Solving Systems of Equations Algebraically

LINEAR – QUADRATIC

Example 1: Algebraically Solve a System of Equations

Solve the system of equations $\begin{aligned} -x^2 + x + y + 6 &= 0 \\ -x + y + 3 &= 0 \end{aligned}$

Solution:

Method 1 – Substitution $\begin{aligned} &1. \quad -x^2 + x + y + 6 = 0 \\ &2. \quad -x + y + 3 = 0 \end{aligned}$	**Method 2 – Elimination** $\begin{aligned} &1. \quad -x^2 + x + y + 6 = 0 \\ &2. \quad -x + y + 3 = 0 \end{aligned}$
Isolate y in the linear equation. y = _____ Substitute x – 3 for y in the quadratic equation, simplify, and solve for x.	Write the equations so that like terms align. Eliminate the y term by subtracting the second equation from the first. Solve for x.

Substitute these x-values into the original linear equation $-x + y + 3 = 0$, or $y = x - 3$, to determine the corresponding values of y.

When x = 3, y = _____ When x = –1, y = _____

Therefore, the solutions to the system of equations are (3, _____) and (–1, _____).

Verify your solutions in the original quadratic equation $-x^2 + x + y + 6 = 0$.

Example 2: Algebraically Solve a System of Equations

Solve the system of equations $\begin{cases} 5x - y = 10 \\ x^2 + x - 2y = 0 \end{cases}$

Solution:

Method 1 – Substitution $\begin{cases} 1. \quad 5x - y = 10 \\ 2. \quad x^2 + x - 2y = 0 \end{cases}$	**Method 2 – Elimination** $\begin{cases} 1. \quad 5x - y = 10 \\ 2. \quad x^2 + x - 2y = 0 \end{cases}$
Isolate y in the linear equation. y = _____ Substitute 5x – 10 for y in the quadratic equation, simplify, and solve for x.	Write the equations so that like terms align. To eliminate the y term, multiply the linear equation by _____ and then _____ the equations. Solve for x.

Substitute these values into the original linear equation $5x - y = 10$, or $y = 5x - 10$, to determine the corresponding values of y.

When x = 4, y = _____ When x = 5, y = _____

Therefore, the solutions to the system of equations are (4, _____) and (5, _____).

Example 3: Solve a Problem Involving a Linear-Quadratic System.

A Canadian cargo plane drops a crate of emergency supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h, in metres above the ground t seconds after leaving the aircraft is given by the following two equations:

$h = -4.9t^2 + 700$ represents the height of the crate during the free fall.
$h = -5t + 650$ represents the height of the crate when the parachute is open.

a. How long after the crate leaves the aircraft does the parachute open? Express your answer to the nearest hundredth of a second.

b. What height above the ground is the crate when the parachute opens? Express you answer to the nearest metre.

c. Verify your solution.

Solution:

The moment when the parachute opens corresponds to the point of intersection of the two heights. Solve the system of equations by substitution.

The parachute opens about _____seconds after the crate leaves the plane.

The crate is about _____metres above the ground when the parachute opens.

Example 4: Model a Situation with a System of Equations

Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93.

Solution:

Let x = the smaller number
y = the larger number

Equation 1. _____

Equation 2. _____

Solve the system of equations by elimination.

Since the numbers have to be integers, the smaller number must be _____.

Determine the value of the larger integer:

Therefore, the two integers are _____ and _____.

QUADRATIC – QUADRATIC

Example 5: Algebraically Solve a System of Equations

Solve the system of equations $\begin{aligned} 6x^2 - x - y &= -1 \\ 4x^2 - 4x - y &= -6 \end{aligned}$

Solution:

Solve by Substitution 1. $6x^2 - x - y = -1$ 2. $4x^2 - 4x - y = -6$	
	Isolate y in one of the equations.
	Replace y in the other equation with this expression and solve for x.
Therefore, the solutions to the system of equations are **(_____ , _____) and (_____ , _____).**	Substitute these values of x into one of the original equations to determine the corresponding values for y.

Example 6: Algebraically Solve a System of Equations

Solve the system of equations $\begin{aligned} -4x^2 - 2x &= y - 5 \\ 3x^2 - 4y - 46x - 37 &= 0 \end{aligned}$

Solution:

Solve by Elimination 1. $-4x^2 - 2x = y - 5$ 2. $3x^2 - 4y - 46x - 37 = 0$	
1. 2.	Write the equations so that like terms align.
1. 2. _____	To eliminate the y term, we can multiply the first equation by _____ and then _____ the equations.
	Solve the quadratic equation.
Therefore, the solutions to the system of equations are **(_____ , _____) and (_____ , _____).**	Substitute these values of x into one of the original equations to determine the corresponding values for y.

Example 7: Solve a Problem Involving a Quadratic-Quadratic System

During a basketball game, Daniel completes an impressive "alley-oop". From one side of the hoop, his teammate Sawyer lobs a perfect pass toward the basket. Daniel jumps up, catches the ball and tips it into the basket. The path of the ball thrown by Sawyer can be modeled by the equation $d^2 - 2d + 3h = 9$, where d is the horizontal distance of the ball from the centre of the hoop, in metres, and h is the height of the ball above the floor, in metres. The path of Daniel's jump can be modeled by the equation $5d^2 - 10d + h = 0$, where d is his horizontal distance from the centre of the hoop, in metres, and h is the height of his hands above the floor, in metres.

Solve the system of equations algebraically. Give your answer to the nearest tenth.

Interpret your result.

Solution: