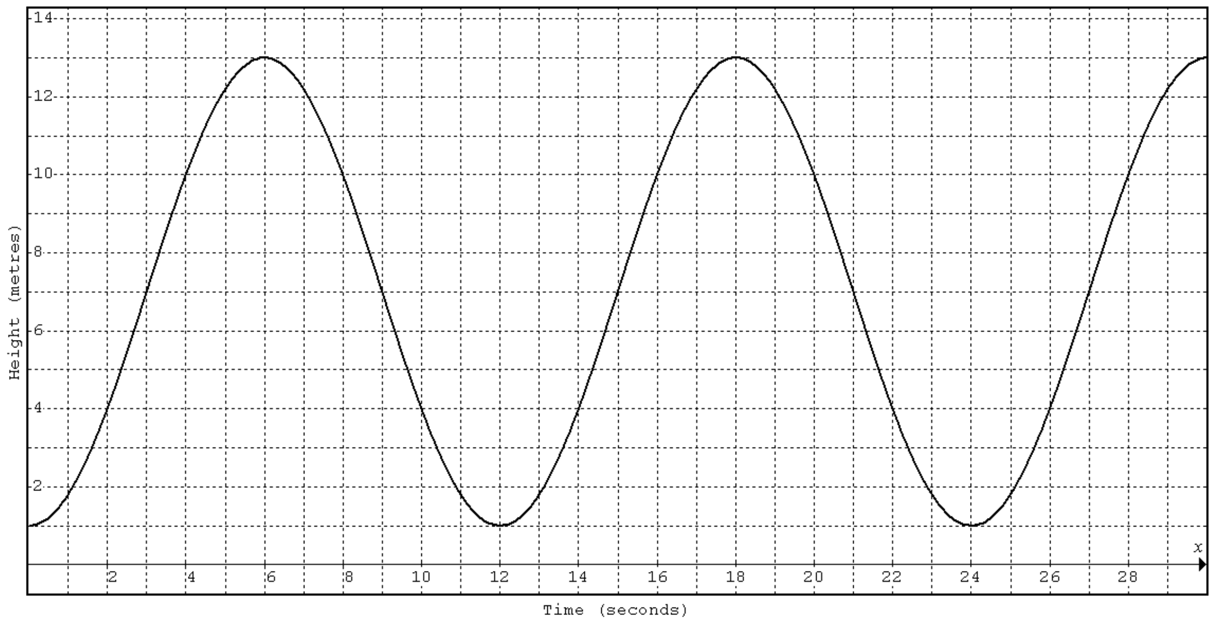
Applications of Trigonometric Functions

Introduction:

Victoria rode on a Ferris wheel at Cluney Amusements. The graph models Victoria's height above the ground in metres in relation to time in seconds. The data were recorded while the ride was in progress.



a. What is the height of the axle on the Ferris wheel? __________

b. What is the radius of the Ferris wheel? __________

c. What is the maximum height of the Ferris wheel? __________

d. How long does it take for the Ferris wheel to complete one revolution? __________

e. Victoria boards the Ferris wheel at its lowest point. How high above the ground is this? __________

f. Within the first 20 seconds, how many times is Victoria at a height of 7 m above the ground? _________

g. What is Victoria's approximate height above ground at 16 seconds? __________

h. What is Victoria's approximate height above the ground at 57 seconds? __________

i. Write an equation that models Victoria's height as a function of time.

Example 1:

A Ferris wheel has a radius of 20 m and travels at a rate of 6 revolutions per minute. You board the bottom chair from a platform that is 2 metres above the ground.

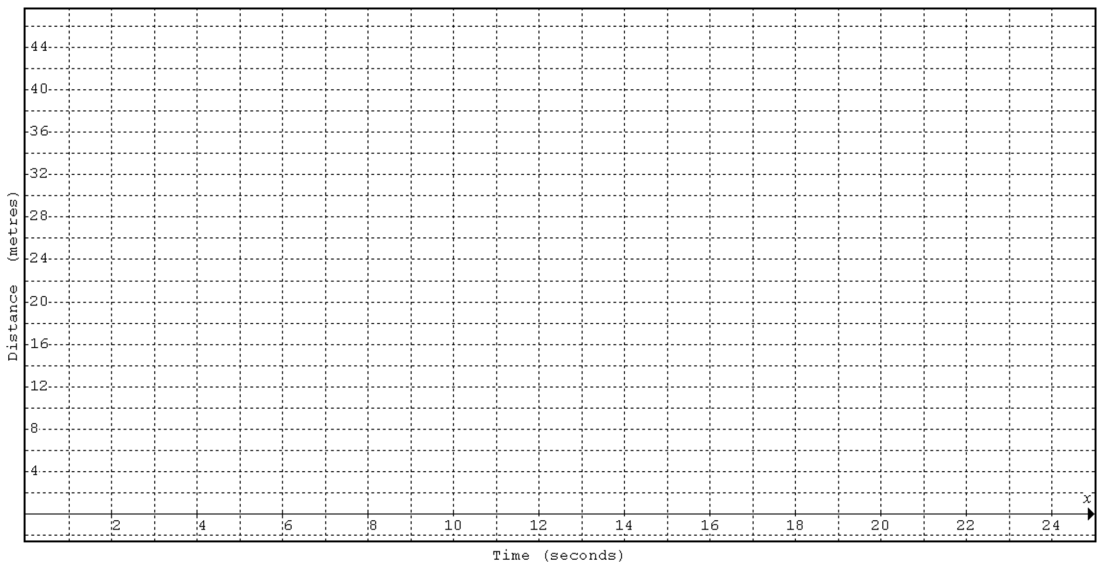
a. Sketch a graph showing how your height above the ground varies during the first two cycles.

b. Write an equation which expresses your height as a function of the elapsed time.

c. Calculate your height above the ground at 12 seconds.

Solution:

a.



b. Equation:

c. Height above the ground at 12 seconds:

Example 2:

Nick is floating in an inner-tube in a wave pool. His height above the bottom of the pool varies sinusoidally with time. He is 1.5 m from the bottom of the pool when he is at the trough of a wave. A stopwatch starts timing at this point. In 1.25s, he is on the next crest of a wave, 2.1m from the bottom of the pool.

a. Sketch a graph that represents Nick's height above the bottom of the pool as a function of time. Show two complete periods.

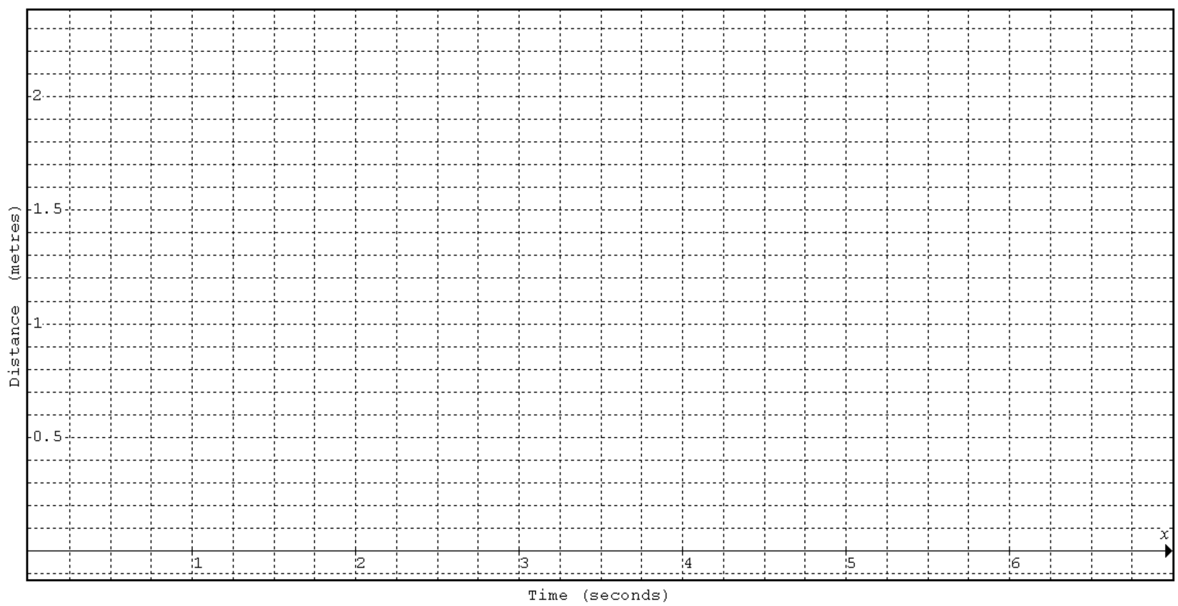
b. Write an equation that expresses Nick's distance from the bottom of the pool as a function of time.

c. Use your equation to determine Nick's height above the bottom of the pool at time t = 10 seconds.

d. How many complete cycles are there within the first 40 seconds?

Solution:

a.



b. Equation:

c. Height above the bottom of the pool at 10 seconds:

d. Number of complete cycles in 40 seconds:

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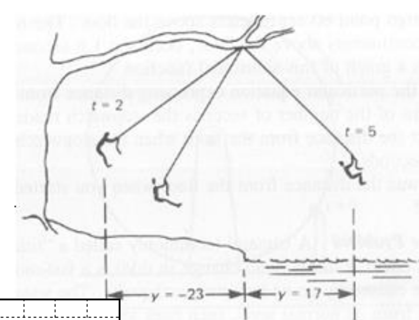
Example 3:

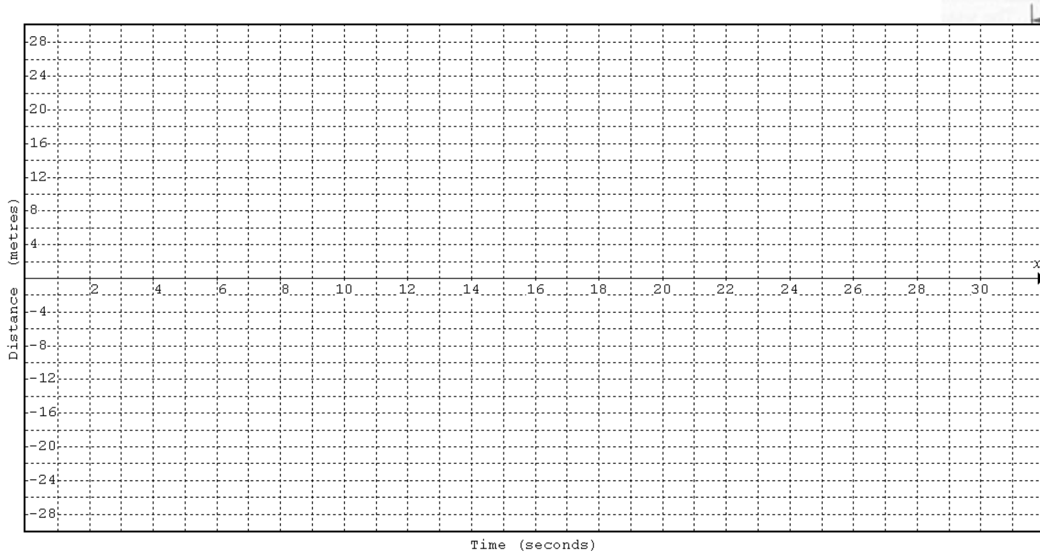
Tarzan is swinging back and forth on his grapevine. As he swings, he moves back and forth across the riverbank alternately over land and water. His distance from the edge of the riverbank varies sinusoidally with time. Assume his distance is positive when he is over the water and negative when he is over land. Jane decides to model his motion using her stopwatch. Jane finds that, after 2 seconds, Tarzan is at one end of his swing, where d = –23 m. After 5 seconds, he reaches the other end of his swing, where d = 17 m.



a. Sketch two periods of the graph of this sinusoidal function.
b. Write an equation expressing distance from the edge of the riverbank as a function of time.
c. Calculate Tarzan's position when t = 15 seconds
d. Where was Tarzan when Jane started her watch?

Solution:

a.



b. Equation:

c. Tarzan's position at t = 15 seconds:

d. Tarzan position when Jane started her watch:

Example 4:

A small plane is flying at a constant altitude of 6000m directly toward an observer. Assume that the ground is flat in the region close to the observer.

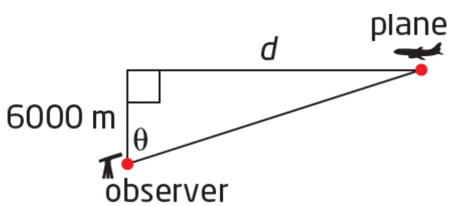
a. Determine the relation between the horizontal distance, in metres, from the observer to the plane and the angle, in degrees, formed from the vertical to the plane.

b. Sketch the graph of the function.

c. Where are the asymptotes located in this graph? What do they represent?

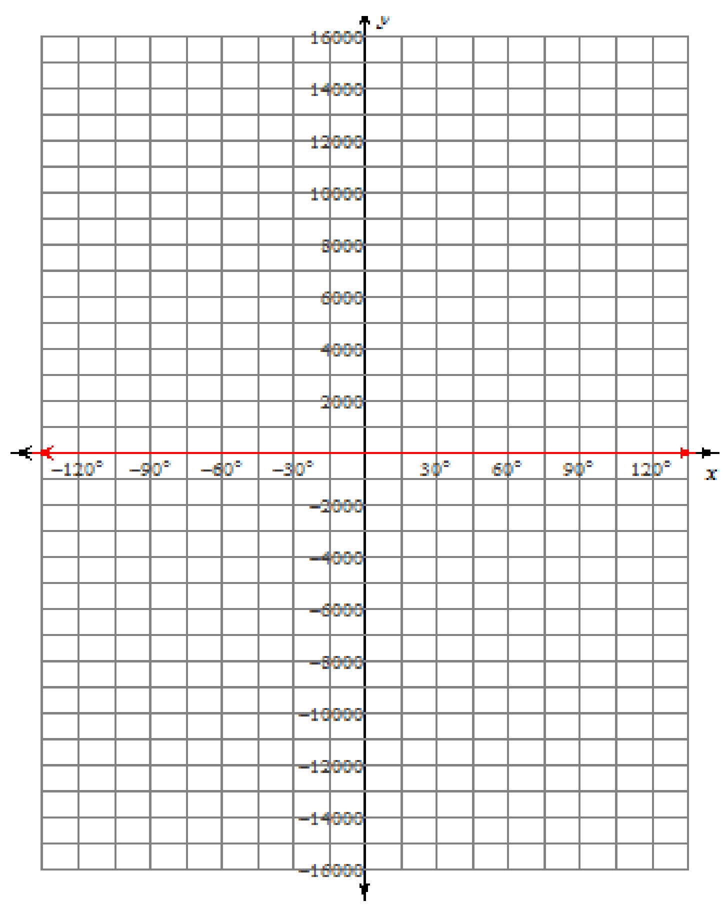
d. Explain what happens when the angle is equal to $0°$.

Solution:



a. Let d = horizontal distance from the observer to the plane and θ = angle formed by the vertical and the line of sight to the plane.

b. Sketch the graph of the function.



c. Where are the asymptotes located? What do they represent?

d. What happens when the angle is equal to $0°$?