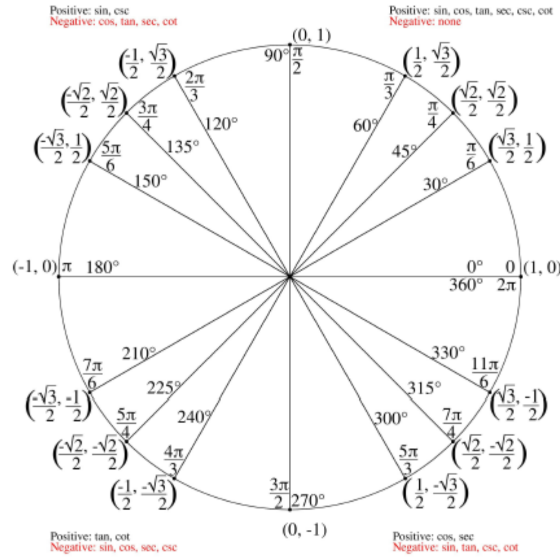
1. Solve for the *trigonometric ratio*.
2. Determine the *reference* angle.
   - If the trigonometric ratio is associated with "special" angles in the unit circle, then state the *exact* value of θ_R, otherwise, use your calculator to determine the *approximate* value of θ_R.
   - If you are using your calculator, make sure that it is in the correct "mode" (degrees or radians) before you start solving.
3. Use the sign (positive or negative) of the trigonometric ratio to determine in which possible *quadrants* the terminal arm of angle θ will lie (use the CAST rule).
4. For your chosen quadrants, use the reference angle to determine the *associated* angles within one full rotation.
5. State:
   - all possible solutions, expressed in *general* form, $\theta \pm 2\pi n,\ n \in W$ or $\theta + 2\pi n,\ n \in I$

     and/or
   - all solutions within a given domain, such as $0 \le \theta < 2\pi$

Steps to Solving Second-Degree Trigonometric Equations

1. Deal with the quadratic nature of the equation first. A second degree equation can be solved by:
   - isolating the quadratic term (if possible) and then taking the square root of both sides of the equation to determine the two possible trigonometric ratios.

     or
   - rearranging the equation so that the quadratic expression equals zero and then factoring or using the quadratic formula to solve for the two possible trigonometric ratios.
2. Determine the *reference* angle for *each* trigonometric ratio and proceed as described above (steps 2 – 5).



Example 1: Solving First-Degree Trigonometric Equations

Solve each trigonometric equation. State the *general solution* and then the solution in the *specified domain*.

a. $2\sin\theta - \sqrt{3} = 0,\ 0 \le \theta < 2\pi$
b. $\sec\theta + 2 = 0,\ 0 \le \theta < 2\pi$
c. $5\sin\theta + 3 = 3\sin\theta + 5,\ 0° \le \theta < 360°$
d. $3\tan\theta - 5 = 0,\ 0 \le \theta < 2\pi$
e. $-17 + 3\cot\theta = -29,\ 0° \le \theta < 360°$
f. $3\csc x - 4 = 0,\ 0 \le x < 2\pi$

Solution:

a. $2\sin\theta - \sqrt{3} = 0,\ 0 \le \theta < 2\pi$	b. $\sec\theta + 2 = 0,\ 0 \le \theta < 2\pi$	c. $5\sin\theta + 3 = 3\sin\theta + 5,\ 0° \le \theta < 360°$
d. $3\tan\theta - 5 = 0,\ 0 \le \theta < 2\pi$	e. $-17 + 3\cot\theta = -29,\ 0° \le \theta < 360°$	f. $3\csc x - 4 = 0,\ 0 \le x < 2\pi$

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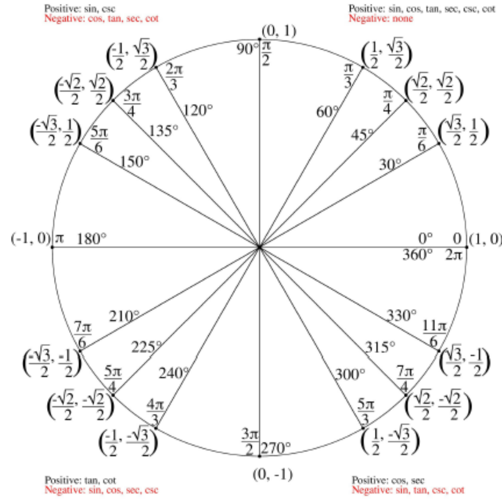

Example 2: Solving Second-Degree Trigonometric Equations

Solve for θ in *general form* and then in the *specified domain*.

a. $2\cos^2\theta + \sqrt{2}\cos\theta = 0,\ 0 \le \theta < 4\pi$
b. $\tan^2\theta - 4\tan\theta + 3 = 0,\ 0° \le \theta < 720°$
c. $\tan^2 x - 3 = 0,\ \ -180° \le x < 180°$
d. $2\sin^2\theta - \sin\theta - 1 = 0,\ \ -2\pi < \theta < 2\pi$

Solution:

a. $2\cos^2\theta + \sqrt{2}\cos\theta = 0,\ 0 \le \theta < 4\pi$	b. $\tan^2\theta - 4\tan\theta + 3 = 0,\ 0° \le \theta < 720°$
c. $\tan^2 x - 3 = 0,\ \ -180° \le x < 180°$	d. $2\sin^2\theta - \sin\theta - 1 = 0,\ \ -2\pi < \theta < 2\pi$