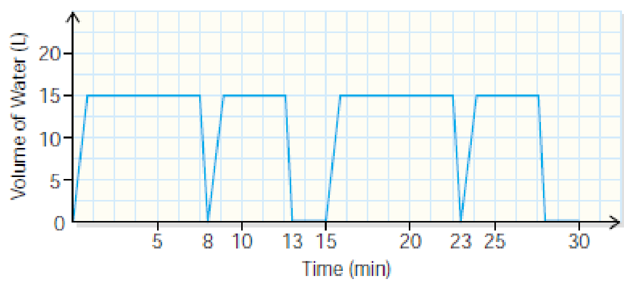
Graphing Sine and Cosine Functions

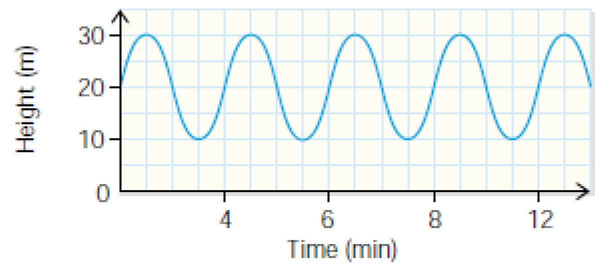
PERIODIC FUNCTION:
A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

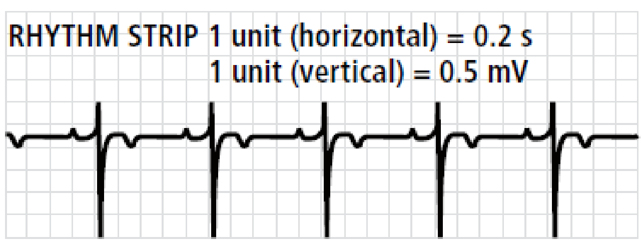
Examples:



The automatic dishwasher in a school cafeteria runs constantly through lunch. The graph shows the amount of water used as a function of time.

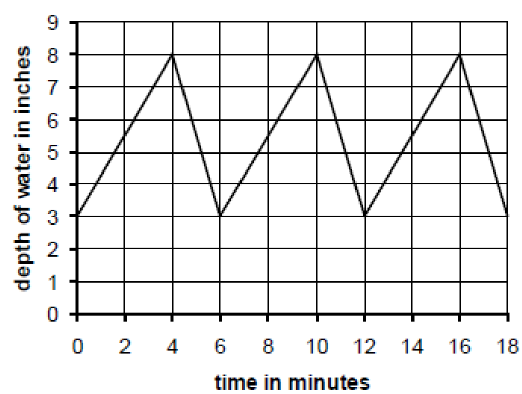
An individual's height above the ground as a function of time as they ride the Ferris wheel.



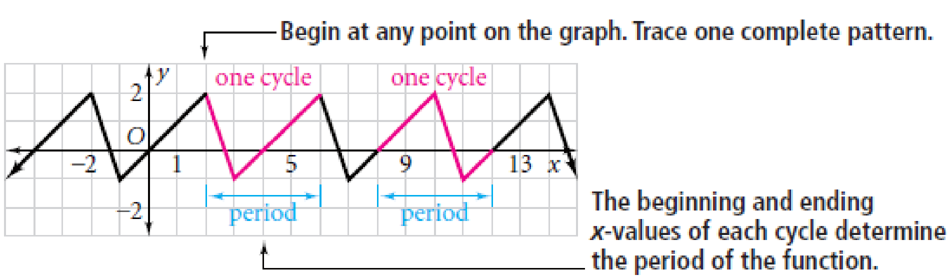


An ECG measures the electrical activity of a person's heart in millivolts over time.

Rebecca has a submersible pump in her basement that is situated such that it can remove water that collects below her foundation. There is a hole in the basement floor and the pump sits in the bottom of that hole. During a particularly heavy rain storm, the pump kept turning off and on at regular intervals as it attempted to drain the excess water below the foundation. The relationship between the depth of water in the hole and time is shown in the graph.

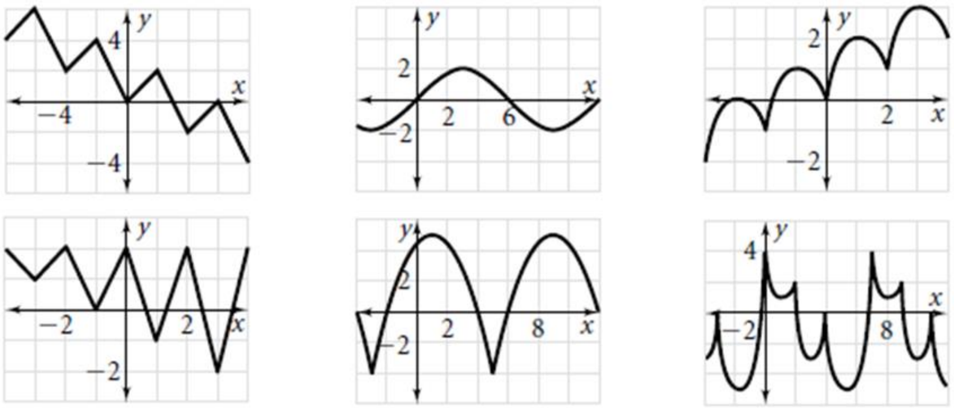


PERIOD:
The horizontal length of one cycle on the graph of a periodic function.



Each cycle is 4 units long. The period of the function is 4.

Determine whether each function shown is periodic or not.

