Exploring Reciprocal Functions

Reciprocal Function: For any function $f(x)$, the reciprocal function is $\frac{1}{f(x)}$. The reciprocal function is not defined when the denominator is 0, so $f(x) \neq 0$.

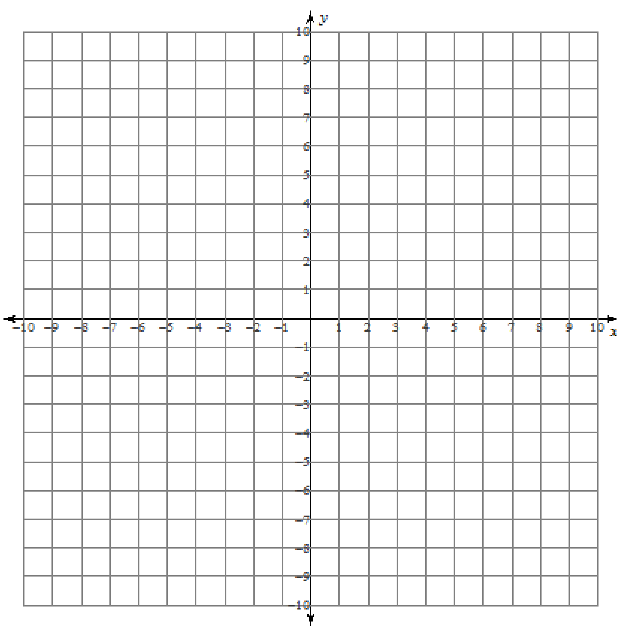
Example 1: Comparing the Graphs of a Function and its Reciprocal

If $f(x) = x$, sketch the graph of $y = f(x)$ and its reciprocal $y = \frac{1}{f(x)}$. Examine the behaviour of $y = \frac{1}{f(x)}$ as the value of x approaches $\pm\infty$, at x = 0, and as the value of x approaches 0 from the left and from the right. State the domain, range, equations of the vertical and horizontal asymptotes, and any invariant points for the two functions.

Solution:

Complete the table of values for y = x and $y = \frac{1}{x}$ and graph the functions.

x	–10	–5	–2	–1	$\frac{-1}{2}$	$\frac{-1}{5}$	$\frac{-1}{10}$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	1	2	5	10
y=x															
$y = \frac{1}{x}$															



For the function $y = \frac{1}{x}$:

- As the value of x approaches $\pm\infty$, the value of y approaches ______.
- At x = 0, the value of y is ____________________________.
- As the value of x approaches 0 *from the left*, the value of y approaches ________.
- As the value of x approaches 0 *from the right*, the value of y approaches ________.

Function	y = x	$y = \frac{1}{x}$
Equation of the Vertical Asymptote		
Equation of the Horizontal Asymptote		
Domain		
Range		
Invariant Points	(_____ , 1) and (_____ , –1)	

Consider the following when graphing $y = \dfrac{1}{f(x)}$ given the graph of $y = f(x)$*:

* For polynomial functions y=f(x)

Vertical asymptotes: A vertical asymptote is a vertical line, x=*a*, that the graph of a function approaches, without bound, as x approaches *a*. The *reciprocal*, 1/f(x), of *any* function, f(x), will have a vertical asymptote at any *non-permissible* value of x. Consider the behaviour of the reciprocal function near the vertical asymptotes.

Horizontal asymptote: A horizontal asymptote is a horizontal line that the graph of a function approaches as the value of x approaches $\pm\infty$. The *reciprocal*, 1/f(x), of *any polynomial* function, f(x), will have a horizontal asymptote at y = 0 since, for all polynomial functions, as x approaches $\pm\infty$, f(x) approaches $\pm\infty$, and therefore 1/f(x) becomes increasingly close to 0.

Invariant Points: For any function, f(x), and its reciprocal, 1/f(x), the invariant points exist at f(x)= ± 1.

