Transformations of Exponential Functions

- To graph an exponential function of the form $y = a(c)^{b(x-h)} + k$, apply transformations to the graph of the base function, $y = c^x$, where $c > 0$.

Example 1: Apply Transformations and Sketch a Graph

Consider the base function $y = 3^x$. For each transformed function,

- State the parameters and describe the corresponding transformations.
- Write the mapping rule.
- Graph the base function and the transformed function on the same grid.
- State the domain, range, intercepts, and equation of the horizontal asymptote.

a. $y = \frac{1}{3}(3)^{x+4}$ b. $y = 2(3)^{-2(x-1)} - 5$

Solution:

a. $y = \frac{1}{3}(3)^{x+4}$

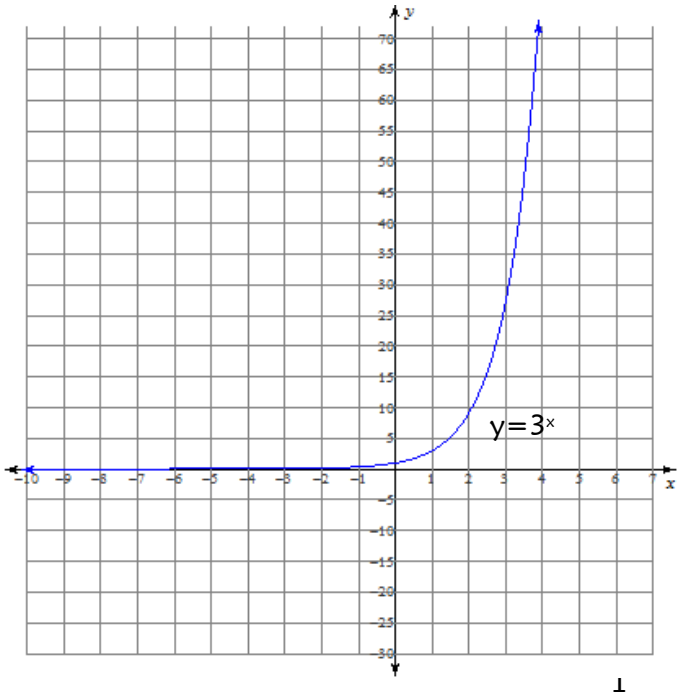
- Compare the function $y = \frac{1}{3}(3)^{x+4}$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.

a = _____, which corresponds to a _____ by a factor of _____.
b = _____, which corresponds to _____.
h = _____, which corresponds to a _____ of _____ units _____.
k = _____, which corresponds to _____.

- Mapping rule: _____
- Complete each table of values and sketch the graph of the function $y = \frac{1}{3}(3)^{x+4}$.

$y = 3^x$	
x	y
–2	
–1	
0	
1	
2	
3	
4	

$y = \frac{1}{3}(3)^{x+4}$	
x	y



- For the function $y = \frac{1}{3}(3)^{x+4}$:

Domain: _____ Range: _____

x-intercept: _____ y-intercept: _____

Equation of the horizontal asymptote : _____

b. $y = 2(3)^{-2(x-1)} - 5$

- Compare the function $y = 2(3)^{-2(x-1)} - 5$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.

  a = _____, which corresponds to a _____ by a factor of _____.

  b = _____, which corresponds to a _____ by factor of _____,

  and a _____ in the _____.

  h = _____, which corresponds to a _____ of _____ unit _____.

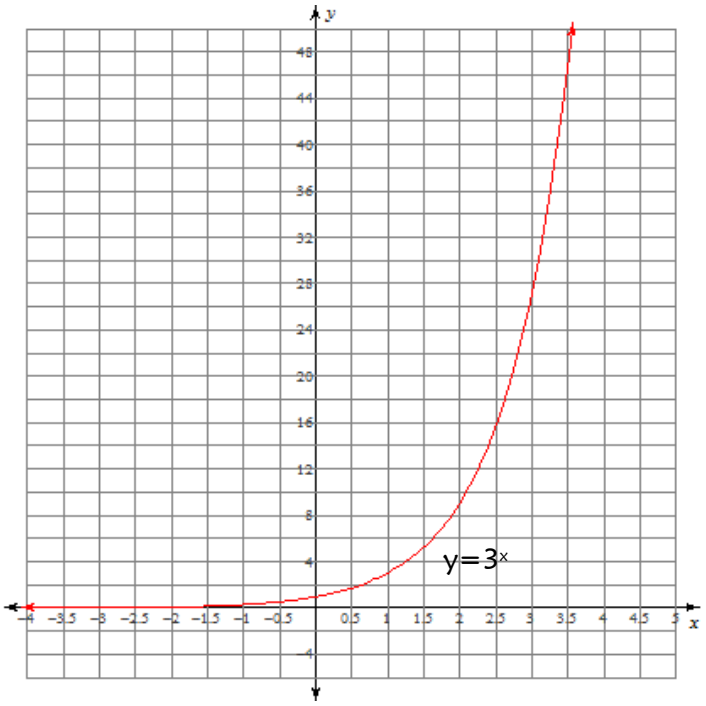
  k = _____, which corresponds to a _____ of _____ units _____.

- Mapping rule: _____

- Complete each table of values and sketch the graph of the function $y = 2(3)^{-2(x-1)} - 5$.

$y = 3^x$	
x	y
–2	
–1	
0	
1	
2	
3	
4	

$y = 2(3)^{-2(x-1)} - 5$	
x	y



- For the function $y = 2(3)^{-2(x-1)} - 5$:

- Domain: _____ Range: _____

  x-intercept: 0.58 y-intercept: _____

Note *– In the next unit, we will learn an algebraic method of solving exponential equations that will enable us to determine the value of the x-intercept.*

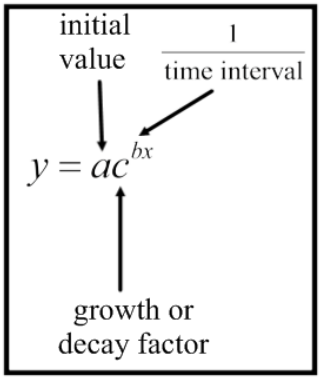
Equation of the horizontal asymptote : _____

Example 2: Use Transformations of an Exponential Function to Model a Situation

An initial population of 2000 insects is expected to triple every 5 days.

a. Write an exponential function in the form $y = a(c)^{bx}$ to model this situation.
b. Use your equation to calculate the insect population in 21 days.

Solution:



a. Determine the exponential function $y = a(c)^{bx}$:

b. Insect population in 21 days:

Example 3: Use Transformations of an Exponential Function to Model a Situation

A hockey card that was purchased for $250 is expected to increase in value by 12% every 3 years.

a. Write an exponential function in the form $y = a(c)^{bx}$ to model this situation.
b. Use your equation to calculate the value of the card in 5 years.

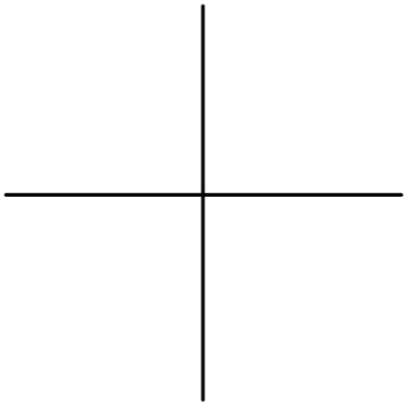
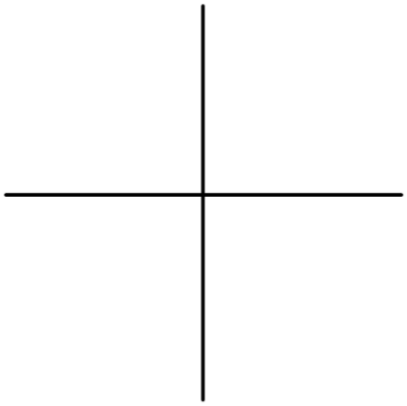
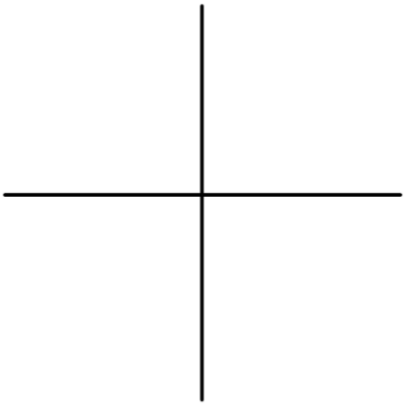
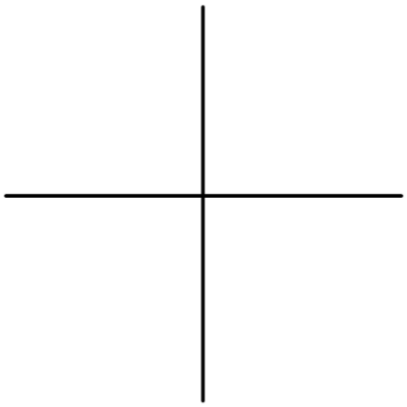
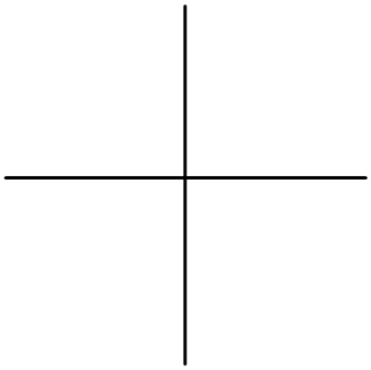
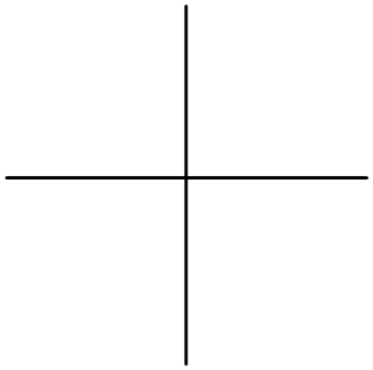
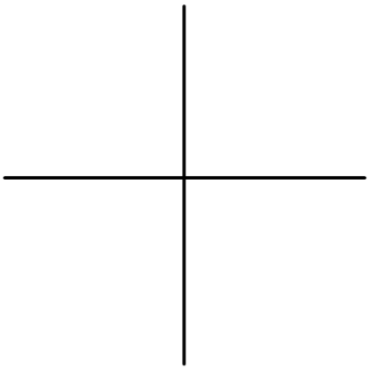
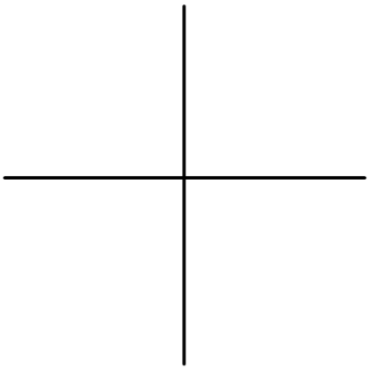
Solution:

a. Determine the exponential function $y = a(c)^{bx}$:

b. Value of the card in 5 years:

Example 4: Sketch Graphs of Transformed Exponential Functions

Without using a mapping rule or a table of values, sketch each of the exponential functions given below. Include the correct location of the horizontal asymptote. For the point (0, 1) on the base function, determine the coordinates of the corresponding image point on the transformed function (Use a mapping rule for this point only).

$y = 3(2)^{x-4} + 5$ 	$y = -3(2)^{x-4} + 5$ 	$y = 3(2)^{-(x-4)} + 5$ 
$y = -3(2)^{-(x-4)} + 5$ 	$y = 3\left(\frac{1}{2}\right)^{(x+1)} - 2$ 	$y = -3\left(\frac{1}{2}\right)^{(x+1)} - 2$ 
$y = 3\left(\frac{1}{2}\right)^{(-(x+1))} - 2$ 	$y = -3\left(\frac{1}{2}\right)^{(-(x+1))} - 2$ 	$y = -\left(\frac{1}{4}\right)^{(-(x+2))} + 3$