

Radical Equations

A "radical" equation is an equation with radicals that have variables in the radicands.

When solving a radical equation with an index of 2:

- 1. Isolate the radical (or *one* of the radicals if there is more than one in the equation).
- 2. Square *each side* of the equation.
- 3. If you still have a radical left in the equation, repeat steps 1 and 2.
- 4. Solve the resulting equation.
- 5. Check all solutions by substituting into the *original* equation.

Example 1: Solve an Equation with One Radical Term

State any restrictions on x so the radical is a real number and then solve the equation.

a. $\sqrt{x-1}+7=13$

b. $x-\sqrt{4x-15}=3$

Solution:

a. $\sqrt{x-1}+7=13$		b. $x-\sqrt{4x-15}=3$
	Isolate the radical. Square each side of the equation. Solve the resulting equation.	
	Check the solution(s) by substituting into the <i>original</i> equation.	
Solution:		Solution:

Example 2: Radical Equation with an Extraneous Root

An *extraneous solution* is a solution that emerges from the process of solving an equation, but is not a valid solution to the original equation.

State the restrictions on the variable so that the equation involves real numbers and then solve the equation.

a. $x + \sqrt{x - 2} = 4$

b. $3\sqrt{1 - x} - x = -3$

Solution:

a. $x + \sqrt{x - 2} = 4$		b. $3\sqrt{1 - x} - x = -3$
	Isolate the radical. Square each side of the equation. Solve the resulting equation.	
	Check the solution(s) by substituting into the <i>original</i> equation.	
Solution:		Solution:

Example 3: Solve an Equation with Two Radicals

Solve the radical equation.

a. $\sqrt{2x+5} = 2\sqrt{2x} + 1$

b. $\sqrt{5x+1} - \sqrt{3x-5} = 2$

Solution:

a. $\sqrt{2x+5} = 2\sqrt{2x} + 1$		b. $\sqrt{5x+1} - \sqrt{3x-5} = 2$
	Isolate one of the radicals. Square each side of the equation. Isolate the other radical. Square each side of the equation. Solve the resulting equation.	
	Check the solution(s) by substituting into the <i>original</i> equation.	
Solution:		Solution:

Example 4: Solve Problems Involving Radical Equations

A cylindrical container of chocolate drink mix has a volume of 162 cubic inches. The radius r of the container can be found by using the formula $r = \sqrt{\frac{V}{\pi h}}$, where V is the volume of the container and h is the height.

If the radius is 2.5 inches, find the height of the container. Round your answer to the nearest hundredth.

Solution: