Multiplying and Dividing Radical Expressions

Multiplying Radicals

When multiplying radicals, multiply the coefficients and multiply the radicands. You can only multiply radicals if they have the same index. Radicals can be simplified before multiplying.

Example 1: Multiplying Radicals

Multiply. Simplify the product if possible.

a. $(2\sqrt{7})(\sqrt{75})$ b. $(4\sqrt{14x})(2\sqrt{7x^3})$, $x \geq 0$ c. $(\sqrt{3})(-5\sqrt{10}+\sqrt{6})$

d. $(5\sqrt{2x}+\sqrt{5})(-4\sqrt{2x}+\sqrt{5x})$, $x \geq 0$ e. $(7x\sqrt[3]{8xy^2})(3\sqrt[3]{8x^2y^2})$

Solution:

a. $(2\sqrt{7})(\sqrt{75})$

b. $(4\sqrt{14x})(2\sqrt{7x^3})$, $x \geq 0$

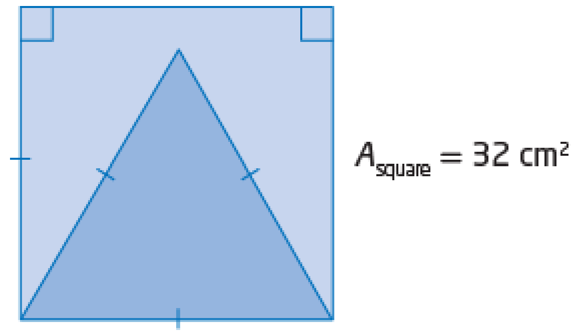
c. $(\sqrt{3})(-5\sqrt{10}+\sqrt{6})$

d. $(5\sqrt{2x}+\sqrt{5})(-4\sqrt{2x}+\sqrt{5x})$, $x \geq 0$

e. $(7x\sqrt[3]{8xy^2})(3\sqrt[3]{8x^2y^2})$

Example 2: Applying Radical Multiplication

The following questions refer to the diagram shown.



a. Determine the exact *perimeter* of the triangle.

b. Determine the exact *height* of the triangle.

c. What is the exact *area* of the triangle?

Solution:

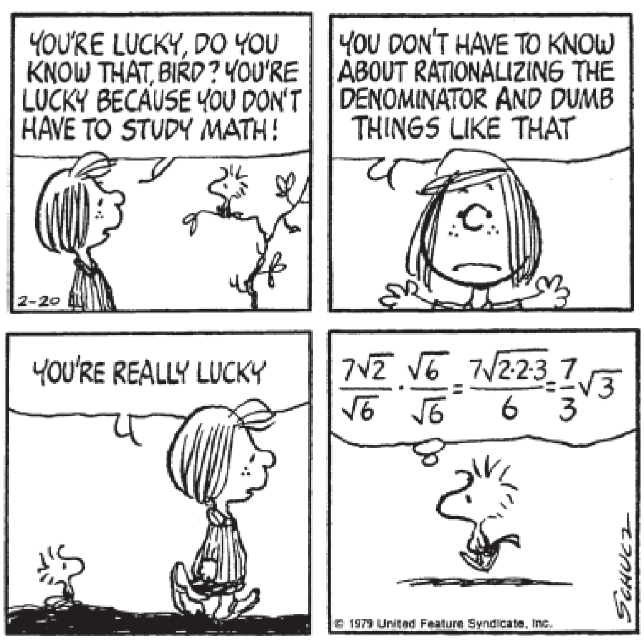
Dividing Radicals

When dividing radicals, divide the coefficients and divide the radicands. You can only divide radicals that have the same index.

Rationalizing the Denominator

NEVER leave a radical in the denominator. To simplify an expression that has a radical in the denominator, we *rationalize* the denominator. This means to convert the denominator to a rational number without changing the value of the expression.

Monomial Square-Root Denominator	Binomial Square-Root Denominator
$\begin{aligned}&\frac{2}{3\sqrt{5}}\\ &=\frac{2}{3\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}\\ &=\frac{2\sqrt{5}}{15}\end{aligned}$ Multiply the numerator and denominator by the radical term in the denominator.	$\begin{aligned}&\frac{2\sqrt{3}}{5-\sqrt{8}}\\ &=\frac{2\sqrt{3}}{5-\sqrt{8}}\times\frac{5+\sqrt{8}}{5+\sqrt{8}}\\ &=\frac{10\sqrt{3}+2\sqrt{24}}{25-8}\\ &=\frac{10\sqrt{3}+4\sqrt{6}}{17}\end{aligned}$ Multiply the numerator and denominator by the *conjugate* of the denominator.



Example 3: Dividing Radicals

Simplify each expression.

a. $\dfrac{20\sqrt[3]{6}}{5\sqrt[3]{3}}$ b. $\dfrac{14\sqrt{15xy}}{2\sqrt{5x}}$, $x>0$, $y\geq 0$ c. $\dfrac{8\sqrt{5}}{2\sqrt{3}}$ d. $\dfrac{7}{5\sqrt{3}-\sqrt{2}}$ e. $\dfrac{4+\sqrt{5}}{3-\sqrt{5}}$ f. $\dfrac{2\sqrt[3]{4}}{y\sqrt[3]{6}}$, $y\neq 0$

Solution:

a. $\dfrac{20\sqrt[3]{6}}{5\sqrt[3]{3}}$

b. $\dfrac{14\sqrt{15xy}}{2\sqrt{5x}}$, $x>0$, $y\geq 0$

c. $\dfrac{8\sqrt{5}}{2\sqrt{3}}$

d. $\dfrac{7}{5\sqrt{3}-\sqrt{2}}$

e. $\dfrac{4+\sqrt{5}}{3-\sqrt{5}}$

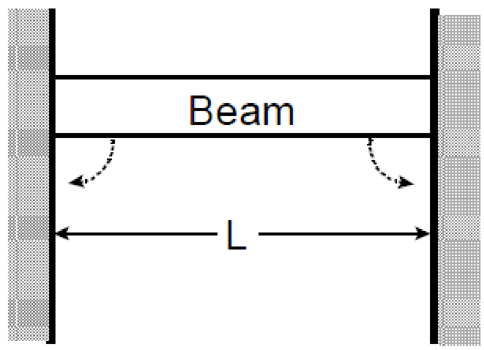
f. $\dfrac{2\sqrt[3]{4}}{y\sqrt[3]{6}}$, $y\neq 0$

Example 4: Applying Radical Division

In stress analysis, the bending moment, M (a measure of its tendency to twist where it is attached to its supports) of a beam of length L (in metres) which is fixed at both ends is given by

$$M = \frac{WL^2}{12}$$

where W is the mass in kilograms of a one metre length of beam. Rearrange this formula to give solve for L, expressed in simplest form.



Solution: