

# Rational Expressions

A rational expression is an algebraic fraction of the form  $\frac{p}{q}$ , where  $p$  and  $q$  are polynomials and  $q \neq 0$ .

Examples :

$$\frac{-5}{a} \qquad \frac{6r^2p^3}{4rp^4} \qquad \frac{x}{x^2 - 5x} \qquad \frac{x - 4}{x^2 + 3x - 28}$$

Whenever we are working with rational expressions, we must identify the *non-permissible values* of the variables. A non-permissible value is a value of the variable that causes the expression to be undefined. For rational expressions, this occurs when the denominator is equal to 0.

## Example 1: Determine Non-Permissible Values

For each rational expression, determine all non-permissible values.

a.  $\frac{5z}{7xy^2}$                       b.  $\frac{-4x}{5x^2 - 4x}$                       c.  $\frac{x^2 - 9x + 14}{x^2 - 25}$

**Solution:**

Remember that *denominators* cannot equal zero.

a.  $\frac{5z}{7xy^2}$

b.  $\frac{-4x}{5x^2 - 4x}$

c.  $\frac{x^2 - 9x + 14}{x^2 - 25}$

## Example 2: Simplify a Rational Expression

To simplify a rational expression, factor the numerator and denominator. Then divide both the numerator and denominator by any factors that are common to both. When a rational expression is in simplest form, the numerator and denominator will have no common factors other than 1. **Consider the factors of the denominator to find the non-permissible values *before* simplifying the expression.**

Simplify each rational expression. State the non-permissible values.

a.  $\frac{4x^2y^3}{20x^3y}$

b.  $\frac{x^2 - 3x - 18}{x^2 - 10x + 24}$

c.  $\frac{4x^2 - 1}{2x^2 - 5x - 3}$

**Solution:**

a.  $\frac{4x^2y^3}{20x^3y}$

b.  $\frac{x^2 - 3x - 18}{x^2 - 10x + 24}$

c.  $\frac{4x^2 - 1}{2x^2 - 5x - 3}$

### Example 3: Recognize Additive Inverses

Simplify each rational expression and state the non-permissible values.

a.  $\frac{9-x}{x^2-81}$

b.  $\frac{10-2x}{x^2-x-20}$

**Solution:**

a.  $\frac{9-x}{x^2-81} =$

Note that  $(9-x)$  in the numerator is the *additive inverse* of the factor  $(x-9)$  in the denominator. In other words,  $(9-x)$  and  $(x-9)$  are negatives of each other and you can, therefore, cancel these factors with a “-1” remainder .

b.  $\frac{10-2x}{x^2-x-20} =$

### Example 4: Rational Expressions with Pairs of Non-Permissible Values

Consider the expression  $\frac{4x^2y+10xy}{6xy^2-8x^2y}$  .

- Simplify the rational expression and state the non-permissible values of  $x$  and  $y$ .
- Evaluate the expression for  $x = 2.6$  and  $y = 1.2$ .

**Solution:**

- Simplify the rational expression and state the non-permissible values of  $x$  and  $y$ .

Examples of non-permissible values are  $(0, \underline{\hspace{1cm}})$  ,  $(\underline{\hspace{1cm}}, 1)$  ,  $(2, \underline{\hspace{1cm}})$

- Evaluate the expression for  $x = 2.6$  and  $y = 1.2$ . Show two ways to determine the answer.