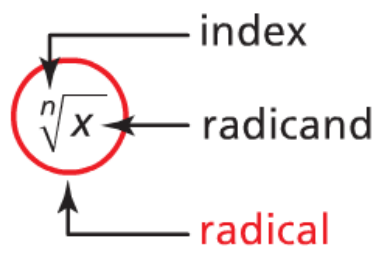


# Working with Radicals



- Radicals with the same radicand and index are called *like* radicals.
- If entire radicals have the same index, the radicands can be compared.
- If the index is even, the radicand must be positive or zero.
- If the index is odd, the radicand may be any real number.

## Example 1: Converting Mixed Radicals to Entire Radicals

Express each mixed radical as an entire radical.

a)  $5\sqrt{2}$

b)  $3x^4\sqrt{x}$

c)  $-4b\sqrt[3]{3b^2}$

**Solution:**

a) $5\sqrt{2}$	Write the coefficient 5 as a square root.  Multiply the radicands of the square roots.
b) $3x^4\sqrt{x}$  <u>Note:</u> For the radical in the original expression to be a real number, x (the radicand) must be greater than or equal to zero.	Write the coefficient $3x^4$ as a square root.  Multiply the radicands of the square roots.
c) $-4b\sqrt[3]{3b^2}$	Write the coefficient $-4b$ as a cube root.  Multiply the radicands of the cube roots.

**Example 2: Express Radicals as Mixed Radicals in Simplest Form**

Convert each radical to a mixed radical in simplest form: a)  $3\sqrt{200}$  b)  $\sqrt[3]{x^7}$  c)  $\sqrt{48y^5}$  d)  $\sqrt[3]{-108x^5}$

**Solution:**

a) $3\sqrt{200}$	<p>The perfect-square factors of 200 are 1, 4, 25 and 100. Write <math>\sqrt{200}</math> as product of square roots using the <i>greatest</i> perfect-square factor.</p> <p>Simplify.</p>
b) $\sqrt[3]{x^7}$	<p>Write <math>\sqrt[3]{x^7}</math> as a product of cube roots using the greatest perfect-cube factor.</p> <p>Simplify.</p>
c) $\sqrt{48y^5}$	<p>Write <math>\sqrt{48y^5}</math> as a product of square roots using the greatest perfect-square factor.</p> <p>Simplify.</p> <p><u>Note:</u> For the radical in the original expression to be a real number, y must be greater than or equal to zero.</p>
d) $\sqrt[3]{-108x^5}$	<p>Write <math>\sqrt[3]{-108x^5}</math> as a product of cube roots using the greatest perfect-cube factor.</p> <p>Simplify.</p>

### Example 3: Compare and Order Radicals

Without the aid of a calculator, arrange the following real numbers in ascending order.

$$\sqrt{149} \qquad 13 \qquad 4\sqrt{10} \qquad 2(42)^{\frac{1}{2}} \qquad 3\sqrt{19}$$

**Solution:**

Express each number as an entire radical. Since these radicals have the same index, they can be compared.

$$\sqrt{149} \qquad 13 = \qquad 4\sqrt{10} = \qquad 2(42)^{\frac{1}{2}} = \qquad 3\sqrt{19} =$$

The numbers in ascending order are \_\_\_\_\_.

### Example 4: Adding and Subtracting Radicals

Add and/or subtract the following radicals, as indicated.

$$\text{a) } \sqrt{27} + 2\sqrt{12} \qquad \text{b) } 7\sqrt{8} - 3\sqrt{18} + \sqrt{3} + \sqrt{75} \qquad \text{c) } \sqrt{4c} - 4\sqrt{9c}, c \geq 0$$

