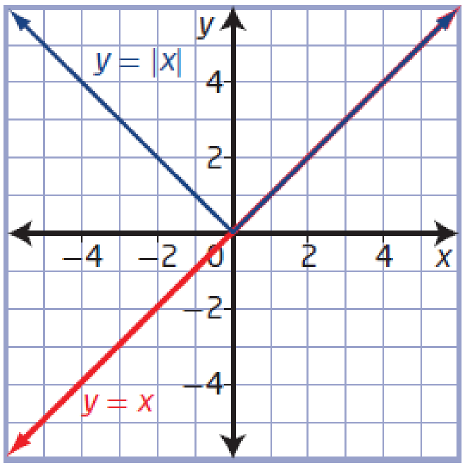
Absolute Value Functions

An absolute value function is a function that involves the absolute value of a variable expression.



Consider the graph of the absolute value function $y = |x|$:

The vertex (0,0) divides the graph of the absolute value function $y = |x|$ into two distinct pieces.

For all values of x greater than or equal to zero, the y-value is x.
For all values of x less than zero, the y-value is –x.

Since the function is defined by two different rules for each interval in the domain, you can define $y = |x|$ as the piecewise function:

$$y = \begin{cases} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{cases}$$

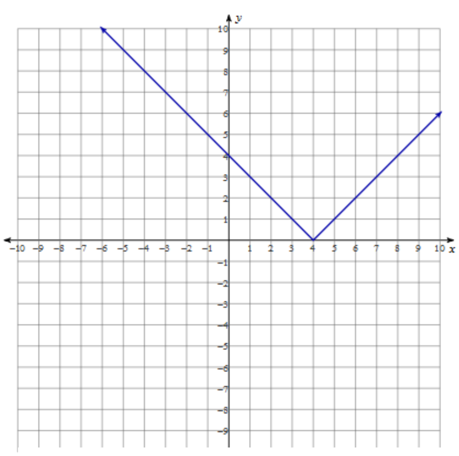
Piecewise function: A function composed of two or more separate functions or pieces, each with its own specific domain, that combine to define the overall function.

The graphs show how $y = |x|$ is related to the graph of $y = x$. Since $|x|$ cannot be negative, the part of the graph of $y = x$ that is *below* the x-axis is reflected in the x-axis to become the line $y = -x$ in the interval x < 0. The part of the graph of that is *on or above* the x-axis remains unchanged as the line $y = x$ in the interval x ≥ 0.

Example 1: Defining Absolute Value Functions of the Form $y = |ax + b|$ as Piecewise Functions

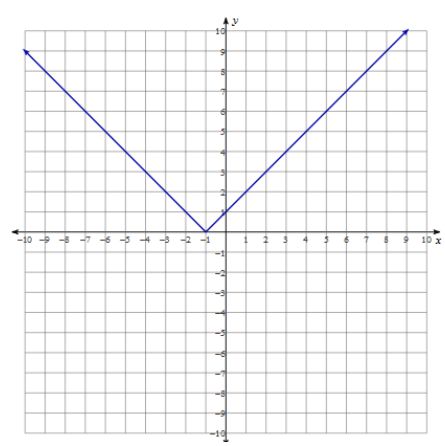
1. Define each of the following absolute value functions as piecewise functions by considering their graphs.

a. $y = |x - 4|$



$y = \{$

b. $y = |-x - 1|$



$y = \{$

2. Define each of the following absolute value functions as piecewise functions by considering the algebraic definition of absolute value.

   a. $y = |5x + 10|$

   $$y = \begin{cases} \\ \end{cases}$$

   b. $y = |-2x - 8|$

   $$y = \begin{cases} \\ \end{cases}$$

Example 2: Graph an Absolute Value Function of the Form $y = |ax + b|$

Consider the absolute value function $y = |2x - 1|$.

a. Determine the y-intercept and the x-intercept.
b. Sketch the graph.
c. State the domain and range.
d. Express the absolute value function as a piecewise function.

Solution:

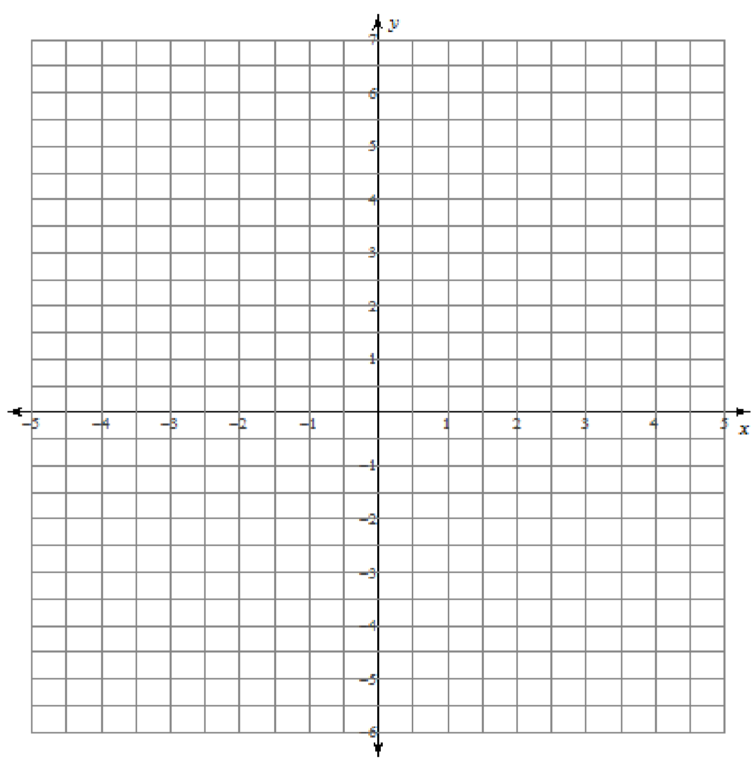
a. To determine the y-intercept, let x =0 and solve for y:

   To determine the x-intercept, let y =0 and solve for x:

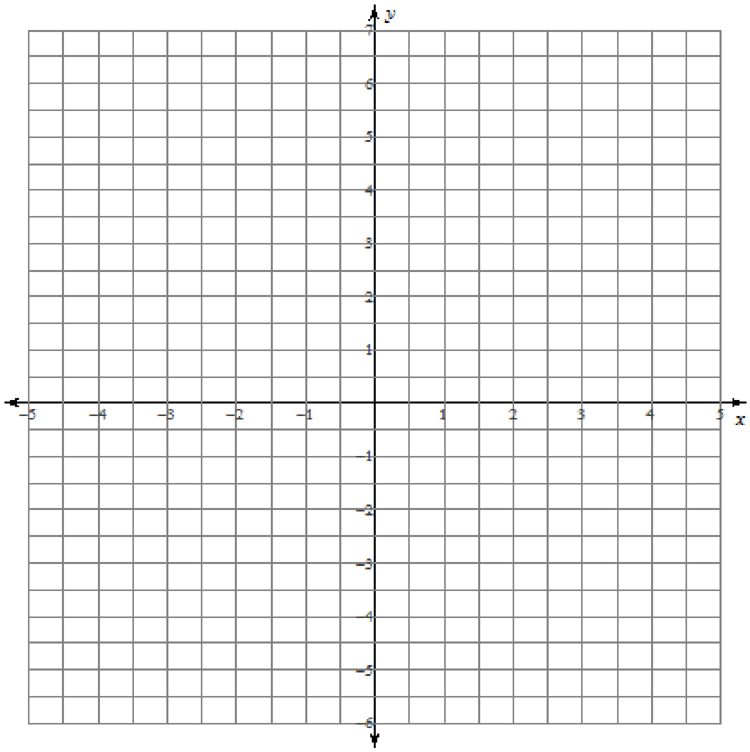
b. **Method 1: Sketch Using a Table of Values**

   Create a table of values using the x-intercept and values to the right and left of it. Sketch the graph using the points in the table.

x	$y = \|2x - 1\|$
-1	
0	
$\frac{1}{2}$	
1	
2	



Method 2: Sketch Using the Graph of the Related Linear Function



Sketch the graph of y = 2x – 1.

The x-intercept of the linear function is the x-intercept of the corresponding absolute value function. This point can be referred to as an *invariant point.*

Invariant point: A point that remains unchanged when a transformation is applied to it.

Reflect in the x-axis the part of the graph of y = 2x – 1 that is *below* the x-axis to obtain the final graph of $y = |2x - 1|$.

c. **Domain:** ______________________ **Range:**__________________________

d. The V-shaped graph of the absolute value function $y = |2x - 1|$ is composed of two separate linear functions, each with its own domain.

- For $x \geq \frac{1}{2}$, the graph of $y = |2x - 1|$ is the graph of y = ____________________
- For $x < \frac{1}{2}$, the graph of $y = |2x - 1|$ is the graph of y = ____________________

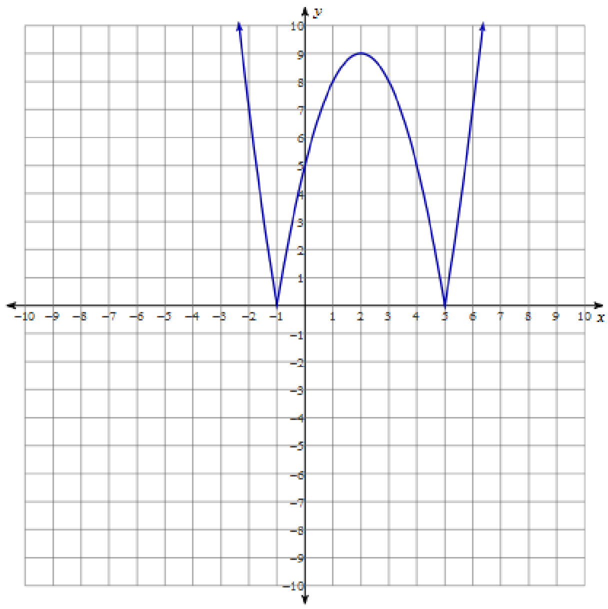
We can combine these two linear functions with their domains to define the absolute value function $y = |2x - 1|$ as the piecewise function:

$$y = \begin{cases} \\ \end{cases}$$

Example 3: Defining Absolute Value Functions of the Form $y = |ax^2 + bx + c|$ as Piecewise Functions

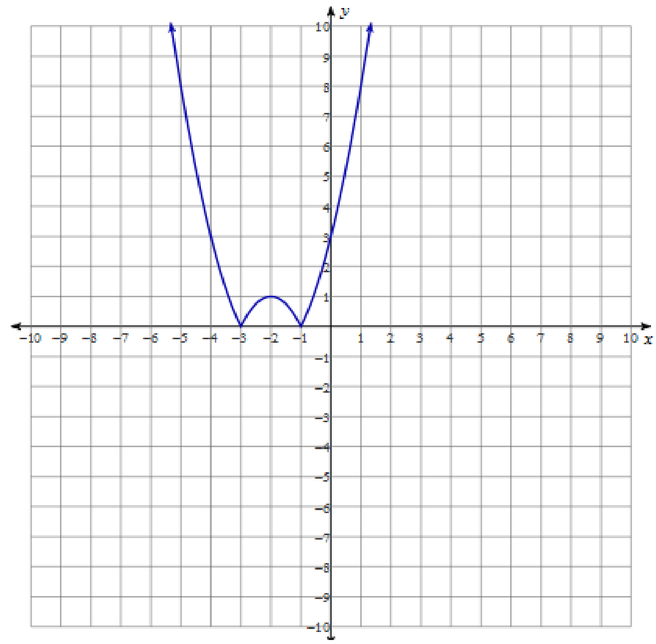
1. Define each of the following absolute value functions as piecewise functions by considering their graphs.

