

Characteristics of Exponential Functions

Exponential function: A function of the form $y = c^x$, where c is a constant ($c > 0$) and x is a variable.

- If $c > 1$, then the function is increasing
- If $0 < c < 1$, then the function is decreasing
- If $c = 1$, then the function is neither increasing nor decreasing

Example 1: Analyze the Graph of an Exponential Function

Graph each exponential function. Then identify the following:

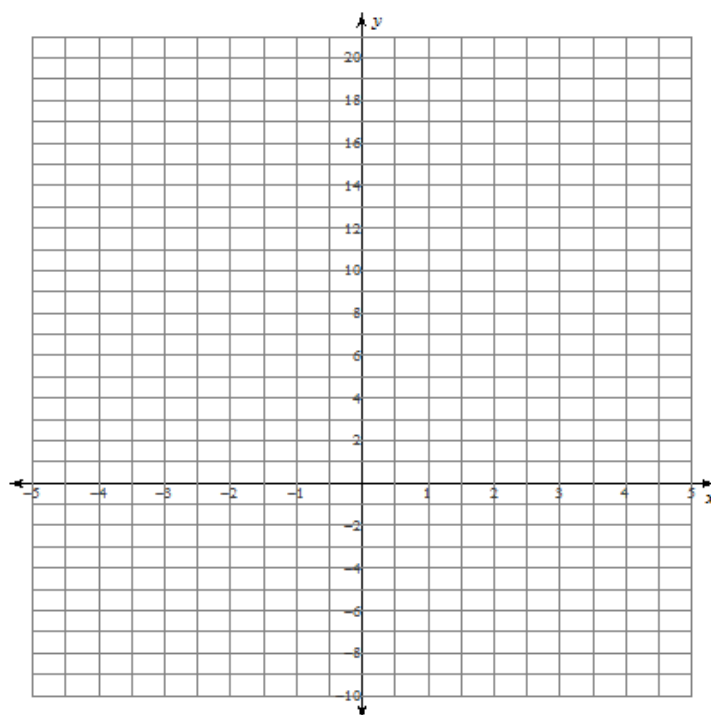
- the domain and range
- the x-intercept and y-intercept, if they exist
- whether the graph represents an increasing or decreasing function
- the equation of the horizontal asymptote

a. $y = 2^x$ b. $f(x) = \left(\frac{1}{4}\right)^x$

Solution:

a. $y = 2^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	
4	



Domain: _____ Range: _____

x-intercept: _____ y-intercept: _____

Is the function increasing or decreasing?: _____

*Note: Exponential functions of the form $y = c^x$, where $c > 1$, can be used to model **exponential growth**.*

Equation of the **horizontal asymptote** *: _____ *As values of x get very small, the graph approaches this line.

b. $f(x) = \left(\frac{1}{4}\right)^x$

x	y
-2	
-1	
0	
1	
2	
3	

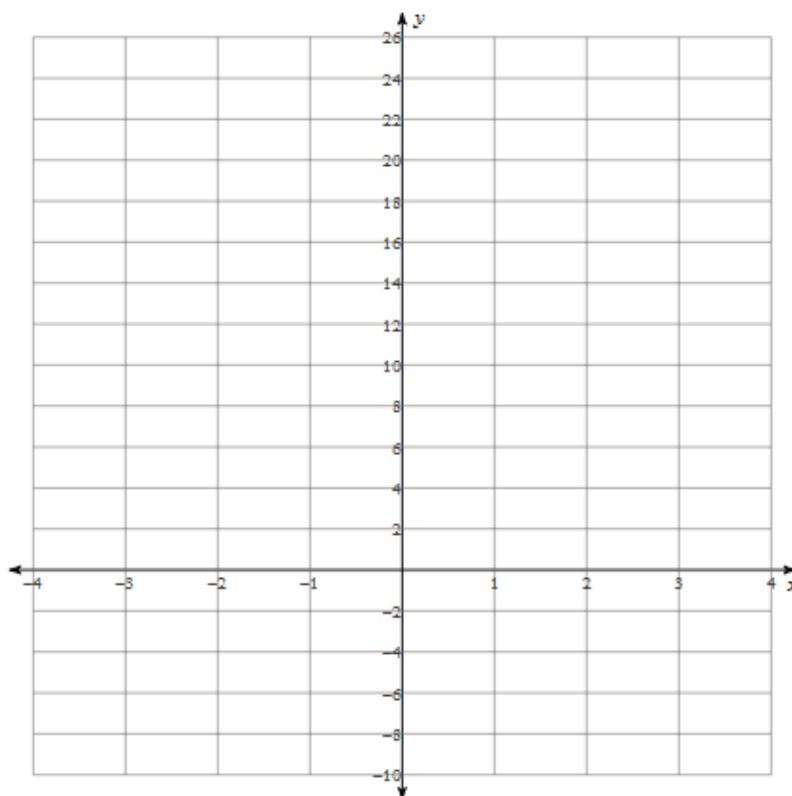
Domain: _____

Range: _____

x-intercept: _____

y-intercept: _____

Is the function increasing or decreasing?

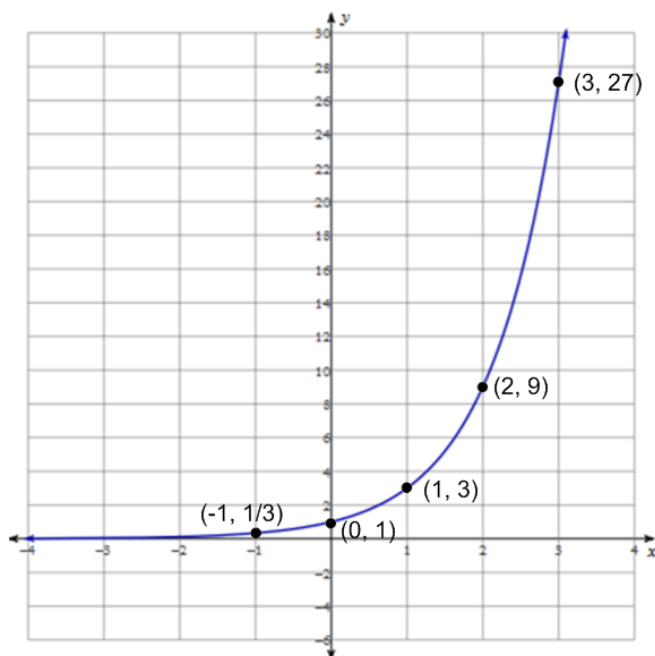


*Note: Exponential functions of the form $y = c^x$, where $0 < c < 1$, can be used to model **exponential decay**.*

Equation of the horizontal asymptote: _____

Example 2: Write the Exponential Function Given its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?



Solution:

Look for a pattern in the ordered pairs from the graph.

x	y
-1	1/3
0	1
1	3
2	9
3	27

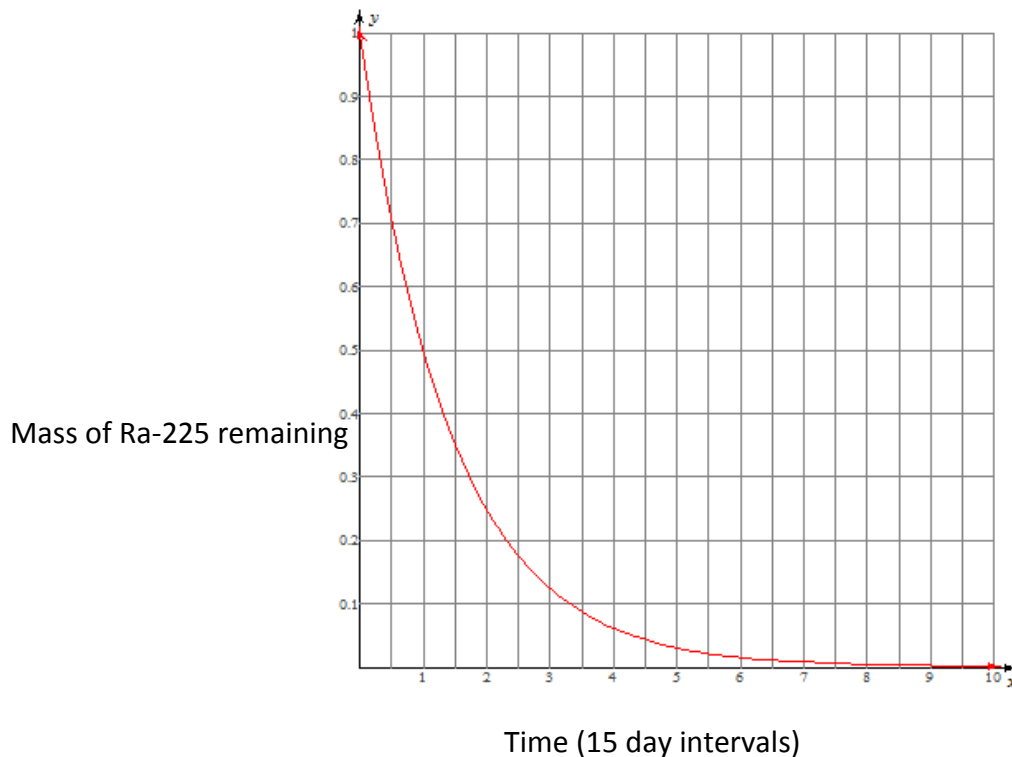
Equation of the function: _____

Choose a point other than (0, 1) to substitute into the equation to verify that the function is correct:

Example 3: Application of an Exponential Function

A radioactive sample of radium (Ra-225) has a half-life of 15 days. The mass, m , in grams, of Ra-225 remaining over time, t , in 15 day intervals, can be modeled using the exponential graph shown.

Half-life: The length of time for an unstable element to spontaneously decay to one half its original mass



- What is the initial mass of Ra-225 in the sample? _____
- What value does the remaining mass of Ra-225 approach as time passes? _____
- Determine the domain and range of this function.
Domain: _____
Range: _____
- Write the exponential decay model that relates the mass of Ra-225 remaining to time, in 15-day intervals.
- Estimate how many days it would take for Ra-225 to decay to $1/30$ of its original mass.