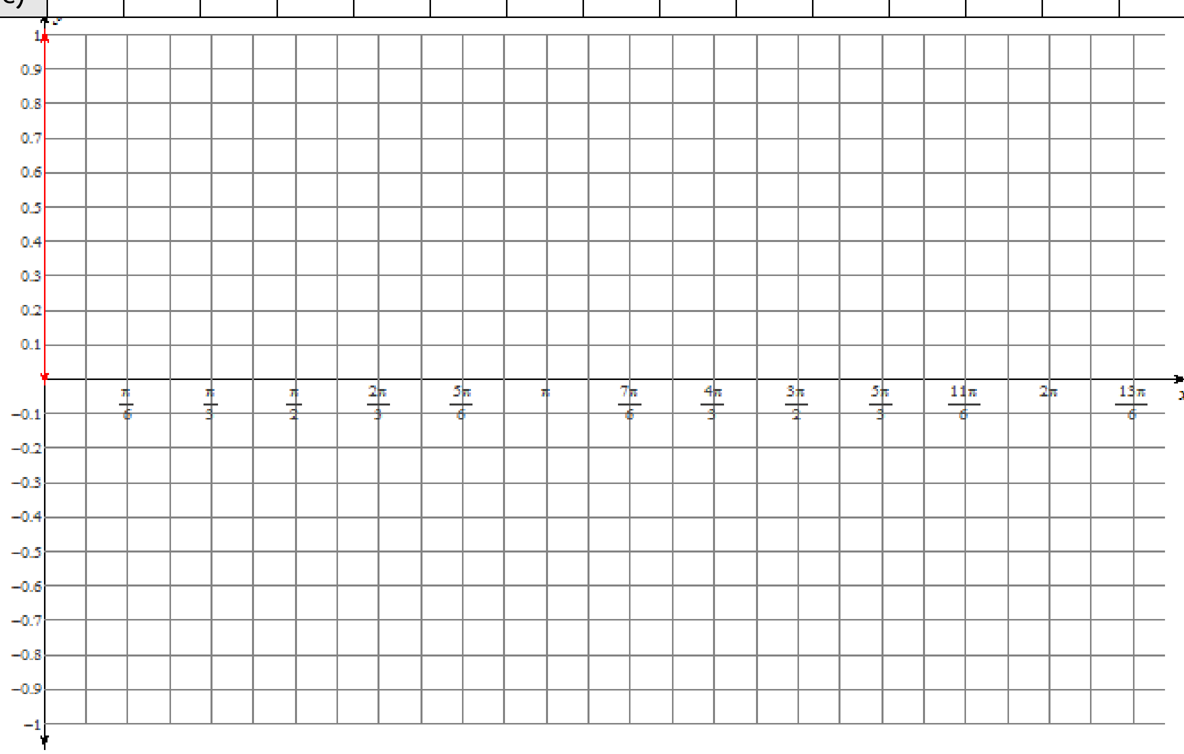


THE SINE FUNCTION

Sketch the graph of $y = \sin\theta$ for $0° \le \theta \le 360°$ or $0 \le \theta \le 2\pi$. Describe its characteristics.

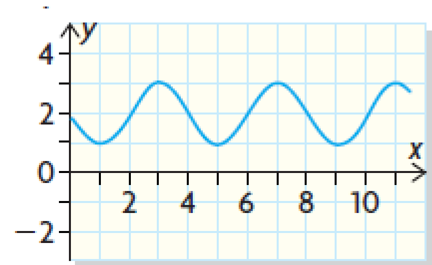
Solution:
Complete the following table of values for $y = \sin\theta$. Plot the points and join them with a smooth curve.

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin\theta$ (exact)			$\frac{\sqrt{2}}{2}$														
$\sin\theta$ (approximate)			0.71														



SINUSOIDAL FUNCTION:

A periodic function whose graph looks like smooth symmetrical waves, where any portion of the wave can be horizontally translated onto another portion of the curve.



EQUATION OF THE SINUSOIDAL AXIS:
The equation of the horizontal line halfway between the maximum and minimum.

AMPLITUDE:
The vertical distance from the function's axis to the maximum or minimum value.

Using your graph of $y = \sin\theta$, determine each of the following:

DOMAIN:	RANGE:	MAXIMUM VALUE: MINIMUM VALUE:	EQUATION OF THE SINUSOIDAL AXIS:
AMPLITUDE:	PERIOD:	Y-INTERCEPT:	θ-INTERCEPTS :

Determine the 5 key points that would enable you to draw the sine curve quickly.

θ (degrees)	θ (radians)	y
0°	0	
90°	$\frac{\pi}{2}$	
180°	π	
270°	$\frac{3\pi}{2}$	
360°	2π	

TRANSFORMATIONS OF THE SINE FUNCTION

Sketch the graphs of the following functions over the given intervals. Identify the amplitude, period, vertical translation, phase shift, domain, range, maximum and minimum values, and the equation of the sinusoidal axis.

a. $y = 2\sin(x - 45°) + 3;\ x \in [-360°, 360°]$

b. $y = -3\sin 2\left(\theta + \frac{\pi}{4}\right);\ \theta \in [-2\pi, 2\pi]$

Solution:

a. $y = 2\sin(x - 45°) + 3;\ x \in [-360°, 360°]$

Mapping Rule: $(x, y) \rightarrow$ ____________________

$y = \sin x$	
x	y
0°	
90°	
180°	
270°	
360°	

$y = 2\sin(x - 45°) + 3$	
x	y

Amplitude:

Period:

Vertical Translation:

Phase Shift (Horizontal Translation):

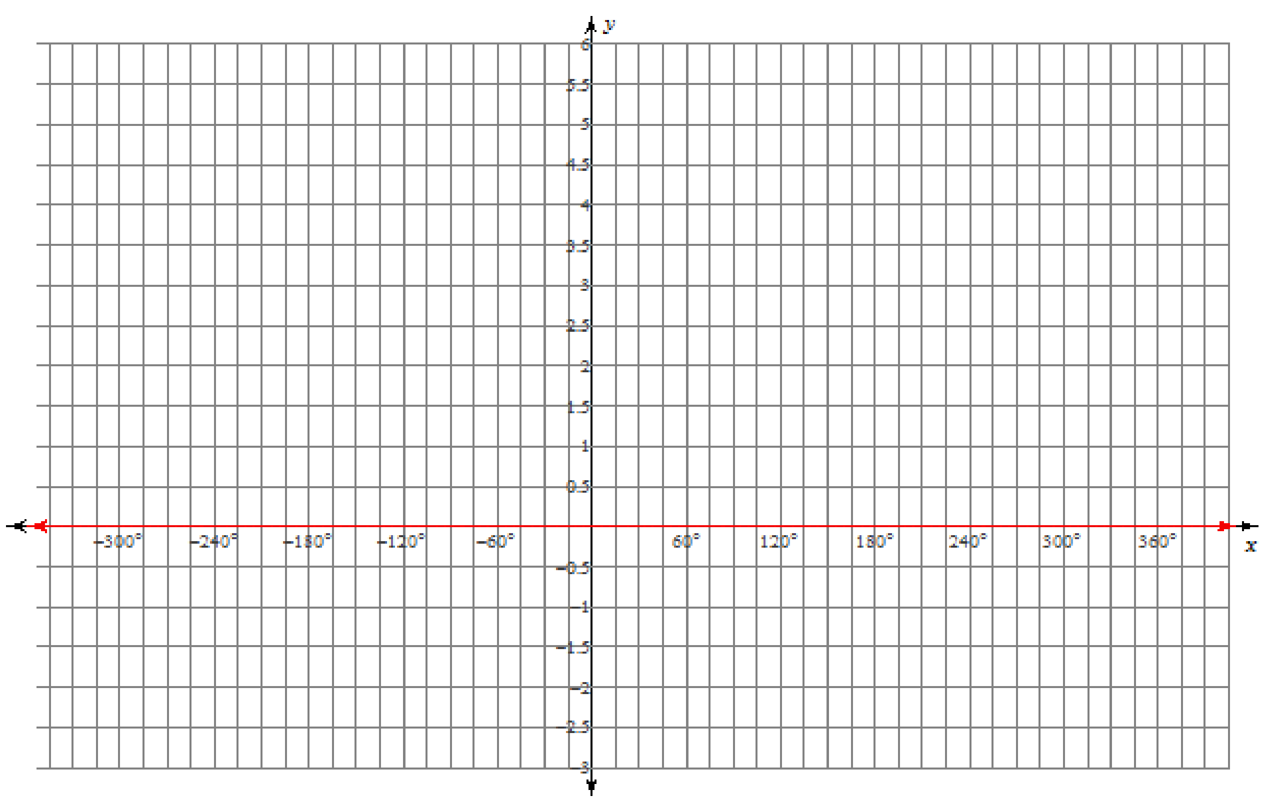
Domain:

Range:

Maximum Value:

Minimum Value:

Equation of the Sinusoidal Axis:



b. $y = -3\sin 2\left(\theta + \frac{\pi}{4}\right)$; $\theta \in [-2\pi, 2\pi]$

Mapping Rule: $(\theta, y) \rightarrow$ ______________________

$y = \sin\theta$	
θ	y
	0
	1
	0
	–1
	0

$y = -3\sin 2\left(\theta + \frac{\pi}{4}\right)$	
θ	y

Amplitude:

Period:

Vertical Translation:

Phase Shift (Horizontal Translation):

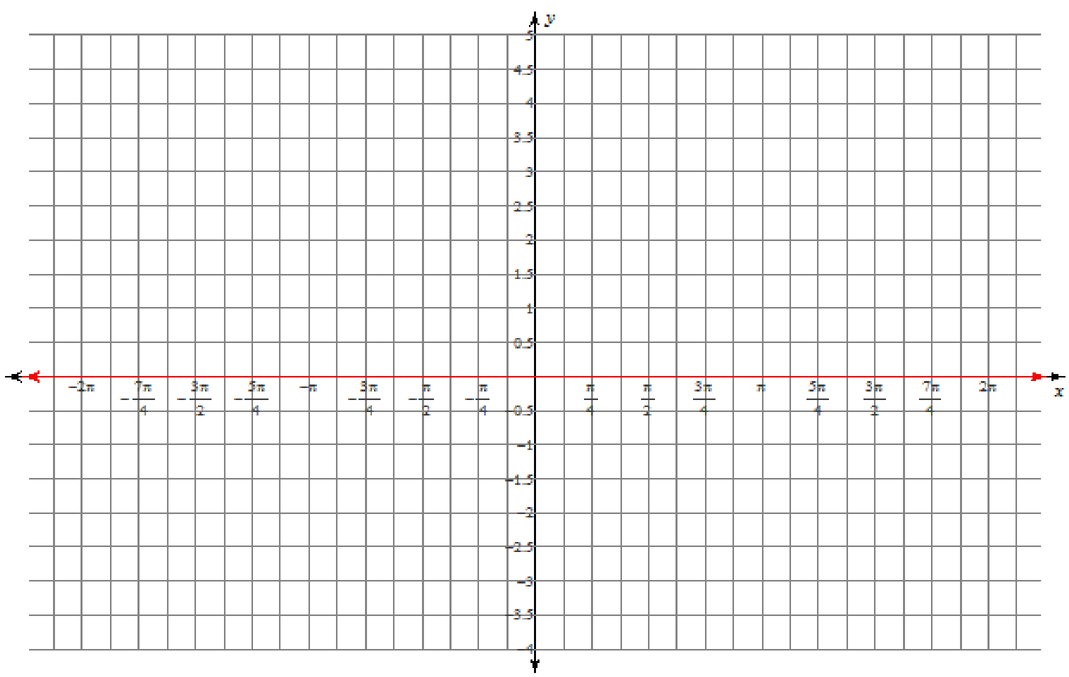
Domain:

