Absolute Value Equations

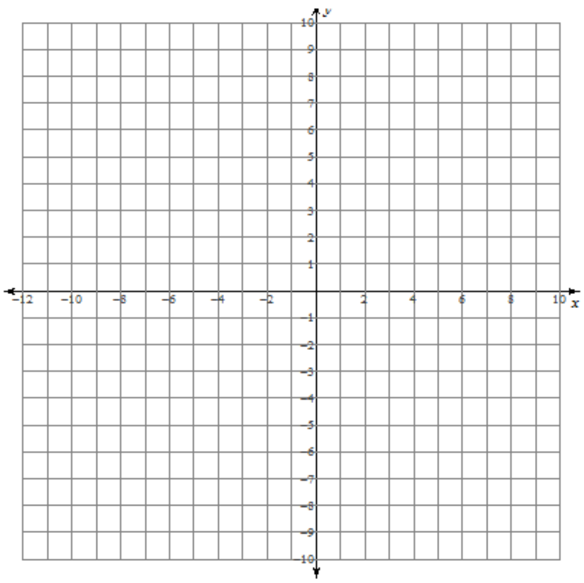
An absolute value equation includes the absolute value of a variable expression.

Example 1: Solve an Absolute Value Equation

Solve $|x+3|=8$.

Method 1: Use a Graph

Graph the functions $f(x)=|x+3|$ and $g(x)=8$ on the same coordinate grid to see where they intersect.



The graphs intersect at (______ , ______) and (______ , ______).

Thus, the solutions to the equation $|x+3|=8$ are x = ______ and x = ______.

Verify the solutions algebraically by substitution.

$|x+3|=8$

$|x+3|=8$

Method 2: Use an Algebraic Method

Use the definition of absolute value to set up the cases: $|x+3|=\begin{cases} x+3, \text{ if } x\geq -3 \\ -(x+3), \text{ if } x<-3 \end{cases}$

Case 1	Case 2
The value x=_______ does / does not satisfy the condition __________.	**The value x = _______ does / does not satisfy the condition __________.**

Both values of x satisfy the given conditions, therefore, the solutions to the equation $|x+3|=8$ are x = ______ and x = ______.

Example 2: Absolute Value Equation with an Extraneous Solution

Solve $|3x+2| = 4x+5$.

Solution:
Use the definition of absolute value to set up the cases: $|3x+2| = \begin{cases} 3x+2, \textit{ if } \_\_\_\_\_\_\_\_\_\_ \\ -(3x+2), \textit{ if } \_\_\_\_\_\_\_\_\_\_ \end{cases}$

Case 1	**Case 2**
The value x=_______ does / does not satisfy the condition __________.	**The value x = _______ does / does not satisfy the condition __________.**

Only x = _______ satisfies the given conditions, therefore the solution to the equation $|3x+2| = 4x+5$ is x = _______.

Verify these results algebraically by substitution.

$|3x+2| = 4x+5$

$|3x+2| = 4x+5$

Example 3: Absolute Value Equation with No Solution.

Solve $|5x+1|+7=3$.

Solution:
Isolate the absolute value expression.

Since the absolute value of a number is always greater than or equal to zero, by inspection this equation has _______ solution.

Example 4: Solve an Absolute Value Problem

A musical group's new single is released. Weekly sales S (in thousands) increase steadily for a while and then decrease as given by the function $S = -2|t - 20| + 40$ where t is the time in weeks. In which week(s) will 10 000 singles be sold?

Solution:

Substitute S = _______ into the function. Isolate the absolute value expression and solve the equation.

Example 5: Solve an Absolute Value Equation Involving Linear & Quadratic Expressions

Solve $3|x-10|+30x=3x^2$.

Solution:
Isolate the absolute value expression:

Use the definition of absolute value to set up the cases: $|x-10| = \begin{cases} \\ \end{cases}$

Case 1	Case 2
The value x=_______ does / does not satisfy the condition __________. The value x=_______ does / does not satisfy the condition __________.	The value x=_______ does / does not satisfy the condition __________. The value x=_______ does / does not satisfy the condition __________.

In case 1, only x = _______ satisfies the given conditions, so x = _______ is an extraneous root. In case 2, only x = _______ satisfies the given conditions, but x = _______ satisfies the conditions in case 1, so the solutions to are x = _______ and _______.

Verification:

$|x-10|=x^2-10x$ $\qquad$ $|x-10|=x^2-10x$ $\qquad$ $|x-10|=x^2-10x$

Example 6: Solve an Absolute Value Equation Involving a Quadratic Expression

Solve $\left|x^2 - 6x\right| = 8$.

Solution:
Use the definition of absolute value to set up the cases: $\left|x^2 - 6x\right| = \begin{cases} x^2 - 6x, \textit{ if } \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ -(x^2 - 6x), \textit{ if } \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$

Case 1	Case 2
The value x=____________ does / does not satisfy the condition __________. The value x=____________ does / does not satisfy the condition __________.	The value x=_______ does / does not satisfy the condition __________. The value x=_______ does / does not satisfy the condition __________.

All values of x satisfy the given conditions, therefore, the solutions to the equation $\left|x^2 - 6x\right| = 8$ are x = _______________, _______, and _______.

Verification :

$\left|x^2 - 6x\right| = 8$ $\quad\left|x^2 - 6x\right| = 8$ $\quad\left|x^2 - 6x\right| = 8$ $\quad\left|x^2 - 6x\right| = 8$

Example 7: Solve an Absolute Value Equation Involving a Quadratic Expression

Solve $|2x^2 + 3x - 1| = 3x + 5$.

Solution:
Use the definition of absolute value to set up the cases:

Case 1	Case 2
The value x=____________ does / does not satisfy the condition __________. The value x=____________ does / does not satisfy the condition __________.	The value x=_______ does / does not satisfy the condition __________. The value x=_______ does / does not satisfy the condition __________.

The solutions to the equation $|2x^2 + 3x - 1| = 3x + 5$ are x = _______ and _______.

Verification :

$|2x^2 + 3x - 1| = 3x + 5$ $|2x^2 + 3x - 1| = 3x + 5$ $|2x^2 + 3x - 1| = 3x + 5$ $|2x^2 + 3x - 1| = 3x + 5$

Extra Practice:

Solve the following equations. Remember to first *isolate* the absolute value expression and then use the *definition of absolute value* to set up the cases.

1. $\left|x^2+2x-8\right|+x-2=0$ **(solutions: x = –5, –3, 2)**

2. $-2\left|-x^2+8x-15\right|-8x=-24$ **(solutions: x = 1, 3)**

3. $\left|2x^2+4x-9\right|=4x-1$ **(solutions: x = 1, 2)**

4. $\left|-2x^2-5x+8\right|=5x-4$ **(solutions: x = 1, $\sqrt{2}$)**

5. $\left|6x^2-3x-8\right|=3x+4$ **(solutions: x = –1, 2, $\pm\sqrt{\frac{2}{3}}$)**