**Solution:**

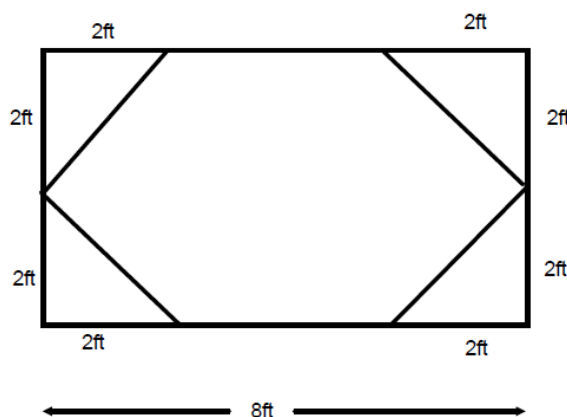
When adding and subtracting radicals, only *like* radicals can be combined. Radicals with the same radicand and index are called *like* radicals. Simplify the radicals first, and then combine the like radicals.

$$\text{a) } \sqrt{27} + 2\sqrt{12} \qquad \text{b) } 7\sqrt{8} - 3\sqrt{18} + \sqrt{3} + \sqrt{75} \qquad \text{c) } \sqrt{4c} - 4\sqrt{9c}, c \geq 0$$

### Example 5: Apply Addition of Radical Expressions

Four corners are cut from a 4 ft by 8 ft sheet of plywood as shown. Determine the perimeter of the remaining piece of plywood.

**Solution:**



## EXTRA PRACTICE: ADDING AND SUBTRACTING RADICALS

Simplify each of the following:

1)  $2\sqrt{5} + 2\sqrt{5} + 2\sqrt{5}$

2)  $-3\sqrt{6} - 3\sqrt{3} - 2\sqrt{3}$

3)  $-3\sqrt{2} + 3\sqrt{5} + 3\sqrt{5}$

4)  $-2\sqrt{6} - \sqrt{3} - 3\sqrt{6}$

5)  $-2\sqrt{6} - 2\sqrt{6} - \sqrt{6}$

6)  $-3\sqrt{3} + 2\sqrt{3} - 2\sqrt{3}$

7)  $3\sqrt{6} + 3\sqrt{5} + 2\sqrt{5}$

8)  $-\sqrt{5} + 2\sqrt{3} - 2\sqrt{3}$

9)  $2\sqrt{2} - 3\sqrt{18} - \sqrt{2}$

10)  $-\sqrt{54} - 3\sqrt{6} + 3\sqrt{27}$

11)  $-3\sqrt{6} - \sqrt{12} + 3\sqrt{3}$

12)  $-\sqrt{5} - \sqrt{5} - 2\sqrt{54}$

13)  $3\sqrt{2} + 2\sqrt{8} - 3\sqrt{18}$

14)  $2\sqrt{20} + 2\sqrt{20} - \sqrt{3}$

15)  $3\sqrt{18} - \sqrt{2} - 3\sqrt{2}$

16)  $-3\sqrt{27} + 2\sqrt{3} - \sqrt{12}$

17)  $-3\sqrt{6} - 3\sqrt{6} - \sqrt{3} + 3\sqrt{6}$

18)  $-2\sqrt{2} - \sqrt{2} + 3\sqrt{8} + 3\sqrt{6}$

19)  $-2\sqrt{18} - 3\sqrt{8} - \sqrt{20} + 2\sqrt{20}$

20)  $-3\sqrt{18} - \sqrt{8} + 2\sqrt{8} + 2\sqrt{8}$

21)  $-2\sqrt{24} - 2\sqrt{6} + 2\sqrt{6} + 2\sqrt{20}$

22)  $-3\sqrt{8} - \sqrt{5} - 3\sqrt{6} + 2\sqrt{18}$

23)  $3\sqrt{24} - 3\sqrt{27} + 2\sqrt{6} + 2\sqrt{8}$

24)  $2\sqrt{6} - \sqrt{54} - 3\sqrt{27} - \sqrt{3}$

25)  $-2\sqrt[3]{16} + 2\sqrt[3]{16} + 2\sqrt[3]{2}$

26)  $3\sqrt[3]{135} - \sqrt[3]{81} - \sqrt[3]{135}$

27)  $2\sqrt[4]{243} - 2\sqrt[4]{243} - \sqrt[4]{3}$

28)  $-3\sqrt[4]{4} + 3\sqrt[4]{324} + 2\sqrt[4]{64}$

29)  $3\sqrt[4]{2} - 2\sqrt[4]{2} - \sqrt[4]{243}$

30)  $2\sqrt[4]{6} + 2\sqrt[4]{4} + 3\sqrt[4]{6}$

31)  $-\sqrt[4]{324} + 3\sqrt[4]{324} - 3\sqrt[4]{4}$

32)  $-2\sqrt[4]{243} - \sqrt[4]{96} + 2\sqrt[4]{96}$

33)  $2\sqrt[4]{2} + 2\sqrt[4]{3} + 3\sqrt[4]{64} - \sqrt[4]{3}$

34)  $2\sqrt[4]{48} - 3\sqrt[4]{405} - 3\sqrt[4]{48} - \sqrt[4]{162}$

35)  $-3\sqrt[5]{6} - \sqrt[5]{64} + 2\sqrt[5]{192} - 2\sqrt[5]{64}$

36)  $-3\sqrt[7]{3} - 3\sqrt[7]{768} + 2\sqrt[7]{384} + 3\sqrt[7]{5}$

37)  $2\sqrt[5]{160} - 2\sqrt[5]{192} - \sqrt[5]{160} - \sqrt[5]{-160}$

38)  $-2\sqrt[7]{256} - 2\sqrt[7]{256} - 3\sqrt[7]{2} - \sqrt[7]{640}$

39)  $-\sqrt[6]{256} - 2\sqrt[6]{4} - 3\sqrt[6]{320} - 2\sqrt[6]{128}$

## SOLUTIONS

1)  $6\sqrt{5}$

2)  $-3\sqrt{6} - 5\sqrt{3}$

3)  $-3\sqrt{2} + 6\sqrt{5}$

4)  $-5\sqrt{6} - \sqrt{3}$

5)  $-5\sqrt{6}$

6)  $-3\sqrt{3}$

7)  $3\sqrt{6} + 5\sqrt{5}$

8)  $-\sqrt{5} + \sqrt{3}$

9)  $-8\sqrt{2}$

10)  $-6\sqrt{6} + 9\sqrt{3}$

11)  $-3\sqrt{6} + \sqrt{3}$

12)  $-2\sqrt{5} - 6\sqrt{6}$

13)  $-2\sqrt{2}$

14)  $8\sqrt{5} - \sqrt{3}$

15)  $5\sqrt{2}$

16)  $-9\sqrt{3}$

17)  $-3\sqrt{6} - \sqrt{3}$

18)  $3\sqrt{2} + 3\sqrt{6}$

19)  $-12\sqrt{2} + 2\sqrt{5}$

20)  $-3\sqrt{2}$

21)  $-4\sqrt{6} + 4\sqrt{5}$

22)  $-\sqrt{5} - 3\sqrt{6}$

23)  $8\sqrt{6} - 9\sqrt{3} + 4\sqrt{2}$

24)  $-\sqrt{6} - 10\sqrt{3}$

25)  $2^3\sqrt{2}$

26)  $6^3\sqrt{5} - 3^3\sqrt{3}$

27)  $-\sqrt[4]{3}$

28)  $10^4\sqrt{4}$

29)  $\sqrt[4]{2} - 3^4\sqrt{3}$

30)  $5^4\sqrt{6} + 2^4\sqrt{4}$

31)  $6^4\sqrt{3} - 3^4\sqrt{4}$

32)  $-6^4\sqrt{3} + 2^4\sqrt{6}$

33)  $2^4\sqrt{2} + \sqrt[4]{3} + 6^4\sqrt{4}$

34)  $-2^4\sqrt{3} - 9^4\sqrt{5} - 3^4\sqrt{2}$

35)  $\sqrt[5]{6} - 6^5\sqrt{2}$

36)  $\sqrt[7]{3} - 6^7\sqrt{6} + 3^7\sqrt{5}$

37)  $4^5\sqrt{5} - 4^5\sqrt{6}$

38)  $-11^7\sqrt{2} - 2^7\sqrt{5}$

39)  $-4^6\sqrt{4} - 6^6\sqrt{5} - 4^6\sqrt{2}$