a. $y = |x^2 - 4x - 5|$



$y = \{$

b. $y = |-x^2 - 4x - 3|$



$y = \{$

2. Define each of the following absolute value functions as piecewise functions by considering the algebraic definition of absolute value.

a. $y = |x^2 - 25|$

$y = \{$

b. $y = |-x^2 + 12x - 27|$

$y = \{$

Example 4: Graph an Absolute Value Function of the Form $y = |ax^2 + bx + c|$

Consider the absolute value function $y = |-x^2 - 2x + 8|$.

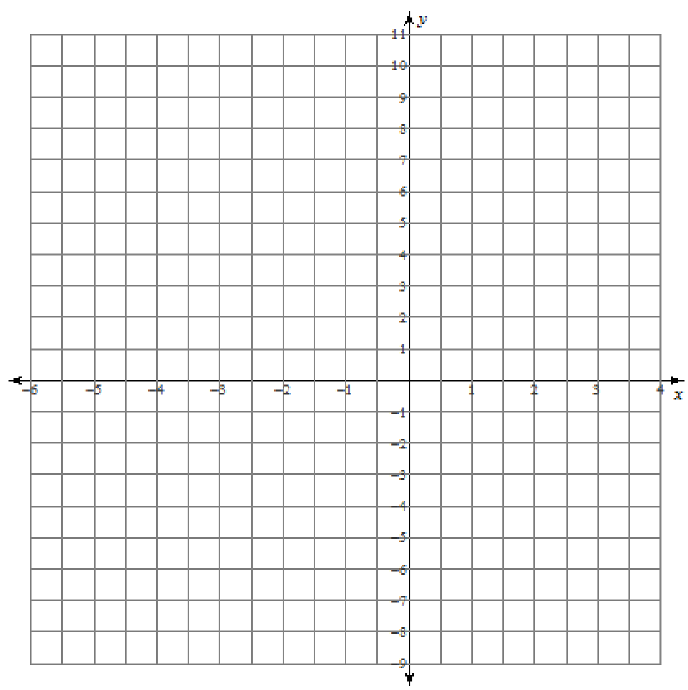
a. Determine the y-intercept and the x-intercepts.
b. Sketch the graph.
c. State the domain and range.
d. Express the absolute value function as a piecewise function.

Solution:

a. To determine the y-intercept, let x =0 and solve for y:

To determine the x-intercept, let y =0 and solve for x:

b. Sketch the graph of $y = |-x^2 - 2x + 8|$ using the graph of the *related* quadratic function $y = -x^2 - 2x + 8$.



State each of the following for $y = -x^2 - 2x + 8$:

- x- intercepts: ____________________
- y-intercept: ____________________
- vertex: ____________________
- direction of opening: ____________________
- other points: (–5, _____) , (3, _____)

Using the information above, sketch the graph of $y = -x^2 - 2x + 8$. Include the point symmetrical to the y-intercept.

Reflect in the x-axis the part of the graph of $y = -x^2 - 2x + 8$ that lies *below* the x-axis to obtain the final graph of $y = |-x^2 - 2x + 8|$.

c. Domain: ________________________ Range: ________________________

d. The graph of $y = |-x^2 - 2x + 8|$ is composed of two separate quadratic functions.

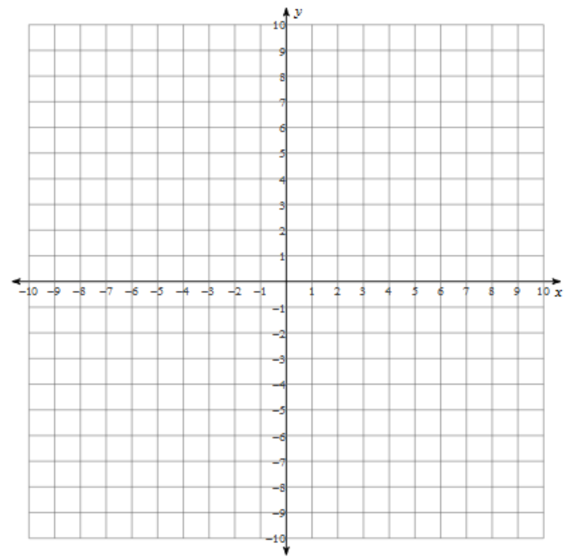
- When $-4 \le x \le 2$, the graph of $y = |-x^2 - 2x + 8|$ is the graph of y = ____________________.
- When $x < -4$ *or* $x > 2$, the graph of $y = |-x^2 - 2x + 8|$ is the graph of y = ____________________.

We can combine these two quadratic functions with their domains to define the absolute value function $y = |-x^2 - 2x + 8|$ as the piecewise function:

$$y = \begin{cases} & \\ & \end{cases}$$

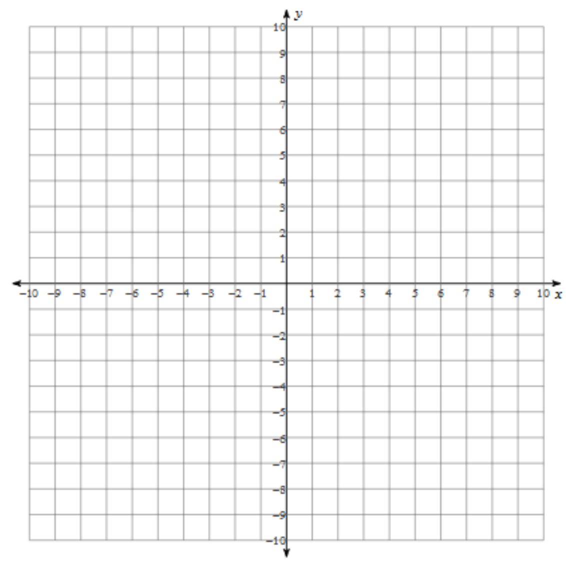
EXTRA PRACTICE:

1. Consider the absolute value function $y = \left|\frac{2}{3}x - 4\right|$.



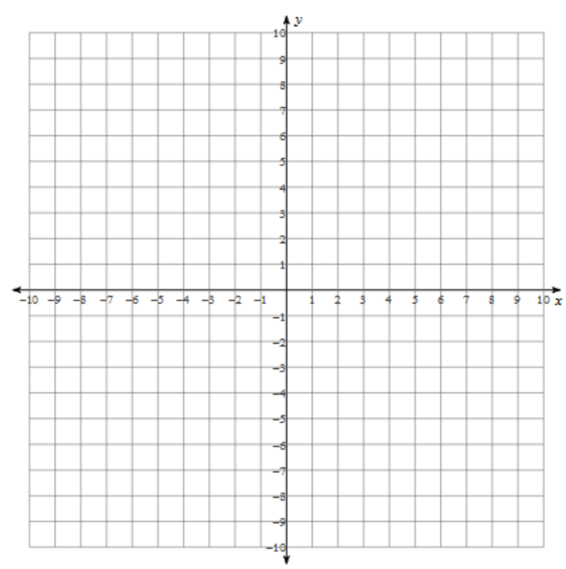
   a. Determine the y-intercept and the x-intercept.
   b. Sketch the graph.
   c. State the domain and range.
   d. Express the absolute value function as a piecewise function.

2. Consider the absolute value function $y = \left|-\frac{3}{4}x - 3\right|$.



   a. Determine the y-intercept and the x-intercept.
   b. Sketch the graph.
   c. State the domain and range.
   d. Express the absolute value function as a piecewise function.

3. Consider the absolute value function $y = \left|(x-3)^2 - 4\right|$.



   a. Determine the y-intercept and the x-intercepts.
   b. Sketch the graph.
   c. State the domain and range.
   d. Express the absolute value function as a piecewise function.