

Adding and Subtracting Rational Expressions

To add or subtract rational expressions, follow procedures similar to those used in adding and subtracting rational numbers.

$$\frac{2}{3} + \frac{4}{5} = \frac{2(5)}{3(5)} + \frac{4(3)}{5(3)} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15}$$

All fractions must have a common denominator before you can add or subtract.

Example 1: Add or Subtract Rational Expressions with Non-Variable Denominators

To Add or Subtract Rational Expressions with Non-Variable Denominators
1. Determine the LCD.
2. Rewrite each fraction as an equivalent fraction with the LCD. This is done by multiplying both the numerator and denominator of each fraction by any factors needed to obtain the LCD.
3. Add or subtract the numerators while maintaining the LCD.
4. Simplify.

Determine each sum or difference. Express each answer in simplest form.

a. $\frac{3x}{4} + \frac{x}{5} - \frac{7x}{10}$	b. $\frac{5a-1}{3} - \frac{4a+5}{4}$ <i>Note: When subtracting rational expressions, the entire numerator of the second fraction must be subtracted, not just the first term.</i>	c. $\frac{7x+3}{2} + 3x - \frac{x-3}{8}$
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To Add or Subtract Rational Expressions with a Common Denominator

1. Add or subtract the numerators.
2. Place the sum or difference of the numerators over the common denominator.
3. Simplify the fraction, if possible.

a. $\frac{3}{2a} - \frac{5}{2a} + \frac{7}{2a}$

b. $\frac{6}{x-5} - \frac{x+2}{x-5}$

$$\text{c. } \frac{x^2 + 3x - 2}{(x + 5)(x - 2)} + \frac{4x + 12}{(x + 5)(x - 2)}$$

When you add or subtract rational expressions, you begin by finding a common denominator. The lowest common denominator, LCD, of a collection of denominators is the smallest expression that is divisible by each of the given denominators. This is equivalent to saying that the lowest common denominator is the least common multiple, LCM, of all denominators.

1. Factor each denominator completely. Any factor that occurs more than once should be expressed as a power.
2. List all the different factors that appear in any of the denominators. Use the highest power that appears in any of the denominators for each factor.
3. The least common denominator is the product of all the factors listed in the previous step.

$$\frac{2}{3y} + \frac{6}{5}$$

$$\frac{x+7}{2x^2} - \frac{3}{5x}$$

$$\frac{1}{18x^3v} + \frac{5}{27x^2v^3}$$

$$\frac{7}{3v(v-2)} + \frac{x^2}{(v-2)(v-2)}$$

$$3y = 3 \cdot y$$

$$2x^2 = 2 \cdot x^2$$

$$18x^3y = 2 \cdot 3^2 \cdot x^3 \cdot y$$

$$3x(x-2) = 3 \cdot x \cdot (x-2)$$

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Example 4: Add or Subtract Rational Expressions with Unlike Monomial Denominators**To Add or Subtract Two Rational Expressions with Unlike Denominators**

1. Determine the LCD.
2. Rewrite each fraction as an equivalent fraction with the LCD. This is done by multiplying both the numerator and denominator of each fraction by any factors needed to obtain the LCD.
3. Add or subtract the numerators while maintaining the LCD.
4. Simplify the fraction.

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

a. $\frac{2}{3y} + \frac{6}{5}$	b. $\frac{x+7}{2x^2} - \frac{3}{5x}$	c. $\frac{1}{18x^3y} + \frac{5}{27x^2y^3}$
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Example 5: Add or Subtract Rational Expressions with Different Monomial/Binomial Denominators

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

a. $\frac{x-9}{2x} + \frac{3x}{x-4}$	b. $\frac{2x-1}{x-5} - \frac{x-4}{x+1}$
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Example 6: Add or Subtract Rational Expressions with Denominators with Common Binomial Factors

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

a. $\frac{5}{(x+1)(x-2)} + \frac{2}{(x+4)(x-2)}$	b. $\frac{5}{x^2-5x} - \frac{x}{5x-25}$	c. $\frac{7}{3x^2-6x} + \frac{x^2}{x^2-4x+4}$
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Example 7: Add or Subtract Rational Expressions with Trinomial Denominators

Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.

a. $\frac{m}{m^2+6m+5} - \frac{2m}{m^2-3m-4}$	b. $\frac{x^2-x-12}{x^2-8x+16} - \frac{x^2+5x-14}{x^2+10x+21}$
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Example 8: Simplifying Complex Fractions

To Simplify Complex Fractions

1. Find a common denominator in both the numerator and the denominator of the complex fraction.
2. Rewrite both the numerator and the denominator as rational expressions with the common denominators from the previous step.
3. Multiply the expression in the numerator by the reciprocal of the expression in the denominator.
4. Simplify.

Simplify the expression $\frac{\frac{4}{x} + \frac{4}{3}}{\frac{1}{x+12} - \frac{1}{x}}$ and state any non-permissible values.

Solution: