Laws of Logarithms

Logarithmic Identities

Given that b>0,

- $\log_b 1 = 0$ (because $b^0 = 1$)
- $\log_b b = 1$ (because $b^1 = b$)
- $\log_b 0$ is *undefined* (because there is no number n such that $b^n = 0$)
- $\log_b b^n = n$ (because $b^n = b^n$)
- $b^{\log_b n} = n$ (because if $\log_b n = c$, then $b^c = n$, so $b^{\log_b n} = n$)

Example 1: Evaluating Logarithmic Identities

Evaluate the following:

a. $\log_7 7 =$ _____ b. $\log_4 0 =$ _____ c. $\log_5 1 =$ _____ d. $3^{\log_3 9} =$ _____ e. $\log_2 2^5 =$ _____

Laws of Logarithms

Rules for working with logarithms are derived from the rules for working with exponents. Proofs for each of the following rules, or laws, of logarithms can be found in your textbook, pages 394-395.

Name of Law	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$ eg. $\log_2(8\times 4) = \log_2 8 + \log_2 4$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$ eg. $\log_2 \frac{32}{8} = \log_2 32 - \log_2 8$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P\log_c M$ eg. $\log_2 8^2 = 2\log_2 8$	The logarithm of a power of a number is the exponent times the logarithm of the number.

* **Note:** M, N, and c must be positive real numbers with $c \neq 1$. P can be any real number.

Example 2: Use the Laws of Logarithms to Expand Expressions

Expand each expression using the laws of logarithms.

$\log_4 \frac{x^3 y}{4z}$	$\log_5 \sqrt{xy^3}$	$\log \frac{100\sqrt[3]{x^4}}{y^2}$

Example 3: Evaluate Expressions with the Laws of Logarithms

Evaluate each expression.

$\log_6 216\sqrt[4]{36}$	$2\log_2 12 - \left(\log_2 6 + \frac{1}{3}\log_2 27\right)$	$\log_4 \sqrt{12} + \log_4 \sqrt{9} - \log_4 \sqrt{27}$

Example 4: Use the Laws of Logarithms to Simplify Expressions

Rewrite each expression using a single logarithm in simplest form. State the restrictions on the variable.

Remember that for $\log_c x$ *to be defined, it must be the case that x > 0. For the following expressions to be defined, all terms in the expression must be defined.*

$5+\log_2 x$	$\log_2 x^3 - 4\log_2 x - \log_2 \sqrt{x}$
$4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$	$\log_2(x^2-9) - \log_2(x^2-x-6)$

Example 5: Solve a Problem Using a Logarithmic Scale

The Richter scale was developed in 1935 by seismologist Charles F. Richter. It measures the magnitude of an earthquake by comparing the intensity of the earthquake to a reference earthquake. The formula developed by Richter is $M = \log\left(\frac{I}{I_o}\right)$, where I is the intensity of the earthquake under study, I_o is the intensity of a reference earthquake, and M is the Richter value used to measure the magnitude of the earthquake.
An earthquake of magnitude 6 is ten times stronger than a magnitude 5 earthquake.

In October 1989, San Francisco, CA had an earthquake that measured 7.1 on the Richter scale. In January 1994, an earthquake in Northridge, CA measured 6.6 on the Richter scale. Compare the intensities of the two earthquakes.

Solution:

Notice there are two different situations here. There is the San Francisco earthquake and the Northridge quake.

I_{SF} - intensity of the San Francisco earthquake

I_N - intensity of the Northridge earthquake

$$7.1 = \log\left(\frac{I_{SF}}{I_o}\right) \qquad\qquad 6.6 = \log\left(\frac{I_N}{I_o}\right)$$

Expand both equations using one of the laws of logarithms.

Now subtract the two equations to eliminate I_o.

Use one of the laws of logarithms to simplify the equation.

Solve for the ratio $\frac{I_{SF}}{I_N}$ and make the comparison between the earthquake intensities.