

Properties of Graphs of Quadratic Functions

Quadratic Function – A function that can be written in the *standard form* $y = ax^2 + bx + c$, where $a \neq 0$; for example, $y = 2x^2 + 8x + 7$. The degree of all quadratic functions is 2.

Parabola – The symmetrical curve of the graph of a quadratic function.

Vertex of a Parabola: The lowest point on the graph if the graph opens upward or the highest point on the graph if the graph opens downward.

Minimum / Maximum Values: If the vertex is the *lowest* point on the graph, then the y-coordinate of the vertex is called the *minimum value*. It is the smallest value in the range of the function. If the vertex is the *highest* point on the graph, then the y-coordinate of the vertex is called the *maximum value*. It is the greatest value in the range of the function.

Axis of Symmetry: A vertical line through the vertex that divides the graph of a quadratic function into two congruent halves. The x-coordinate of the vertex defines the equation of the axis of symmetry.

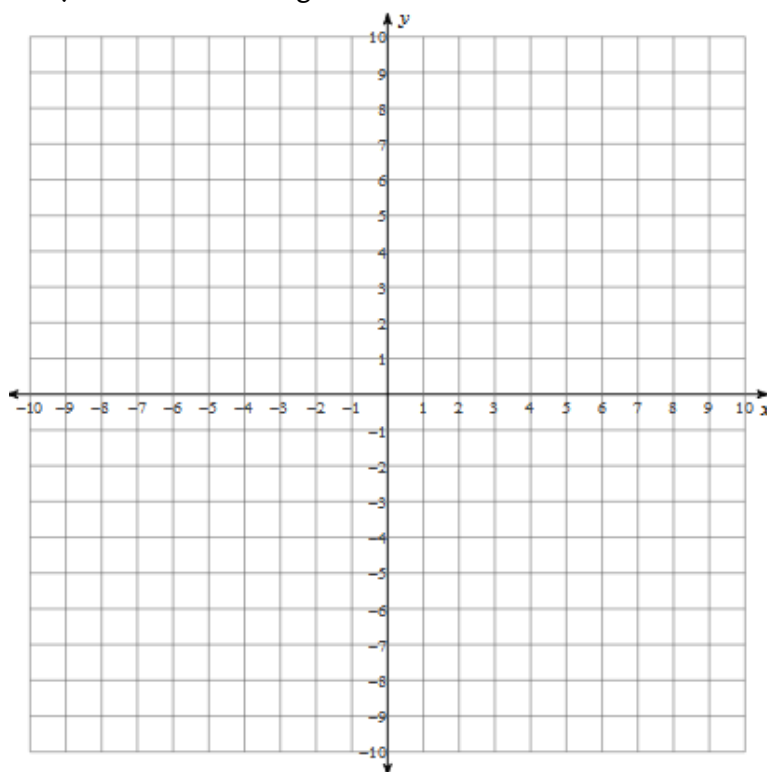
Example 1: Investigate the Basic Quadratic Function $y = x^2$

For the basic quadratic function $y = x^2$, complete the given table of values and sketch the graph on the grid provided.

State the direction of opening, coordinates of the vertex, maximum or minimum value of the function, equation of the axis of symmetry, domain and range of the function.

$y = x^2$	
x	y
-3	
-2	
-1	
0	
1	
2	
3	

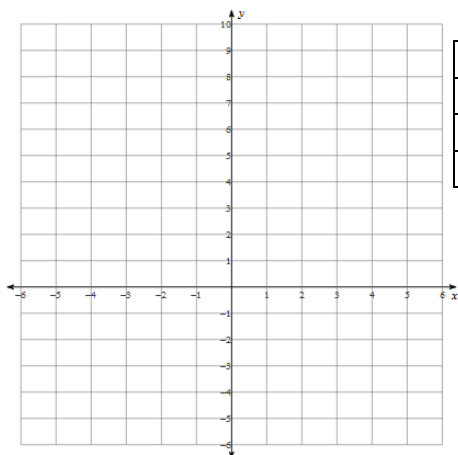
Direction of Opening	
Vertex	
Max / Min Value	
Axis of Symmetry	
Domain	
Range	



Example 2: Identify characteristics and investigate the effects of “a” and “c” for a quadratic function $y = ax^2 + bx + c$

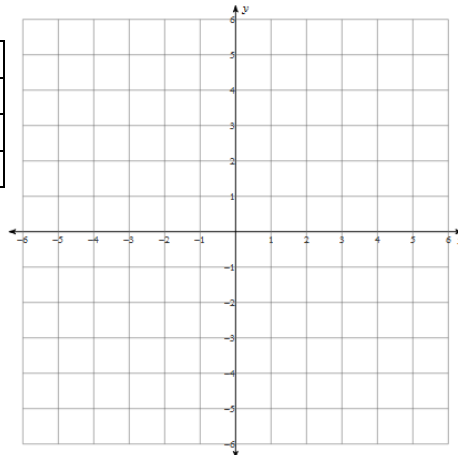
Use Desmos to graph each of the following functions, sketch the graph on the grid provided, and identify the given characteristics.

a. $y = 2x^2 + 8x + 7$



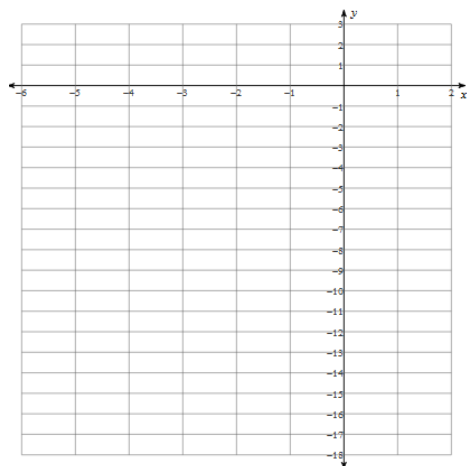
Direction of Opening	
Max / Min Value	
Axis of Symmetry	
y-intercept	

b. $y = -x^2 + 2x + 3$



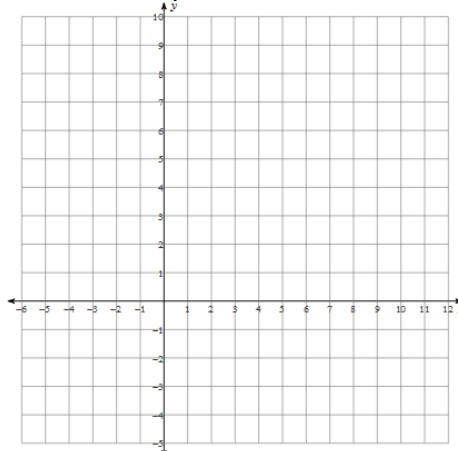
Direction of Opening	
Max / Min Value	
Axis of Symmetry	
y-intercept	

c. $y = -3x^2 - 12x - 15$



Direction of Opening	
Max / Min Value	
Axis of Symmetry	
y-intercept	

d. $y = \frac{1}{4}x^2 - 2x + 5$



Direction of Opening	
Max / Min Value	
Axis of Symmetry	
y-intercept	

Note: For any quadratic function $y = ax^2 + bx + c$,

- The parabola will open *upward* if the value of _____ is _____.
- The parabola will open *downward* if the value of _____ is _____.
- The value of the y-intercept of a parabola is the value of the constant term _____.

Example 3: Determine the Maximum Value Using a Table of Values

At a local splash pad, the water jets spray from ground level. The path of water from one of these jets forms an arch that can be defined by the function $f(x) = -0.12x^2 + 3x$, where $f(x)$ represents the height, in feet, of the water and x represents the horizontal distance, in feet, from the opening in the ground. Determine the maximum height of the arch of water, and how far from the opening in the ground the water can reach.



Solution:

The function $f(x) = -0.12x^2 + 3x$ has degree _____, so this is a _____ function.
The arch must be a _____.

The coefficient, a , of x^2 is negative, so the parabola opens _____. This means that the function has a maximum value, which will be the *maximum height* of the arch of water.

The y-intercept of the function is _____. This confirms that the stream of water shoots from ground level.

We can construct a table of values to help us determine the maximum height of the arch of water, and how far from the opening in the ground the water can reach.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$f(x)$															

Based on symmetry and the table of values, the vertex will be located halfway between _____ and _____.

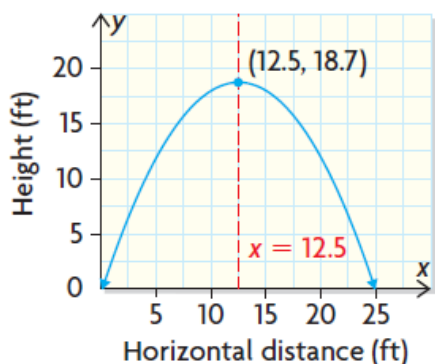
Coordinates of the vertex:

$x =$

$f(x) =$

Maximum horizontal distance: $x =$

Maximum height: At a horizontal distance of $x = 12.5$ ft, $f(x) =$



Example 4: Graph a Quadratic Function Using a Table of Values

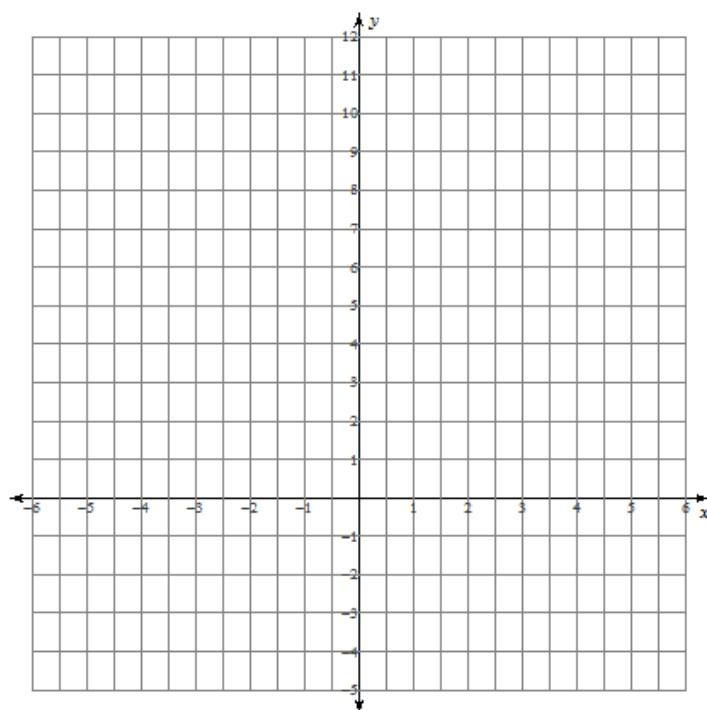
Sketch the graph of the function $y = x^2 + x - 2$. Determine the y-intercept, x-intercepts, equation of the axis of symmetry, coordinates of the vertex, domain and range of the function.

Solution:

Include the y-intercept _____ in a table of values and determine some other points by substituting values of x into the equation $y = x^2 + x - 2$.

x				0		
y				-2		

Sketch the graph:



axis of symmetry:

y-coordinate of the vertex:

The y-intercept is _____.

The x-intercepts are _____ and _____.

The equation of the axis of symmetry is _____.

The coordinates of the vertex are _____.

The domain is _____.

The range is _____.