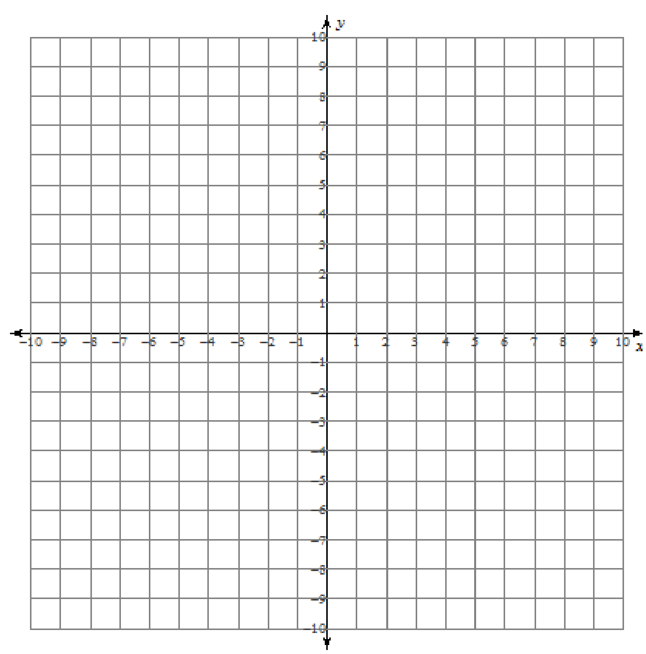
Other Points: For any point (x, y) on the graph of $y = f(x)$, the corresponding point $\left(x, \dfrac{1}{y}\right)$ will be on the graph of $y = \dfrac{1}{f(x)}$.

Example 2: Graph the Reciprocal of a Linear Function

Sketch the graph of $f(x) = x + 5$ and its reciprocal function, $f(x) = \dfrac{1}{x+5}$.

Solution:

- Sketch the graph of the function $f(x) = x + 5$.
- Determine the following characteristics of $f(x) = \dfrac{1}{x+5}$ and sketch its graph.



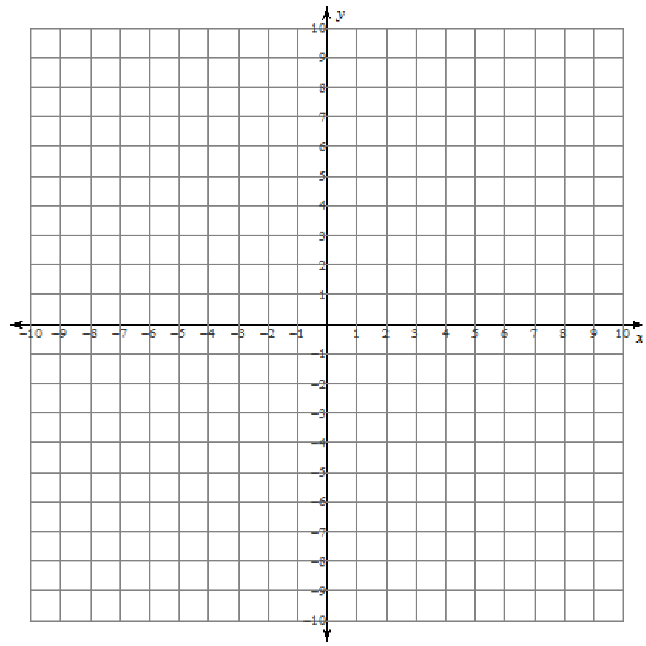
Characteristic	$f(x) = \dfrac{1}{x+5}$
x-intercept(s)	
y-intercept	
Non-permissible value of x	
Equation of the Vertical Asymptote	
Behaviour of the function as x approaches –5 *from the left*	$\lim\limits_{x \to -5^-} f(x) =$
Behaviour of the function as x approaches –5 *from the right*	$\lim\limits_{x \to -5^+} f(x) =$
End Behaviour of the function	$\lim\limits_{x \to \pm\infty} f(x) =$
Equation of the Horizontal Asymptote	
Domain	
Range	
Invariant Points	(_____ , 1) & (_____ , –1)

Example 3: Graph the Reciprocal of a Quadratic Function

Sketch the graph of $f(x) = x^2 - 4$ and its reciprocal function, $f(x) = \dfrac{1}{x^2 - 4}$.

Solution:

- Sketch the graph of the function $f(x) = x^2 - 4$.
- Determine the following characteristics of $f(x) = \dfrac{1}{x^2 - 4}$ and sketch its graph.

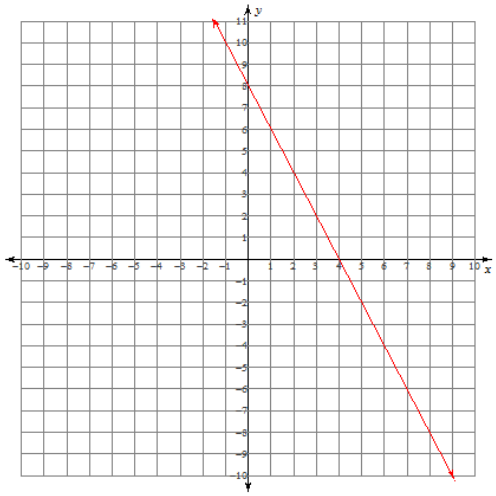


Characteristic	$f(x) = \dfrac{1}{x^2 - 4}$
x-intercept(s)	
y-intercept	
Factored form of $f(x) = \dfrac{1}{x^2 - 4}$	
Non-permissible values of x	
Equations of the Vertical Asymptotes	
Behaviour of the function as x approaches –2 *from the left*	$\lim\limits_{x \to -2^-} f(x) =$
Behaviour of the function as x approaches –2 *from the right*	$\lim\limits_{x \to -2^+} f(x) =$
Behaviour of the function as x approaches 2 *from the left*	$\lim\limits_{x \to 2^-} f(x) =$
Behaviour of the function as x approaches 2 *from the right*	$\lim\limits_{x \to 2^+} f(x) =$
End Behaviour of the function	$\lim\limits_{x \to \pm\infty} f(x) =$
Equation of the Horizontal Asymptote	
Domain	
Range	
Invariant Points	(_____ , 1) & (_____ , 1) (_____ , –1) & (_____ , –1)

Example 4: Sketch the Graph of $y = \dfrac{1}{f(x)}$ Given the Graph of $y = f(x)$

Sketch the graph of the reciprocal function for each of the following.

a.



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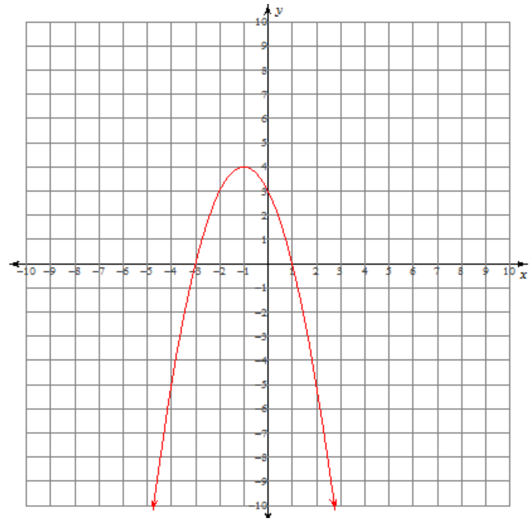
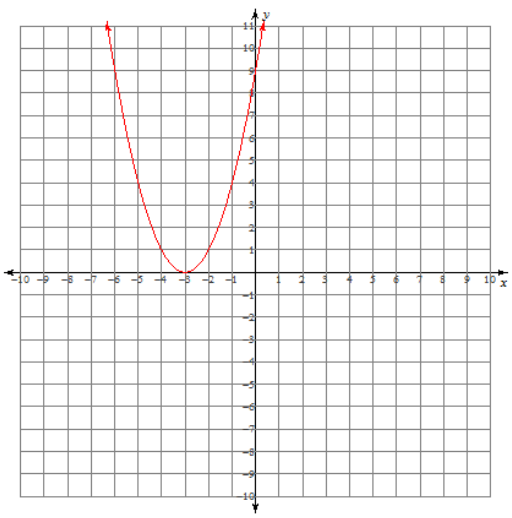
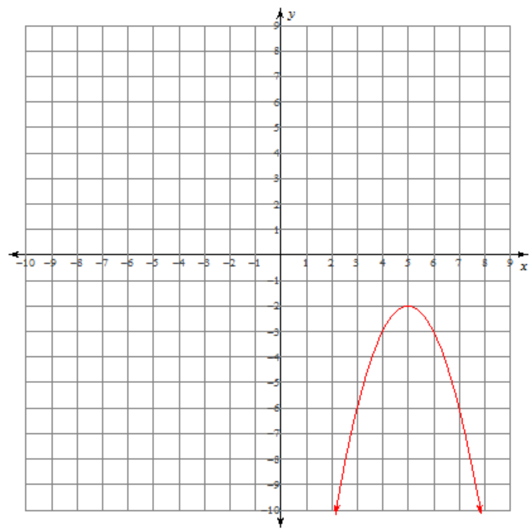


c.



d.



e.