Proving Identities

Example 1: Use the Conjugate to Prove an Identity

Prove $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$ for all permissible values of x.

$\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$	
LHS	**RHS**
$\frac{1-\sin x}{\cos x}$	$\frac{\cos x}{1+\sin x}$

Example 2: Use a Common Denominator to Prove an Identity

Prove $\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = 2\tan^2 x$ for all permissible values of x.

$\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = 2\tan^2 x$	
LHS	**RHS**
$\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1}$	$2\tan^2 x$

Example 3: Use Factoring to Prove an Identity

Prove $\sec^4 x - 1 = \dfrac{\sin^2 x + \sin^2 x \cos^2 x}{\cos^4 x}$ for all permissible values of x.

$\sec^4 x - 1 = \dfrac{\sin^2 x + \sin^2 x \cos^2 x}{\cos^4 x}$	
LHS	**RHS**
$\sec^4 x - 1$	$\dfrac{\sin^2 x + \sin^2 x \cos^2 x}{\cos^4 x}$

Example 4: Prove a More Complicated Identity

Prove $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \frac{\cot x - 1}{\cot x}$ for all permissible values of x.

$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \frac{\cot x - 1}{\cot x}$	
LHS	**RHS**
$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x}$	$\frac{\cot x - 1}{\cot x}$

Example 5: Prove a More Complicated Identity

Prove $\cos x + \tan x \sin x = \dfrac{\tan x \sin x}{1 - \cos^2 x}$ for all permissible values of x.

$\cos x + \tan x \sin x = \dfrac{\tan x \sin x}{1 - \cos^2 x}$	
LHS	**RHS**
$\cos x + \tan x \sin x$	$\dfrac{\tan x \sin x}{1 - \cos^2 x}$

Example 6: Prove a More Complicated Identity

Prove $\frac{\sin x - 2\sin^3 x}{2\cos^3 x - \cos x} = \tan x$ for all permissible values of x.

$\frac{\sin x - 2\sin^3 x}{2\cos^3 x - \cos x} = \tan x$	
LHS	**RHS**
$\frac{\sin x - 2\sin^3 x}{2\cos^3 x - \cos x}$	$\tan x$

Example 7: Prove a More Complicated Identity

Prove $\tan x - \cot x = \dfrac{1-2\cos^2 x}{\cos x \sin x}$ for all permissible values of x.

$\tan x - \cot x = \dfrac{1-2\cos^2 x}{\cos x \sin x}$	
LHS	**RHS**
$\tan x - \cot x$	$\dfrac{1-2\cos^2 x}{\cos x \sin x}$

Example 8: Prove a More Complicated Identity

Prove $1-\frac{\sin^2 x \tan x+\cos^2 x}{\tan x+1}=\sin x \cos x$ for all permissible values of x.

$1-\frac{\sin^2 x \tan x+\cos^2 x}{\tan x+1}=\sin x \cos x$	
LHS	**RHS**
$1-\frac{\sin^2 x \tan x+\cos^2 x}{\tan x+1}$	$\sin x \cos x$

Example 9: Prove a More Complicated Identity

Prove $\sin^2 x \tan x + \cos^2 x \cot x + 2\sin x \cos x = \tan x + \cot x$ for all permissible values of x.

$\sin^2 x \tan x + \cos^2 x \cot x + 2\sin x \cos x = \tan x + \cot x$	
LHS	**RHS**
$\sin^2 x \tan x + \cos^2 x \cot x + 2\sin x \cos x$	$\tan x + \cot x$