Range:

Maximum Value:

Minimum Value:

Equation of the Sinusoidal Axis:



If the equation of a sinusoidal function is given in the form $\begin{array}{l} y = a\sin b(x-c)+d \\ y = a\cos b(x-c)+d \end{array}$, then:

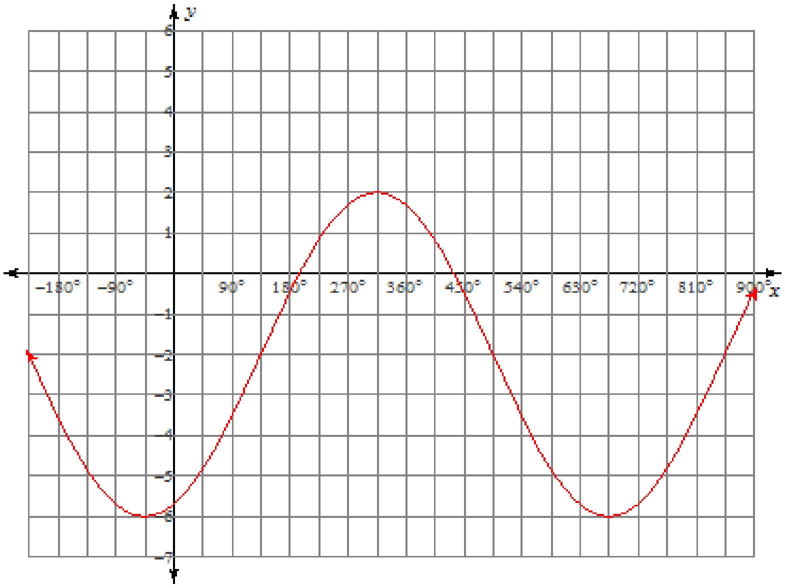
- Vertical stretch factor = $|a|$. If $a < 0$, then the function is reflected in the x-axis. (Note: amplitude = $|a|$)
- Horizontal stretch factor = $\frac{1}{|b|}$. If $a < 0$, then the function is reflected in the y-axis.

  (Note: period = $\frac{1}{|b|} \times 360^\circ$ or $\frac{1}{|b|} \times 2\pi$)
- Horizontal (phase) shift = c units
- Vertical translation = d units

DETERMINE AN EQUATION FROM A GRAPH

Write an equation for each function illustrated below in the form $y = a\sin b(x - c) + d$.

a.



Vertical stretch factor (amplitude) = ________

Reflected in the x-axis: ________

Horizontal stretch factor = $\frac{period}{360^o}$ = ________

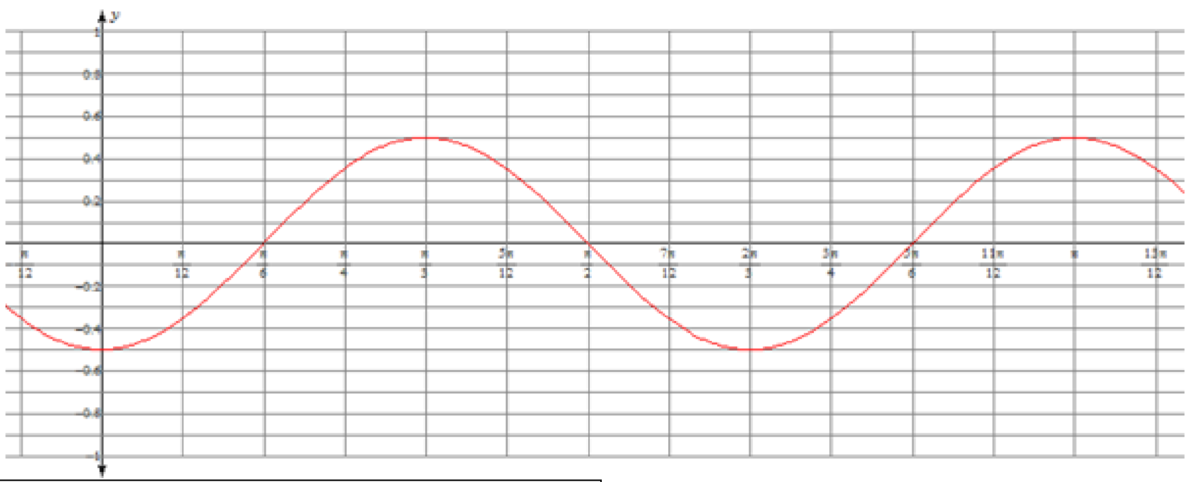
Reflected in the y-axis: ________

Vertical translation = ________________

Horizontal translation = ________________

Equation: ______________________________

b.



Vertical stretch factor (amplitude) = ________

Reflected in the x-axis: ________

Horizontal stretch factor = $\frac{period}{2\pi}$ = ________

Reflected in the y-axis: ________

Vertical translation = ________________

Horizontal translation = ________________

Equation: ______________________________

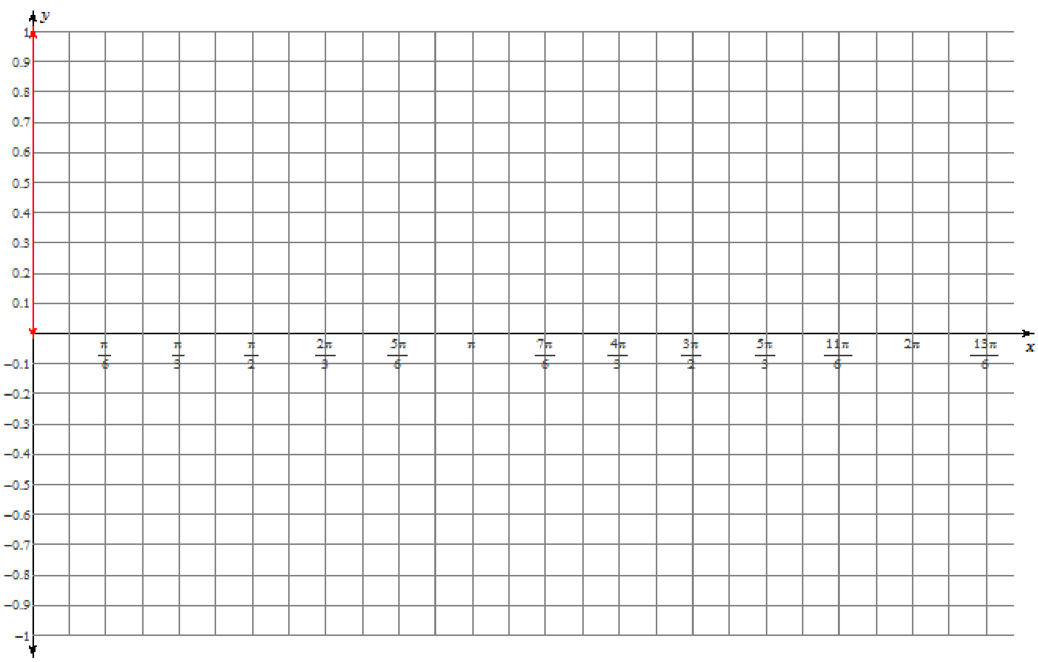
THE COSINE FUNCTION

Sketch the graph of $y = \cos\theta$ for $0° \le \theta \le 360°$ or $0 \le \theta \le 2\pi$. Describe its characteristics.

Solution:

Complete the following table of values. Plot the points and join them with a smooth curve.

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos\theta$ (exact)			$\frac{\sqrt{2}}{2}$														
$\cos\theta$ (approximate)			0.71														



Determine the 5 key points that would enable you to draw the cosine curve quickly.

θ (degrees)	θ (radians)	y
0°	0	
90°	$\frac{\pi}{2}$	
180°	π	
270°	$\frac{3\pi}{2}$	
360°	2π	

Using your graph of $y = \cos\theta$, determine each of the following:

DOMAIN:	RANGE:	MAXIMUM VALUE: MINIMUM VALUE:	EQUATION OF THE SINUSOIDAL AXIS:
AMPLITUDE:	PERIOD:	Y-INTERCEPT:	θ-INTERCEPTS :

TRANSFORMATIONS OF THE COSINE FUNCTION

Sketch the graph of the following functions over the given interval. Identify the amplitude, period, vertical translation, phase shift, domain, range, maximum and minimum values, and the equation of the sinusoidal axis.

a. $y = 3\cos\left(\frac{1}{4}(x+60°)\right)+1;\ x \in [-120°, 1740°]$

b. $y = \frac{-1}{4}\cos\left(2\theta - \frac{\pi}{6}\right);\ \theta \in \left[\frac{-2\pi}{3}, \frac{11\pi}{6}\right]$

Solution:

a. $y = 3\cos\left(\frac{1}{4}(x+60°)\right)+1;\ x \in [-120°, 1740°]$

Mapping Rule: ______________________________

y=cosx	
x	y
0°	
90°	
180°	
270°	
360°	

$y = 3\cos\left(\frac{1}{4}(x+60°)\right)+1$	
x	y

Amplitude:

Period:

Vertical Translation:

Phase Shift (Horizontal Translation):

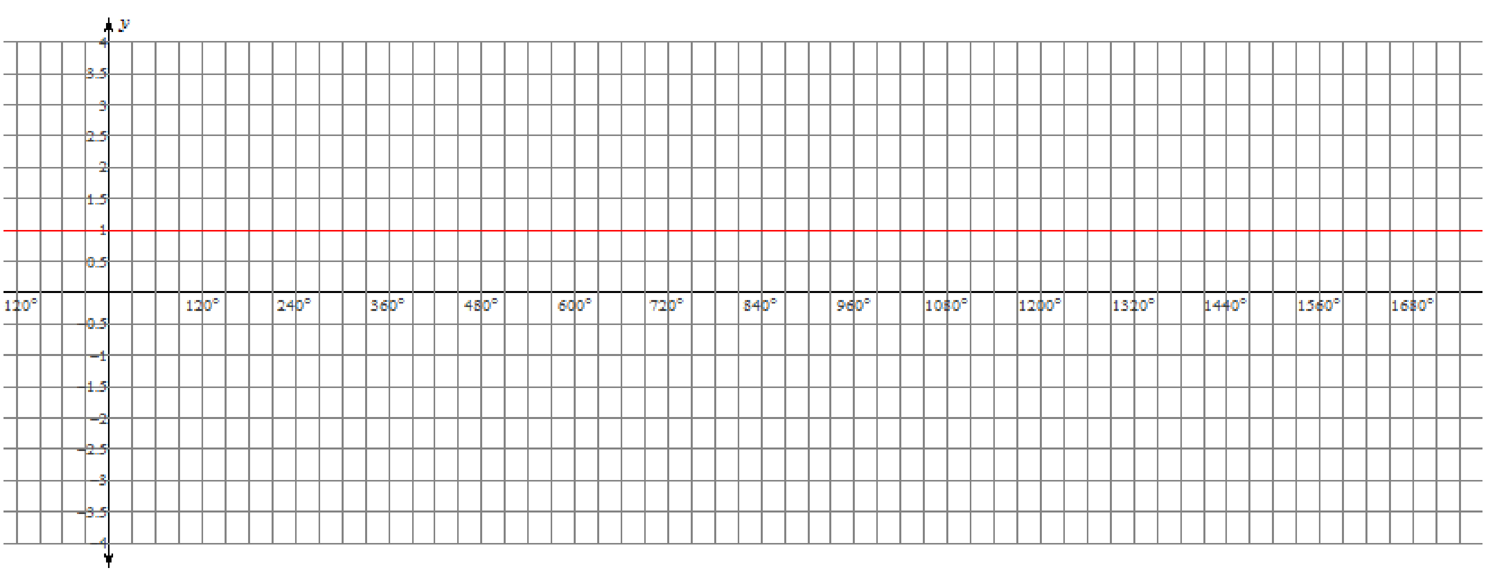
Domain:

Range:

Maximum Value:

Minimum Value:

Equation of the Sinusoidal Axis:



b. $y = \frac{-1}{4}\cos\left(2\theta - \frac{\pi}{6}\right);\ \theta \in \left[\frac{-2\pi}{3}, \frac{11\pi}{6}\right]$

Mapping Rule: ____________________________

$y = \cos\theta$	
θ	y
	1
	0
	–1
	0
	1

$y = \frac{-1}{4}\cos\left(2\theta - \frac{\pi}{6}\right)$	
θ	y

Amplitude:

Period:

Vertical Translation:

Phase Shift (Horizontal Translation):

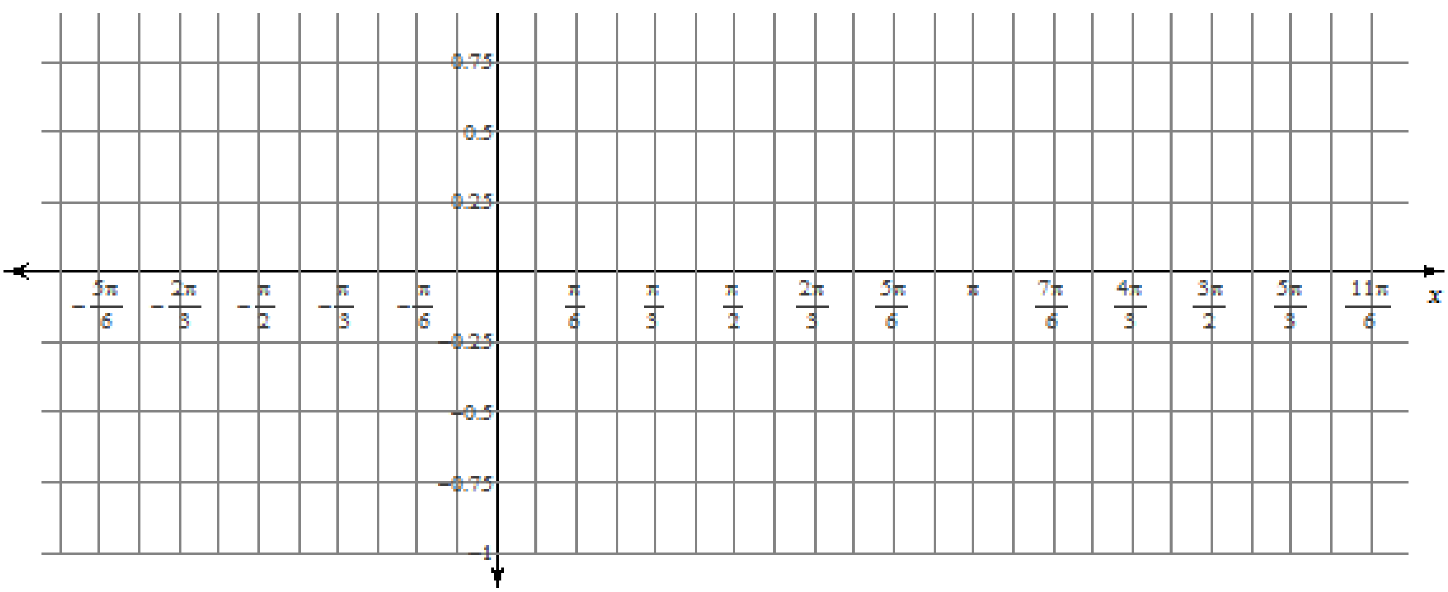
Domain:

Range:

Maximum Value:

Minimum Value:

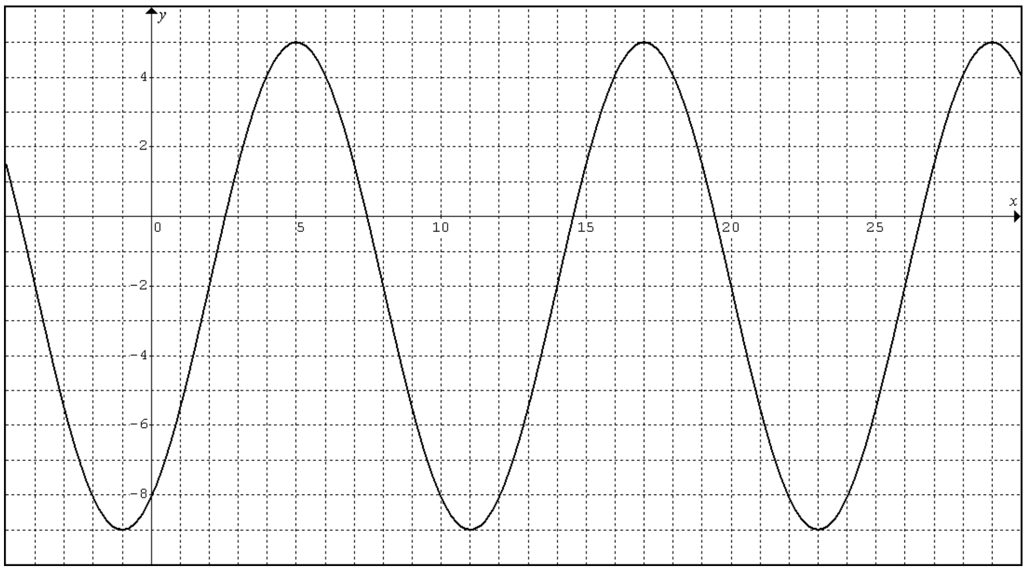
Equation of the Sinusoidal Axis:



DETERMINE AN EQUATION FROM A GRAPH

Write an equation for the function illustrated below in the form:

a. $y = a\cos b(x - c) + d$
b. $y = a\sin b(x - c) + d$



TRANSFORMATION OF Y = COS θ
Vertical stretch factor = __________
Reflected in the x-axis: __________
Horizontal stretch factor = ____________
Vertical translation = ____________________
Horizontal translation = ____________________
EQUATION: ______________________________

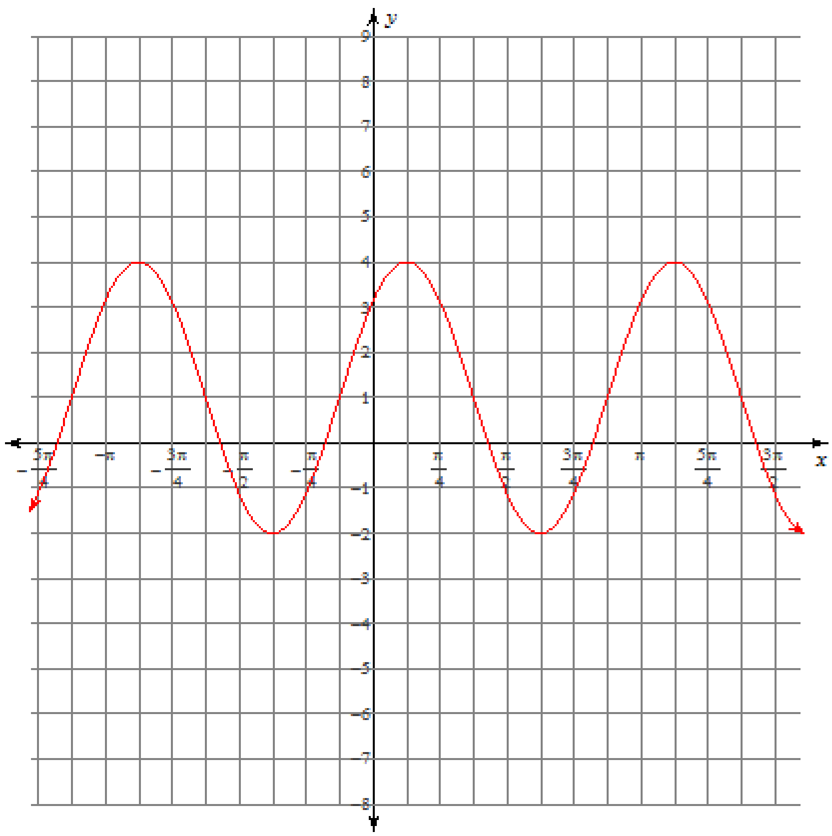
TRANSFORMATION OF Y = SIN θ
Vertical stretch factor = __________
Reflected in the x-axis: __________
Horizontal stretch factor = ____________
Vertical translation = ____________________
Horizontal translation = ____________________
EQUATION: ______________________________

YOUR TURN:

1. Write an equation for the function illustrated below in the form:

   a. $y = a\sin b(x - c) + d$

   b. $y = a\cos b(x - c) + d$



TRANSFORMATION OF Y = SIN θ

Vertical stretch factor = __________

Reflected in the x-axis: __________

Vertical translation = ____________

Horizontal stretch factor= ____________________

Horizontal translation = ____________________

EQUATION: ______________________________

EQUATION (reflected): ______________________

TRANSFORMATION OF Y = COS θ

Vertical stretch factor = __________

Reflected in the x-axis: __________

Vertical translation = ____________

Horizontal stretch factor = ____________________

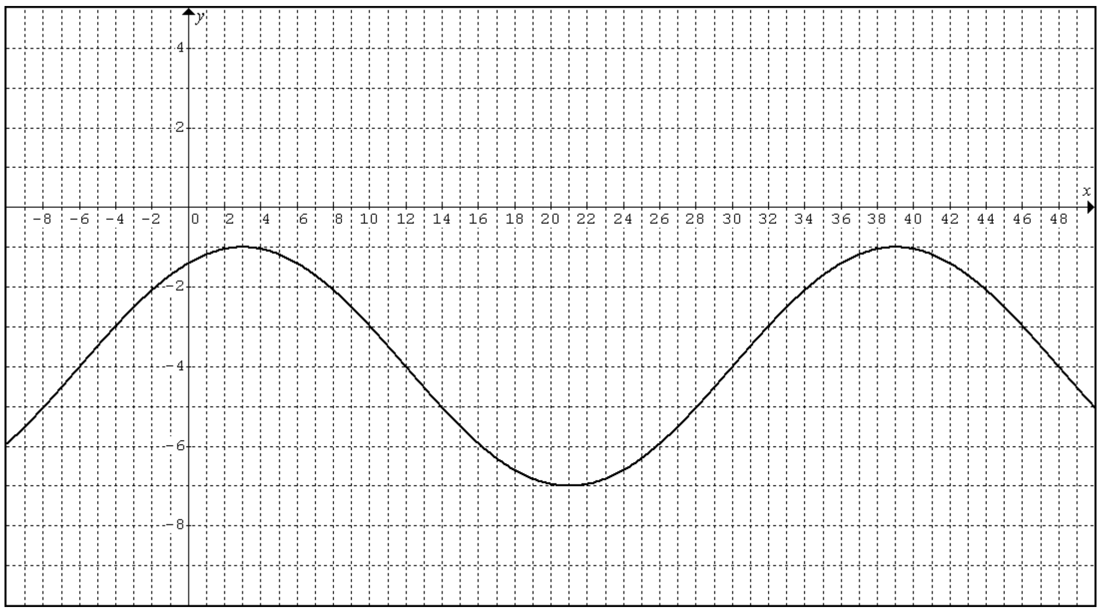
Horizontal translation = ____________________

EQUATION: ______________________________

EQUATION (reflected):_______________________

2. Write an equation for the function illustrated below in the form:

   a. $y = a\sin b(x-c)+d$
   b. $y = a\cos b(x-c)+d$



TRANSFORMATION OF Y = SIN θ

Vertical stretch factor = __________

Reflected in the x-axis: __________

Vertical translation **=** ____________

Horizontal stretch factor= ____________________

Horizontal translation = ____________________

EQUATION: ______________________________

EQUATION (reflected): ______________________

TRANSFORMATION OF Y = COS θ

Vertical stretch factor = __________

Reflected in the x-axis: __________

Vertical translation **=** ____________

Horizontal stretch factor = ____________________

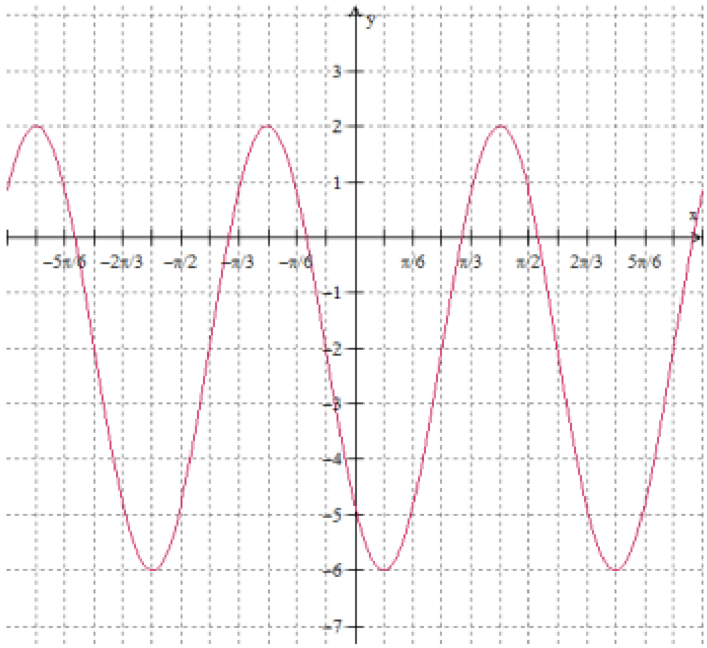
Horizontal translation = ____________________

EQUATION: ______________________________

EQUATION (reflected): _________________________

3. Write an equation for the function illustrated below in the form:

   a. $y = a\sin b(x-c)+d$
   b. $y = a\cos b(x-c)+d$



TRANSFORMATION OF Y = SIN θ	TRANSFORMATION OF Y = COS θ
Vertical stretch factor = ___________	Vertical stretch factor = ___________
Reflected in the x-axis: ___________	Reflected in the x-axis: ___________
Vertical translation = _____________	Vertical translation = _____________
Horizontal stretch factor= ______________________	Horizontal stretch factor = ______________________
Horizontal translation = ______________________	Horizontal translation = ______________________
EQUATION: _________________________________	**EQUATION:** _________________________________
EQUATION (reflected): ________________________	**EQUATION (reflected):** ____________________________