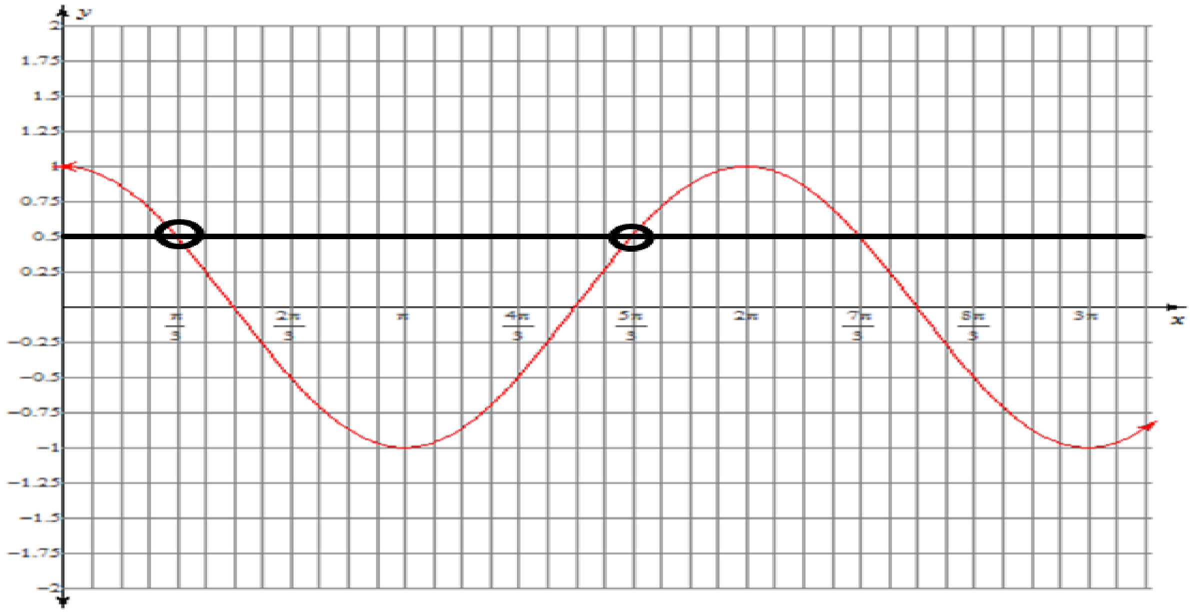
Equations and Graphs of Trigonometric Functions

For any trigonometric equation, we can solve *graphically* as well as *algebraically*.

Example 1: Solving a Trigonometric Equation Graphically and Algebraically

Solve $\cos\theta = \frac{1}{2}$ in the domain $[0, 2\pi)$ and then state the general solution.



GRAPHICALLY:

Consider the graph of the function $y = \cos\theta$ shown. If we draw a line at y = ½, we can see where the line intersects the cosine curve.

Check for solutions in the domain $[0, 2\pi)$: θ = ____________________

The *general* solution would be: θ =

ALGEBRAICALLY:

$\cos\theta = \frac{1}{2}$ Use your calculator and/or unit circle to determine θ_R = ___________.

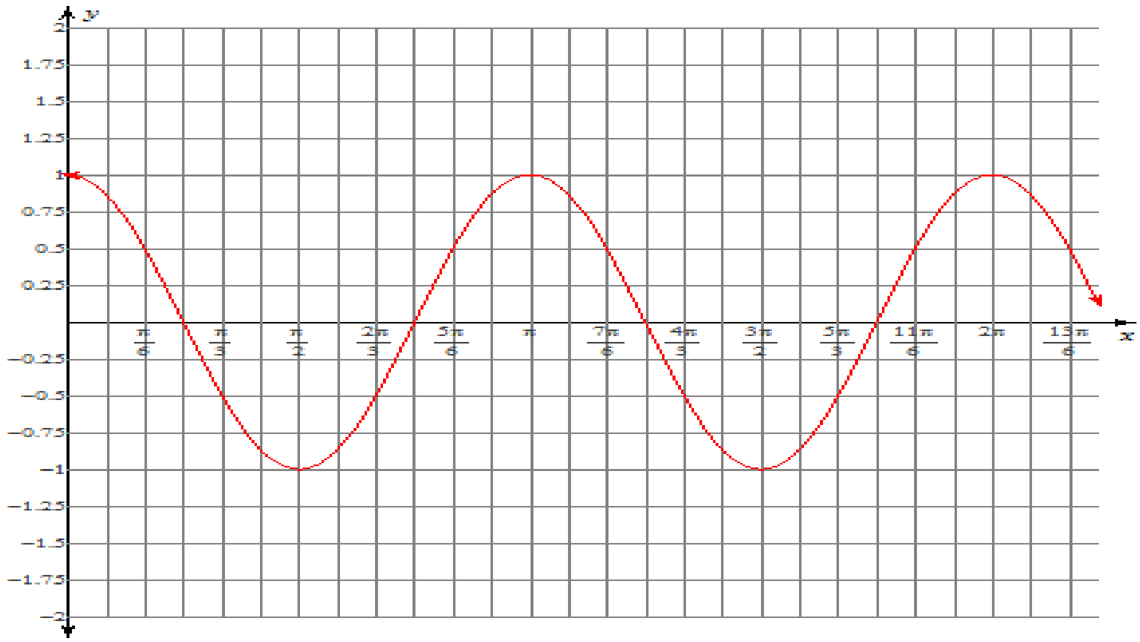
The terminal arm of angle θ lies in either quadrant _____ or _____.

Solutions in the domain $[0, 2\pi)$: θ = ____________________

General solution: θ =

Example 2: Solving a Trigonometric Equation Graphically and Algebraically

Solve $\cos 2\theta = \frac{1}{2}$ in the domain $[0, 2\pi)$ and then state the general solution.



GRAPHICALLY

Consider the graph of the function $y = \cos 2\theta$ shown. If we draw a line at y = ½, we can see where the line intersects the curve.

Check for solutions in the domain $[0, 2\pi)$: θ = ____________________

The *general* solution would be: θ =

ALGEBRAICALLY:

$\cos 2\theta = \frac{1}{2}$

Step 1: Let $a = 2\theta$, then $\cos a = \frac{1}{2}$

Step 2: General solution: $a =$

Step 3: So, $2\theta =$

Step 4: General solution: $\theta =$

Step 5: Solutions in the domain $[0, 2\pi)$: $\theta =$ ___________________________________

Example 3: Solving Trigonometric Equations

Solve for θ in general form and then in the specified domain. Use exact solutions when possible.

a. $\sec 3\theta = \frac{3}{2},\ 0 \le \theta \le 360°$

b. $\sqrt{3}\csc(\theta + 20°) + 2 = 0,\ \ -180° \le \theta < 360°$

c. $3\tan\left(\theta - \frac{\pi}{4}\right) = -\sqrt{3},\ 0 \le \theta \le 3\pi$

d. $16 = 8\cot\frac{1}{3}\theta,\ \ -4\pi \le \theta \le 4\pi$

e. $\sin\left(\frac{1}{2}\theta - 40°\right) = 0,\ \ -720° \le \theta < 720°$

f. $\sqrt{2}\cos\left(2(\theta + 1)\right) + 1 = 0,\ \ -\pi \le \theta \le \pi$

Solution:

$\sec 3\theta = \frac{3}{2},\ 0 \le \theta \le 360°$
$\sqrt{3}\csc(\theta + 20°) + 2 = 0,\ \ -180° \le \theta < 360°$

$3\tan\left(\theta - \frac{\pi}{4}\right) = -\sqrt{3},\ 0 \le \theta \le 3\pi$

$16 = 8\cot\frac{1}{3}\theta,\ \ -4\pi \le \theta \le 4\pi$

$\sin\left(\frac{1}{2}\theta - 40^\circ\right) = 0,\ \ -720^\circ \le \theta < 720^\circ$

$\sqrt{2}\cos\left(2(\theta+1)\right) + 1 = 0,\ \ -\pi \le \theta \le \pi$

Example 4: Solving a Trigonometric Equation Application

The London Eye is a huge Ferris wheel in London, England. It has a diameter of 135 meters and completes one rotation every 30 minutes. The height of a rider on the London Eye can be determined by the equation:

$$h(t) = -67.5\cos\left(\frac{\pi}{15}t\right) + 69.5$$

where h(t) is the height in metres and t is the time in minutes.
In one rotation, for how many minutes is the rider more than 100 metres above ground?

Solution:

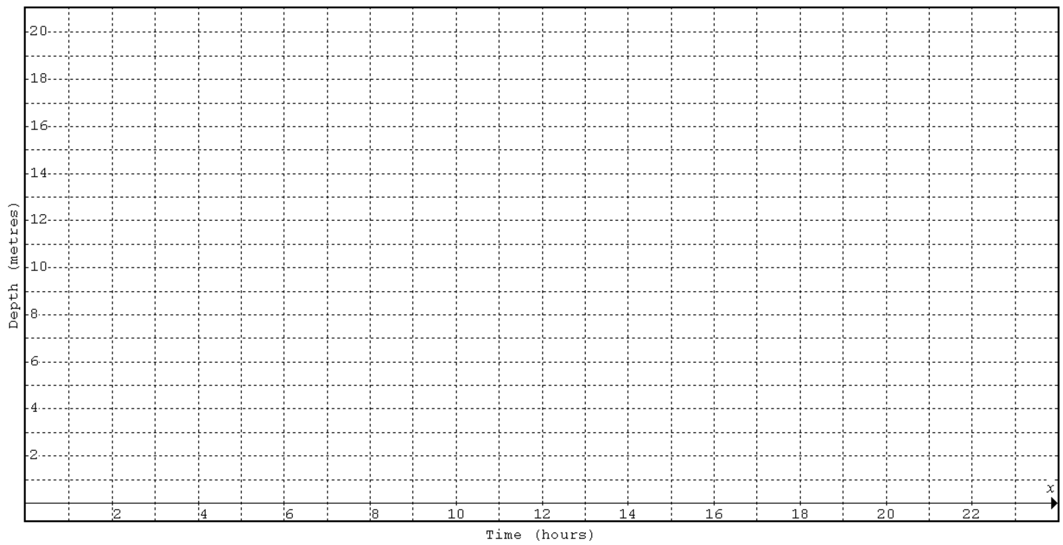
Example 5: Solving a Trigonometric Equation Application

Prince Rupert, BC has the deepest natural harbour in North America. The depth, d, in metres, of the berths for the ships can be approximated by the equation $d(t) = 8\cos\frac{\pi}{6}t + 12$, where t is the time, in hours, after the first high tide.

a. Graph the function for two cycles.
b. What is the *period* of the tide?
c. An ocean liner requires a minimum of 13m of water to dock safely. Determine the number of hours per cycle the ocean liner can safely dock.
d. What is the minimum depth of the water? At what times is the water level at a minimum?

Solution:

a. Graph:



b. Period of the tide: _______________

c. Number of hours per cycle the ocean liner can safely dock:

d. Minimum depth of the water: _______________

Times when depth is a minimum: _____________________________________