Pre-Calculus 110 Review

Trigonometry *(Reference Chapter 2, Sections 2.1 -2.2, pages 74-99)*

Outcomes:

- *Demonstrate an understanding of angles in standard position,* $0° \le \theta \le 360°$
- *Solve problems, using the three primary trigonometric ratios, for angles from* $0°$ *to* $360°$ *in standard position*

1. For each angle listed below
   a. Draw the angle in standard position and indicate in which quadrant the terminal arm is located.
   b. Indicate the reference angle on your diagram.
   c. State two other angles (other than the one in quadrant I) that have the same reference angle.
      i. $245°$ ii. $305°$ iii. $100°$

2. Which angle in standard position has a different reference angle than all the others?
   $105°$ $165°$ $255°$ $285°$

3. Determine the measure of the three angles in standard position, ($90° \le \theta \le 360°$) that have a reference angles of
   a. $65°$ b. $10°$ c. $85°$

4. Each point listed below is situated on the terminal arm of an angle θ in standard position. Determine the exact trigonometric values for $\sin\theta$, $\cos\theta$, and $\tan\theta$.
   a. P (3,-4) b. Q (-7, -24) c. A (-5, 12)

5. Find θ, $0° \le \theta \le 360°$.
   a. $\sin 233° = \sin\theta$ b. $\cos 317° = \cos\theta$ c. $\tan 136° = \tan\theta$
   d. $\sin 98° = \sin\theta$ e. $\cos 261° = \cos\theta$ f. $\tan 76° = \tan\theta$

6. Determine the exact value of
   a. $\sin 120°$ b. $\cos 270°$ c. $\tan 300°$ d. $\sin 330°$ e. $\cos 315°$ f. $\tan 210°$

7. θ is an angle in standard position. Find the two other primary trigonometric values if the terminal arm of θ is in quadrant IV and $\cos\theta = \frac{6}{7}$.

8. Consider $\angle\theta$ in standard position such that $\tan\theta = \frac{-9}{5}$.
   a. Determine the exact values for the other two primary trigonometric ratios.
   b. Determine the possible value(s) of θ, if $0° \le \theta \le 360°$

9. Solve for θ where $0° \le \theta \le 360°$
   a. $\sin\theta = \frac{-\sqrt{3}}{2}$ b. $\cos\theta = \frac{1}{\sqrt{2}}$ c. $\tan\theta = -1$ d. $\sin\theta = 1$ e. $\cos\theta = -\frac{1}{2}$ f. $\tan\theta = \frac{1}{\sqrt{3}}$

10. Solve each equation, where $0° \le \theta \le 360°$. Round to the nearest degree.
    a. $\sin\theta = 0.45$ b. $\cos\theta = -0.36$ c. $\tan\theta = -2.41$

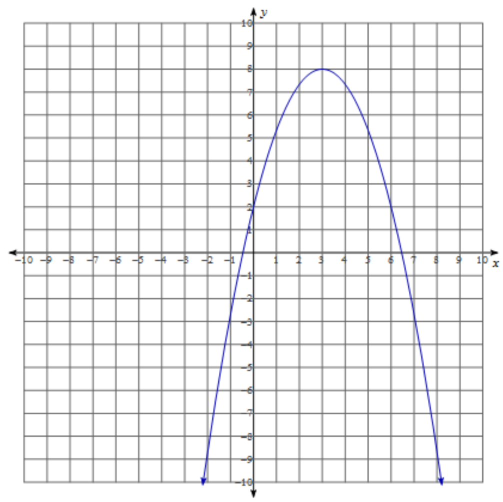
Quadratic Functions

(Reference Chapter 3, Sections 3.1 – 3.3, pages 142-203)

Outcomes:

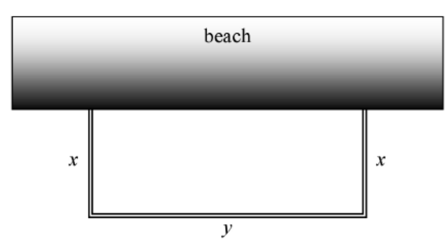
- *Analyze quadratic functions of the form* $y = a(x-p)^2 + q$ and determine the vertex, domain, and range, direction of opening, axis of symmetry, x- and y-intercepts.
- *Analyze quadratic functions of the form* $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems.

1. Indicate the vertex, the equation of the axis of symmetry, direction of opening, the maximum or minimum value, the domain and range for each of the following quadratic functions.
   a. $f(x) = \frac{-2}{3}(x+2)^2 - 9$ b. $g(x) = 2x^2 - 8x + 5$ c. $y = 4 - x^2$ d. x-intercepts at -3 & 7, max value is 1
2. Determine the equation, both in vertex and standard form, for each of the following quadratic functions.
   a. Maximum point is at (-3,-5) and graph passes through the point (-1,-21)
   b. x-intercepts are at -2 and 4, with a minimum of -18
   c. Graph of function is shown to the right



3. By inspection, determine the number of x-intercepts for each quadratic function.
   a. $f(x) = 2(x-5)^2 - 7$ b. $g(x) = -\frac{1}{3}(x+4)^2 - 1$
4. $y = -\frac{1}{2}(x-3)^2 + 8$
   a. Sketch the graph of the quadratic function.
   b. Identify the vertex, domain, range, axis of symmetry, x- and y-intercepts, max/min value.
5. $f(x) = 2x^2 - 12x + 10$
   a. Sketch the graph of the function.
   b. Identify the vertex, domain, range, axis of symmetry, x- and y- intercepts.

6. Determine the vertex form of the following quadratic functions by "completing the square".
   a. $y = x^2 + 5x + 29$ b. $y = -5x^2 + 20x - 6$
7. A lifeguard at a public beach has 400 m of rope available to lay out a restricted swimming area using the straight shoreline as one side of the rectangle. What dimensions will maximize the swimming area?



8. Calculators are sold to students for \$20 each. Three hundred students are willing to buy them at that price. For every \$5 increase in price, there are 30 fewer students willing to buy the calculator. What selling price will produce the maximum revenue and what will the maximum revenue be?
9. A ligting fixture manufacturer has a daily costs of $C(x) = 800 - 10x + 0.25x^2$, where C is the total daily cost in dollars and x is the number of light fixtures produced. What is the minimum cost for daily production for this company? How many fixtures are produced to yield a minimum cost?

Quadratic Equations

(Reference Chapter 4, Sections 4.1-4.4, pages 206-262)

Outcomes:

- *Factor polynomial expressions of the form:*

$ax^2+bx+x, a\neq 0$	$2x^2-x-3$
$a^2x^2-b^2y^2, a\neq 0, b\neq 0$	$25x^2-16y^2$
$a(f(x))^2+b(f(x))+c, a\neq 0$	$2(x-5)^2-3(x-5)-9$
$a^2(f(x))^2+b^2(g(y))^2, a\neq 0, b\neq 0$	$9(2x+1)^2-4(y-2)^2$

- *Solve problems that involve quadratic equations.*

1. Factor.
   a. $5x^2+20x$ b. $x^2-12x+32$ c. $3x^2+14x-5$

2. Factor each of the following. Use substitution as a strategy to aid in the factoring.
   a. $2(x-5)^2-3(x-5)-9$ b. $(x^2+2x)^2-11(x^2+2x)+24$ c. $25(x+3)^2-\frac{16}{49}(y-7)^2$

3. Solve each quadratic equation by factoring.
   a. $4p^2=24p$ c. $0.2x^2-1.4x+2=0$ e. $10q^2=12-26q$
   b. $x^2-5x-36=0$ d. $2t^2+14t=16$ f. $x+\frac{20}{7}=\frac{6}{7}x^2$

4. A park has an area of $2484\ m^2$. The width is 8 m shorter than the length. Determine the dimensions of the park.

5. A mural is to be painted on a wall that is 15 m long and 12 m high. A border of uniform width is to surround the mural. If the mural is to cover 75% of the area of the wall, how wide must the border be?

6. Determine the number and nature of the roots.
   a. $6x^2+5x-6=0$ b. $4x^2+9=12x$ c. $3x^2=2x-10$

7. Solve.
   a. $7x^2=6x+5$ b. $3a^2+4a+11=0$

8. A field is in the shape of a right triangle. The fence around the perimeter of the field measures 40 m. If the length of the hypotenuse is 17m, find the lengths of the other two sides.

9. A diver jumps upward from a platform. The height, h(t), in metres, of the diver above the water is modeled by $h(t)=-5t^2+10t+40$, where t is the time in seconds after she leaves the board.
   a. Find the diver’s maximum height above the water. How long does it take the diver to reach this height?
   b. How long is it before the diver enters the water?
   c. At what time(s) is the diver at a height of 42m?
   d. How high is the platform above the water?

Systems of Equations

(Reference Chapter 8, Sections 8.1-8.2, pages 424 -460)

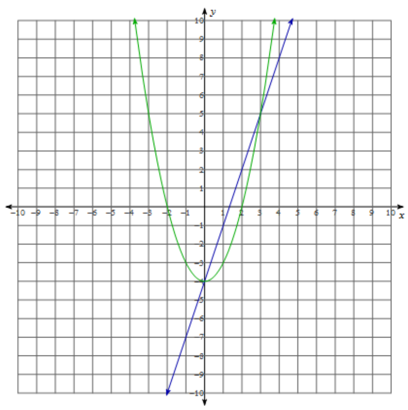
Outcomes:

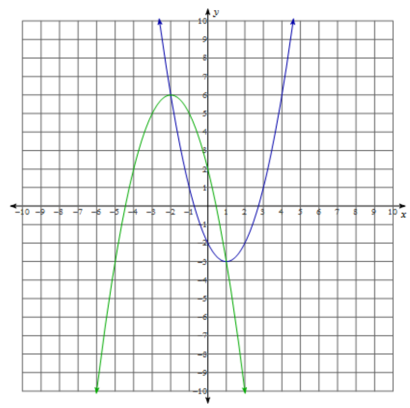
- *Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.*

1. Solve each of the following systems of equations using the corresponding graphs.

   a. $y = 3x - 4$
   $y = x^2 - 4$

   b. $y = x^2 - 2x - 2$
   $y = -x^2 - 4x + 2$





2. Determine the values of m and n if (3, 14) is a solution to the following system of equations: $mx^2 + y = 32$
   $mx^2 - 5y = n$

3. Solve each of the following linear-quadratic systems of equations.

   a. $9x - y = 4$
   $y + 3 = x^2 + 7x$

   b. $y + 2 = 3x$
   $y + x^2 + 3x = 5$

   c. $2x^2 - 4x + y = 3$
   $4x - 2y = -7$

4. The sum of two whole numbers is 17. Thirteen less than three times the square of the smaller yields the larger number. Determine the two numbers.

5. Solve each system of equations algebraically.

   a. $y - x^2 = 7 - 2x$
   $x^2 - 4x + y = 3$

   b. $5x^2 + 3y = -3 - x$
   $2x^2 - x = -4 - 2y$

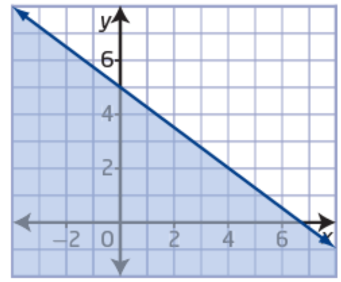
6. Jack is skeet shooting. The height of the skeet is modelled by the equation $h = -5t^2 + 32t + 2$, where h represents the height in metres t seconds after the skeet is released. The path of Jack's bullet is modelled by the equation $h = 31.5t + 1$ with the same units. How long will it take for the bullet to hit the skeet? How high off the ground will the skeet be when it is hit?

Linear and Quadratic Inequalities

(Reference Chapter 9, Sections 9.1-9.3, pages 464-505)

Outcomes:

- *Solve problems that involve linear and quadratic inequalities in two variables.*
- *Solve problems that involve quadratic inequalities in one variable.*



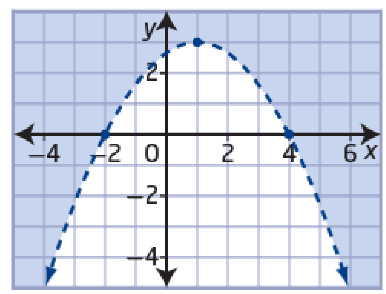
1. Determine the inequality that corresponds to the graph illustrated to the right.

2. Sketch the graph of each of the following linear inequalities.
   a. $y < 2x - 3$ b. $y \geq 6 - 3x$ c. $4x + 3y \leq 12$

3. Solve..
   a. $(x+3)(2x-7) > 0$ b. $-3x^2 \geq -9x + 6$ c. $11x + 10 \leq 6x^2$ d. $2x^2 + 2 < 7x$

4. A firework is shot straight up into the air with an initial velocity of 500 feet per second from 5 feet off the ground. The equation $h(t) = -16t^2 + 500t + 5$ models the path of the firework, where t is the time in seconds and h(t) is the height of the firework in feet. Determine when the firework will be higher than 2000 ft.

   Extension: When will the firework be below 2000 ft?



5. State the inequality represented by the graph to the right. Express your final answer in standard form.

6. Sketch the graphs of the following inequalities.
   a. $y \leq -(x+3)^2 + 4$ b. $y > x^2 + 4x - 5$ c. $y \geq -4x^2 - 4x + 3$

7. A garden is to be in the shape of a rectangle. The length of the garden must be 4 m longer than the width. The gardener wants the area of the garden to be no greater than 480 square meters. Find all the possible dimensions.

Radical Expressions and Equations

(Reference Chapter 5, Sections 5.1-5.3, pages 272 -307)

Outcomes:

- *Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.*
- *Solve problems that involve radical equations (limited to square roots).*

1. Write each radical as an entire radical.

   a. $5\sqrt{3}$ b. $b^3\sqrt{b}$ c. $3k\sqrt{5k^3}$ d. $2r\sqrt[3]{3r^4}$

2. Write each radical in simplest form.

   a. $\sqrt{300}$ b. $\sqrt{450}$ c. $\sqrt[3]{160}$ d. $\sqrt[4]{324}$

3. Simplify.

   a. $6\sqrt{80}-2\sqrt{20}$ b. $5\sqrt{12}-2\sqrt{27}$ c. $\dfrac{5}{6}\sqrt{252}+\dfrac{\sqrt{175}}{8}$ d. $2\sqrt[3]{375}-3\sqrt[3]{648}$

4. State the side length of a square with an area of $1573cm^2$ in simplified radical form.

5. Write $\sqrt[4]{32x^9y^5}$ in simplest form. Identify the values of the variable for which the radical represents a real number.

6. Simplify.

   a. $\sqrt{3}\left(5\sqrt{6}-4\sqrt{24}\right)$ b. $\left(2\sqrt{7}-3\sqrt{5}\right)\left(4\sqrt{5}+\sqrt{7}\right)$ c. $\left(2\sqrt[3]{4}-5\sqrt[3]{2}\right)\left(3\sqrt[3]{2}-\sqrt[3]{4}\right)$

7. Simplify.

   a. $\dfrac{24\sqrt{14}}{8\sqrt{2}}$ b. $\dfrac{9\sqrt{5}}{4\sqrt{3}}$ c. $\dfrac{7\sqrt{5}}{4-2\sqrt{3}}$ d. $\dfrac{5\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}$

8. State the restrictions on x in each of the following:

   a. $\sqrt{2x+1}$ b. $\dfrac{3}{\sqrt{9-x}}$

9. Square the expression $\left(3-2\sqrt{x-5}\right)$.

10. Solve each of the following. Check for extraneous roots.

    a. $4\sqrt{2x-3}=12$ b. $x+2\sqrt{x-1}=9$ c. $\sqrt{p+1}=\sqrt{p+6}-1$

    d. $1+\sqrt{6-2z}=\sqrt{5-4z}$

Rational Expressions and Equations

(Reference Chapter 6, Sections 6.1- 6.4, pages 310 -355)

Outcomes:

- *Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials)*
- *Perform operations on rational expressions (limited to numerators and denominators that are mononials, binomials or trinomials)*
- *Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials)*

1. State the non-permissible value(s) in each rational expression.

   a. $\frac{5w}{3gj^2}$ b. $\frac{3(x+2)}{4x(5-x)}$ c. $\frac{4p+3}{p^2-3p-10}$

2. Simplify each rational expression. State the non-permissible values.

   a. $\frac{3x-12}{32-8x}$ b. $\frac{r^2-2r-3}{2r^2-r-3}$ c. $\frac{4x^2-y^2}{2x^2+5xy-3y^2}$

3. Simplify. State the non-permissible values.

   a. $\frac{x^2-2x}{x+1}\times\frac{x^2-1}{x^2+x-6}$ b. $\frac{x^2+2x-15}{x^3+4x^2}\div\frac{x^2+9x+20}{2x^3+7x^2-4x}$

   c. $\frac{x^2-4}{x^2+2x}\div\frac{x-2}{x^2-3x}\times\frac{x^3}{x^3+3x^2}$

4. Add or subtract. Give answers in simplest form. Identify all non-permissible values.

   a. $\frac{3x+1}{2x^2-2}+\frac{2x+2}{2x^2-8x+6}$ b. $\frac{2x+2}{x^2+4x-12}-\frac{x+1}{x^2-4}$ c. $\frac{5}{x^2-1}-\frac{2}{x^2+4x+3}+\frac{3}{x^2+2x-3}$

5. Simplify each expression. Identify all non-permissible values.

   a. $\dfrac{\frac{4}{3}+\frac{4}{r}}{\frac{r}{r+12}-\frac{1}{r}}$ b. $\dfrac{\frac{-2}{x-7}+\frac{4}{x+7}}{\frac{x}{x^2-49}-\frac{-2}{x-7}}$

6. Simplify each of the following expressions. Identify the non-permissible values.

   a. $\frac{x-3}{x+2}+\frac{8x+4}{x^2+2x-35}\times\frac{x^2+4x-21}{2x^2-5x-3}$ b. $\frac{x-1}{x+5}-\frac{x^3-4x}{x^2-6x-7}\div\frac{3x^3+5x^2-2x}{3x^2+2x-1}$

7. When it was raining, Elliott drove for 120 miles. When the rain stopped, he drove 20 mph faster than he did while it was raining. He drove for 300 miles after the rain stopped. If Elliott drove for a total of 10 hours, how fast did he drive while it was raining?

8. Solve.

a. $\dfrac{2x}{x+1}-\dfrac{5}{x-1}=\dfrac{x^2-17}{x^2-1}$ b. $\dfrac{5}{m-1}+\dfrac{2}{m+1}=-6$ c. $\dfrac{b-24}{b^2-8b}-\dfrac{5-b}{b-8}=\dfrac{2b+3}{b}$

9. A ski resort can manufacture enough machine-made snow in 12 hours to open its steepest run, whereas it would take naturally falling snow 36 hours to provide enough snow. If the resort makes snow at the same time that it is snowing naturally, how long will it take until the run can open?

10. A positive integer is 4 less than another. The sum of the reciprocals of the two integers is $\frac{10}{21}$. Find the integers.

11. Mark is kayaking in a river which flows downstream at a rate of 1mph. He paddles 5 miles downstream and then turns around and paddles 6 miles upstream. The trip takes 3 hours. How fast can Mark paddle in still water?

Absolute Value Functions

(Reference Chapter 7, Sections 7.1- 7.3, pages 358 -391)

Outcomes:

- *Demonstrate an understanding of the absolute value of real numbers.*
- *Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems.*

Short Response:

1. Evaluate
   a. $\left|-5+6^2\right|-\left|8-(-9)\right|+\left|2-5\right|+\left|-4\right|$
   b. $7\left|0.4-5\right|+\left|-2^3\right|$
   c. $\dfrac{-7-\left|-7^2-(-6)\right|}{-2}$

2. Write each of the following as a piecewise function.
   a. $y=\left|3x+5\right|$ b. $y=\left|-4x+7\right|$ c. $y=\left|x^2-3x-10\right|$ d. $y=\left|6-x^2\right|$

3. Sketch the graph of each of the following absolute value functions. State the domain and range of each.
   a. $y=\left|3x+4\right|$ b. $y=\left|-2x+3\right|$ c. $y=\left|x^2-4x-12\right|$ d. $y=\left|\dfrac{-1}{4}x^2+1\right|$

4. Solve.
   a. $\left|3x-8\right|+9=0$ b. $\left|4x+8\right|=-8x+3$ c. $\left|3x+7\right|=x^2+3x+3$
   d. $\left|\dfrac{1}{2}x^2-2x-2\right|=2-x$

SOLUTIONS :

Chapter 2 – Trigonometry

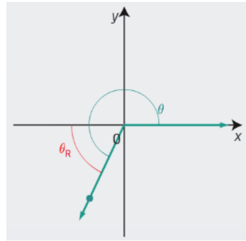
1.

a. $\theta_R = \theta - 180° = 245° - 180° = 65°$

2 other angles are :

$\theta_1 = 180° - \theta_R = 180° - 65° = 115°$

$\theta_2 = 360° - \theta_R = 360° - 65° = 295°$

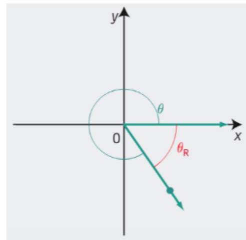


b. $\theta_R = 360° - \theta = 360° - 305° = 55°$

2 other angles are :

$\theta_1 = 180° - \theta_R = 180° - 55° = 125°$

$\theta_2 = 180° + \theta_R = 180° + 55° = 235°$

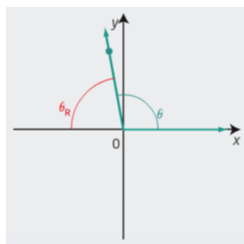


c. $\theta_R = 180° - \theta = 180° - 100° = 80°$

2 other angles are :

$\theta_1 = 180° + \theta_R = 180° + 80° = 260°$

$\theta_2 = 360° - \theta_R = 360° - 80° = 280°$



2. If $\theta = 105°; \theta_R = 180° - \theta = 180° - 105° = 75°$

If $\theta = 165°; \theta_R = 180° - \theta = 180° - 165° = 15°$

If $\theta = 255°; \theta_R = \theta - 180° = 255° - 180° = 75°$

If $\theta = 285°; \theta_R = 360° - \theta = 360° - 285° = 75°$

Conclusion : $\theta = 165°$ has a reference angle that is different from the others.

3.

a. If $\theta_R = 65°$

$\theta_1 = 180° - \theta_R = 180° - 65° = 115°; \theta_2 = 180° + \theta_R = 180° + 65° = 245°; \theta_3 = 360° - \theta_R = 360° - 65° = 295°$

b. If $\theta_R = 10°$

$\theta_1 = 180° - \theta_R = 180° - 10° = 170°; \theta_2 = 180° + \theta_R = 180° + 10° = 190°; \theta_3 = 360° - \theta_R = 360° - 10° = 350°$

c. If $\theta_R = 85°$

$\theta_1 = 180° - \theta_R = 180° - 85° = 95°; \theta_2 = 180° + \theta_R = 180° + 85° = 265°; \theta_3 = 360° - \theta_R = 360° - 85° = 275°$

4.

a. $P(3,-4) \to x = 3; y = -4; r = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$.

Then $\sin\theta = -\dfrac{4}{5}; \cos\theta = \dfrac{3}{5}; \tan\theta = -\dfrac{4}{3}$

b. $P(-7,-24) \to x = -7; y = -24; r = \sqrt{(-7)^2 + (-24)^2} = \sqrt{625} = 25$.

Then $\sin\theta = -\dfrac{24}{25}; \cos\theta = -\dfrac{7}{25}; \tan\theta = \dfrac{24}{7}$

c. $P(-5,12) \to x = -5; y = 12; r = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$.

Then $\sin\theta = \dfrac{12}{13}; \cos\theta = -\dfrac{5}{13}; \tan\theta = -\dfrac{12}{5}$

5.

a. $\theta = 233° \to \theta_R = 233° - 180° = 53°$ and $\theta < 0$ in quadrants III & IV

$\theta = 360° - \theta_R = 360° - 53° = 307°$ so $\sin 233° = \sin 307°$

b. $\theta = 317° \to \theta_R = 360° - 317° = 43°$ and $\cos\theta > 0$ in quadrants I & IV

$\theta = \theta_R 43°$ so $\cos 317° = \cos 43°$

c. $\theta = 136° \to \theta_R = 180° - 136° = 44°$ and $\tan\theta < 0$ in quadrants II & IV

$\theta = 360° - \theta_R = 360° - 44° = 316°$ so $\tan 136° = \tan 316°$

d. $\theta = 98° \to \theta_R = 180° - 98° = 82°$ and $\sin\theta > 0$ in quadrants I & II

$\theta = \theta_R = 82°$ so $\sin 98° = \sin 82°$

e. $\theta = 261° \to \theta_R = 261° - 180° = 81°$ and $\cos\theta < 0$ in quadrants II & III

$\theta = 180° - \theta_R = 180° - 81° = 99°$ so $\cos 261° = \cos 99°$

f. $\theta = 76° \to \theta_R = 76°$ and $\tan\theta > 0$ in quadrants I & III

$\theta = 180° + \theta_R = 180° + 76° = 256°$ so $\tan 76° = \tan 256°$

6.

a. $\theta = 120° \rightarrow \theta$ is in quadrant II where $\sin\theta > 0$ and $\theta_R = 180° - 120° = 60°$.

Since $\sin 60° = \dfrac{\sqrt{3}}{2}$, then $\sin 120° = \dfrac{\sqrt{3}}{2}$.

b. $\cos 270° = \dfrac{x}{r} = \dfrac{0}{r} = 0$.

c. $\theta = 300° \rightarrow \theta$ is in quadrant IV where $\tan\theta < 0$ and $\theta_R = 360° - 300° = 60°$.

Since $\tan 60° = \sqrt{3}$, then $\tan 300° = -\sqrt{3}$.

d. $\theta = 330° \rightarrow \theta$ is in quadrant IV where $\sin\theta < 0$ and $\theta_R = 360° - 330° = 30°$.

Since $\sin 30° = \dfrac{1}{2}$, then $\sin 330° = -\dfrac{1}{2}$.

e. $\theta = 315° \rightarrow \theta$ is in quadrant IV so $\cos\theta > 0$ and $\theta_R = 360° - 315° = 45°$.

Since $\cos 45° = \dfrac{1}{\sqrt{2}}$, then $\cos 315° = \dfrac{1}{\sqrt{2}}$.

f. $\theta = 210° \rightarrow \theta$ is in quadrant III where $\cos\theta < 0$ et $\theta_R = 210° - 180° = 30°$.

Since $\tan 30° = \dfrac{1}{\sqrt{3}}$, then $\tan 210° = \dfrac{1}{\sqrt{3}}$.

7.

a. Since $\cos\theta = \dfrac{x}{r} = \dfrac{6}{7}$ and θ is in quadrant IV $\rightarrow x = 6; r = 7; y = -\sqrt{7^2 - 6^2} = -\sqrt{13}$

Thus $\sin\theta = \dfrac{y}{r} = -\dfrac{\sqrt{13}}{7}$ and $\tan\theta = \dfrac{y}{x} = -\dfrac{\sqrt{13}}{6}$.

8. a. Since $\tan\theta = \dfrac{y}{x} = -\dfrac{9}{5}$ and θ could be in quadrant II $\rightarrow x = -5; y = 9; r = \sqrt{(-5)^2 + 9^2} = \sqrt{106}$

Thus $\sin\theta = \dfrac{y}{r} = \dfrac{9}{\sqrt{106}}$ and $\cos\theta = \dfrac{x}{r} = -\dfrac{5}{\sqrt{106}}$.

θ could also be in quadrant IV, thus $\sin\theta = \dfrac{y}{r} = -\dfrac{9}{\sqrt{106}}$ and $\cos\theta = \dfrac{x}{r} = \dfrac{5}{\sqrt{106}}$

b. $\theta_R = 61°$. Therefore θ=180°-61°=119° or θ=360°-61°=299°

9.

a. $\theta_R = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = 60°$ and since $\sin\theta < 0$, θ is in quadrants III or IV.

Thus $\theta_1 = 180° + \theta_R = 180° + 60° = 240°$ and $\theta_2 = 360° - \theta_R = 360° - 60° = 300°$.

b. $\theta_R = \cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$ and since $\cos\theta > 0$, θ is in quadrants I or IV.

Thus $\theta_1 = \theta_R = 45°$ and $\theta_2 = 360° - \theta_R = 360° - 45° = 315°$.

c. $\theta_R = \tan^{-1}(1) = 45°$ and since $\tan\theta < 0,\ \theta$ is in quadrants II or IV.

Thus $\theta_1 = 180° - \theta_R = 180° - 45° = 135°$ and $\theta_2 = 360° - \theta_R = 360° - 45° = 315°$.

d. Since $\sin 90° = 1,\ \theta = 90°$ is the only solution since 90° is a quadrantal angle.

e. $\theta_R = \cos^{-1}\left(\frac{1}{2}\right) = 60°$ and since $\cos\theta < 0,\ \theta$ is in quadrants II or III.

Thus $\theta_1 = 180° + \theta_R = 180° - 60° = 120°$ and $\theta_2 = 180° + \theta_R = 180° + 60° = 240°$.

f. $\theta_R = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$ and since $\tan\theta > 0,\ \theta$ is in quadrants I or III.

Thus $\theta_1 = \theta_R = 30°$ and $\theta_2 = 180° + \theta_R = 180° + 30° = 210°$.

10.

a. $\theta_R = \sin^{-1}(0{,}45) \approx 27°$ and since $\sin\theta > 0,\ \theta$ is in quadrants I or II.

thus $\theta_1 = \theta_R = 27°$ and $\theta_2 = 180° - \theta_R = 180° - 27° = 153°$.

b. $\theta_R = \cos^{-1}(0{,}36) \approx 69°$ and since $\cos\theta < 0,\ \theta$ is in quadrants II or III.

Thus $\theta_1 = 180° - \theta_R = 180° - 69° = 111°$ and $\theta_2 = 180° + \theta_R = 180° + 69° = 249°$.

c. $\theta_R = \tan^{-1}(2{,}41) \approx 67°$ and since $\tan\theta < 0,\ \theta$ is in quadrants II or IV.

Thus $\theta_1 = 180° - \theta_R = 180° - 67° = 113°$ and $\theta_2 = 360° - \theta_R = 360° - 67° = 293°$.

Chapter 3 – Quadratic Functions

1.

a. $a = -\frac{2}{3};\ p = -2;\ q = -9$ so the vertex is at $(-2,-9)$; A.S.: $x = -2$; opens down

maximum @ $f(-2) = -9$; D: $\{x \in R\}$; R: $\{f(x) \in R \mid f(x) \le -9\}$

b. $a = 2;\ p = -\frac{-8}{2(2)} = 2;\ q = 5 - \frac{(-8)^2}{4(2)} = -3$ so the vertex is at $(2,-3)$; A.S.: $x = 2$; opens up

minimum @ $g(2) = -3$; D: $\{x \in R\}$; R: $\{g(x) \in R \mid g(x) \ge -3\}$

c. $a = -1;\ p = 0;\ q = 4$ so the vertex is at $(0,4)$; A.S.: $x = 0$; opens down

maximum @ $y(0) = 4$; D: $\{x \in R\}$; R: $\{y \in R \mid y \le 4\}$

d. $a < 0;\ p = \frac{-3+7}{2} = 2;\ q = 1$; so the vertex is at $(2,1)$; A.S.: $x = 2$; opens down

maximum @ $y = 1$ when $x = 2$; D: $\{x \in R\}$; R: $\{y \in R \mid y \le 1\}$

2.

a. $(x,y)=(-1,-21);(p,q)=(-3,-5)$

$-21=a\left(-1-(-3)\right)^2-5\rightarrow -16=4a \text{ so } a=-4$

$y=-4(x+3)^2-5$ (vertex form)

standard form: $y=-4(x+3)^2-5=-4\left(x^2+6x+9\right)-5=-4x^2-24x-36-5=-4x^2-24x-41$

b. $(x,y)=(-2,0);\ p=\dfrac{-2+4}{2}=1; q=-18$

$0=a(-2-1)^2-18\rightarrow 18=9a \text{ so } a=2$

$y=2(x-1)^2-18$ (vertex form)

standard form: $y=2(x-1)^2-18=2\left(x^2-2x+1\right)-18=2x^2-4x+2-18=2x^2-4x-16$

c. $(x,y)=(0,2);(p,q)=(3,8)$

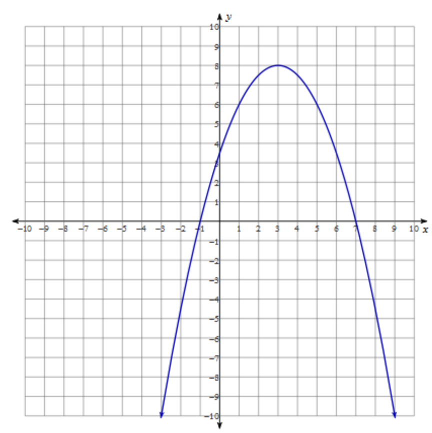
$2=a(0-3)^2+8\rightarrow -6=9a \text{ so } a=-\dfrac{2}{3}$

$y=-\dfrac{2}{3}(x-3)^2+8$ (vertex form)

standard form: $y=-\dfrac{2}{3}(x-3)^2+8=-\dfrac{2}{3}\left(x^2-6x+9\right)+8=-\dfrac{2}{3}x^2+4x-6+8=-\dfrac{2}{3}x^2+4x+2$

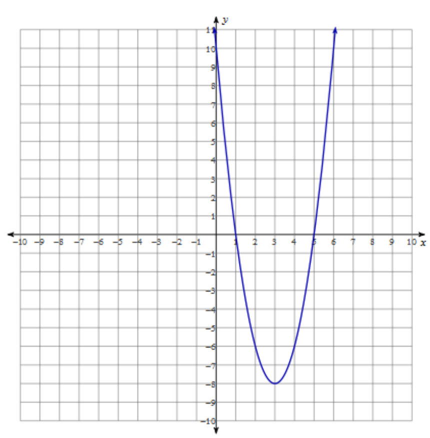
3.

a. Since $a>0$ & $q<0$, $f(x)$ has 2 x-intercepts
b. Since $a<0$ & $q<0$, g(x) has 0 x-intercepts



4. Vertex (3,8), domain $\{x\,|\,x\in R\}$, range $\{y\,|\,y\le 8, y\in R\}$, AS: x=3, x-intercepts -1 and 7, y- intercept $\dfrac{7}{2}$, max value y = 8

5. Vertex (3,-8), domain $\{x\,|\,x\in R\}$, range $\{y\,|\,y\ge -8, y\in R\}$, AS: x=3, x-intercepts 1 and 5, y- intercept 10



6. a.
$$f(x)=x^2+5x+29$$
$$f(x)=\left(x^2+5x\right)+29$$
$$f(x)=\left(x^2+5x+\frac{25}{4}-\frac{25}{4}\right)+29$$
$$f(x)=\left(x+\frac{5}{2}\right)^2+\frac{91}{4}\ ;\ \text{Vertex}\left(\frac{-5}{2},\frac{91}{4}\right)$$

b.
$$f(x)=-5x^2+20x-6$$
$$f(x)=-5\left(x^2-4x\right)-6$$
$$f(x)=-5\left(x^2-4x+4-4\right)-6$$
$$f(x)=-5\left(\left(x^2-4x+4\right)-4\right)\text{-}6$$
$$f(x)=\text{-}5\left(x\text{-}2\right)^2+20-6$$
$$f(x)=-5(x-2)^2+14$$
$$Vertex(2,14)$$

7.
$$x=\text{width}$$
$$y=400-2x\quad\text{length}$$
A = x(400-2x)
$$A=400x-2x^2$$
$$A=-2x^2+400x$$
$$A=-2(x^2-200x)$$
$$A=-2(x^2-200x+10000-10000)$$
$$A=-2(x-100)^2+20000$$
Dimensions are 100 m by 200 m. Maximum enclosed area is $20000m^2$.

8. R = (Selling price) * (Number Sold)

When the selling price is \$20, three hundred students are willing to buy them.

R = (20)*(300)

But for every 5 dollar increase in price (+5x), there is a 30 student decrease (-30x):

R = (20+5x)(300-30x)
$$R=-150x^2+900x+6000$$

Max. R is @ $x=\frac{-b}{2a}=\frac{-900}{-300}=3$; R = 7350

Maximum revenue is \$7350 and it occurs when x = 3 which would bring the price of the calculators up to \$35. There would be 3 price increases of \$5. They would have 210 sales instead of 300.

9.
$$C(x)=800-10x+0.25x^2$$
$$Minimum\ \text{cost @ } x=\frac{-b}{2a}=\frac{10}{0.5}=20$$
$$C(20)=\$700$$

20 light fixtures can be produced for a minimum cost of \$700.

Chapter 4 – Quadratic Equations

1.

a. $5x^2+20x=5x(x+4)$

b. $x^2-12x+32=(x-4)(x-8)$

c. $3x^2+14x-5=(x+5)(3x-1)$

2.

a. Let $a=x-5$

$2(x-5)^2-3(x-5)-9 \;=\; 2a^2-3a-9 \;=\; (a-3)(2a+3)$

$= \;((x-5)-3)(2(x-5)+3) \;=\; (x-5-3)(2x-10+3) \;=\; (x-8)(2x-7)$

b. Let $b=x^2+2x$

$(x^2+2x)^2-11(x^2+2x)+24 \;=\; b^2-11b+24 \;=\; (b-3)(b-8)$

$= \;((x^2+2x)-3)((x^2+2x)-8) \;=\; (x^2+2x-3)(x^2+2x-8) \;=\; (x+3)(x-1)(x+4)(x-2)$

c. Let $\mathrm{c}=x+3$ and $d=y-7$

$25(x+3)^2-\frac{16}{49}(y+7)^2=25c^2-\frac{16}{49}d^2=\left(5c-\frac{4}{7}d\right)\left(5c+\frac{4}{7}d\right)=\left(5(x+3)-\frac{4}{7}(y-7)\right)\left(5(x+3)+\frac{4}{7}(y-7)\right)$

$=\left(5x+15-\frac{4}{7}y+4\right)\left(5x+15+\frac{4}{7}y-4\right)=\left(5x-\frac{4}{7}y+19\right)\left(5x+\frac{4}{7}y+11\right)$

3.

a. $4p^2=24p$

$4p^2-24p=0$

$4p(p-6)=0$

$p=0$ or $p=6$

b. $x^2-5x-36=0$

$(x-9)(x+4)=0$

$x=9$ or $x=-4$

c. $0.2x^2-1.4x+2=0$

$0.2(x^2-7x+10)=0$

$0.2(x-5)(x-2)=0$

$x=5$ or $x=2$

d. $2t^2+14t=16$

$2t^2+14t-16=0$

$2(t^2+7t-8)=0$

$2(t+8)(t-1)=0$

$t=-8$ or $t=1$

e. $10q^2=12-26q$

$10q^2+26q-12=0$

$2(5q^2+13q-6)=0$

$2(q+3)(5q-2)=0$

$q=-3$ or $q=\frac{2}{5}$

f. $x+\frac{20}{7}=\frac{6}{7}x^2$

$\frac{6}{7}x^2-x-\frac{20}{7}=0$

$\frac{1}{7}(6x^2-7x-20)=0$

$\frac{1}{7}(2x-5)(3x+4)=0$

$x=\frac{5}{2}$ or $x=-\frac{4}{3}$

4. Let x be the length and $x - 8$ be the width of the park. The area of the park is given by

$$A = x(x-8) = x^2 - 8x = 2484$$

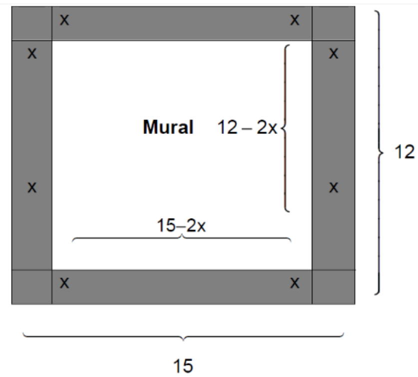
$$x^2 - 8x - 2484 = 0$$

$$(x-54)(x+46) = 0$$

$$x = 54 \text{ or } x = -46$$

The length of the park is 54 m and the width is $54 - 8 = 46$ m

5.



$$\text{new area} = 75\% \text{ of old area}$$

$$(15 - 2x)(12 - 2x) = (0.75)(15)(12)$$

$$180 - 30x - 24x + 4x^2 = 135$$

$$4x^2 - 54x + 180 = 135$$

$$4x^2 - 54x + 45 = 0$$

$$a = 4,\ b = -54,\ c = 45$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-54) \pm \sqrt{(-54)^2 - 4(4)(45)}}{2(4)}$$

$$x = \frac{54 \pm \sqrt{2916 - 720}}{8}$$

$$x = \frac{54 \pm \sqrt{2196}}{8}$$

$$x \doteq \frac{54 \pm 46.8615}{8}$$

$$x \doteq \frac{54 - 46.8615}{8} \text{ or } x \doteq \frac{54 + 46.8615}{8}$$

$$x \doteq \frac{7.1385}{8} \text{ or } x \doteq \frac{100.8615}{8}$$

$$x \doteq 0.8923 \text{ or } x \doteq 12.6077 \text{ (reject)}$$

$$\boxed{x \doteq 0.89 \text{ m}}$$

The width of the mural is approximately 0.89 m.

6.

a. $a = 6; b = 5; c = -6; D = b^2 - 4ac = 5^2 - 4(6)(-6) = 25 + 144 = 169 > 0$

2 real and distinct roots

b. $a = 4; b = -12; c = 9; D = b^2 - 4ac = (-12)^2 - 4(4)(9) = 144 - 144 = 0$

2 real and equal roots

c. $a = 3; b = -2; c = 10; D = b^2 - 4ac = (-2)^2 - 4(3)(10) = 4 - 120 = -116 < 0$

2 non-real roots

7.

a. $a=7; b=-6; c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{(-6)^2-4(7)(-5)}}{2(7)}=\frac{6\pm\sqrt{176}}{14}=\frac{6\pm4\sqrt{11}}{14}=\frac{3\pm2\sqrt{11}}{7}$$

b. $a=3; b=4; c=11$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-4\pm\sqrt{4^2-4(3)(11)}}{2(3)}=\frac{-4\pm\sqrt{-116}}{6}$$ (No *real* solutions)

8.

Let x represent the length of one leg of the right triangle. Then, the other leg is $23-x$.

$$x^2+(23-x)^2=17^2$$
$$x^2+529-46x+x^2=289$$
$$2x^2-46x-240=0$$
$$2(x^2-23x-120)=0$$
$$2(x-8)(x-15)=0$$
$$x-8=0 \quad \text{or} \quad x-15=0$$
$$x=8 \qquad x=15$$

The length of the legs of the right triangle are 8 cm and 15 cm.

‘

9.

a. $x=\frac{-b}{2a}=\frac{-10}{10}=1$. The diver reaches the maximum height after 1 second. The maximum height is 45 m.

b. $-5t^2+10t+40=0$

$-5(t^2-2t-8)=0$ It takes the diver 4 seconds to reach the water.

$-5(t-4)(t+2)=0$

$t=4$ and t = -2

c. $a=-5; b=7; c=4$

$-5t^2+10t+40=42$

$-5t^2+10t-2=0$

$t=\frac{-10\pm\sqrt{100-40}}{-10}=0.225$ and 1.77

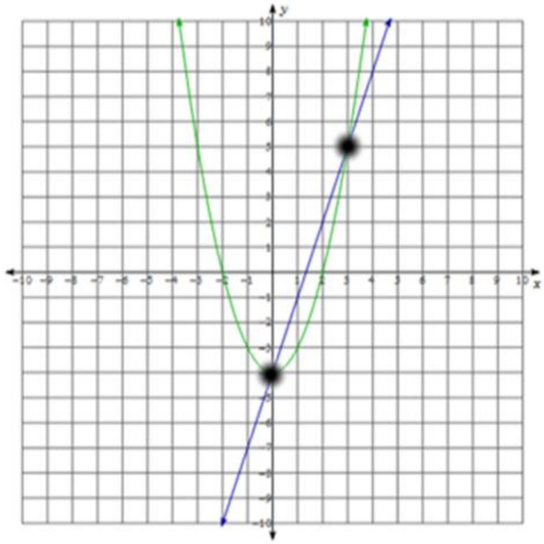
The diver reaches a height of 42m at 0.225 s & again at 1.77 s.

d. $h(0)=40$ m

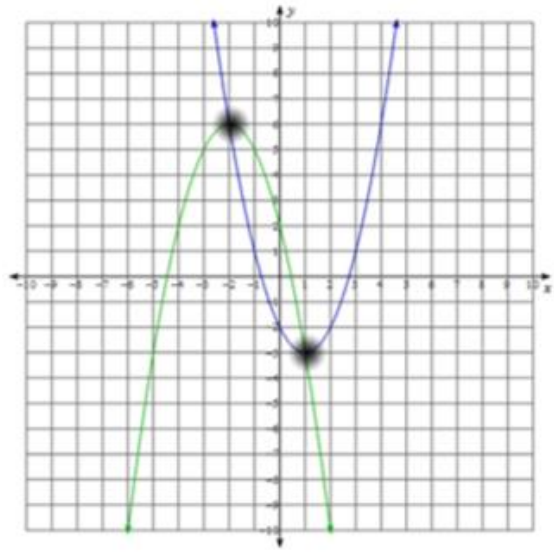
Chapter 8 – Systems of Equations

1.

a. $(x_1, y_1) = (0, -4)$ and $(x_2, y_2) = (3, 5)$



b. $(x_1, y_1) = (-2, 6)$ and $(x_2, y_2) = (1-, 3)$



2.

$$\begin{aligned} mx^2 + y &= 32 \\ 9m + 14 &= 32 \\ m &= 2 \end{aligned} \qquad \begin{aligned} mx^2 - 6y &= n \\ 2(9) - 5(14) &= n \\ -52 &= n \end{aligned}$$

3.

a.

$$\begin{cases} 9x - y = 4 \\ y + 3 = x^2 + 7x \end{cases} \rightarrow \begin{cases} y = 9x - 4 \\ y = x^2 + 7x - 3 \end{cases}$$

$$9x - 4 = x^2 + 7x - 3$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

If $x = 1$

$$y = 9(1) - 4 = 5$$

Solution: $(1,5)$

b.

$$\begin{cases} y + 2 = 3x \\ y + x^2 + 3x = 5 \end{cases} \rightarrow \begin{cases} y = 3x - 2 \\ y = -x^2 - 3x + 5 \end{cases}$$

$$3x - 2 = -x^2 - 3x + 5$$

$$x^2 + 6x - 7 = 0$$

$$(x+7)(x-1) = 0$$

$x = -7$ or $x = 1$

If $x = -7$

$$y = 3(-7) - 2 = -23$$

If $x = 1$

$$y = 3(1) - 2 = 1$$

Solutions: $(-7, -23)$ et $(1,1)$

c. $$\begin{aligned}2x^2-4x+y&=3\\4x-2y&=-7\end{aligned}$$

$$\begin{aligned}2(2x^2-4x+y&=3)\\4x-2y&=-7\end{aligned}$$

$$\begin{aligned}4x^2-8x+2y&=6\\4x-2y&=-7\end{aligned}$$

$$4x^2-4x=-1$$
$$4x^2-4x+1=0$$
$$(2x-1)(2x-1)=0$$
$$x=\frac{1}{2}$$

$$4x-2y=-7$$
$$4\left(\frac{1}{2}\right)-2y=-7$$
$$y=\frac{9}{2}$$

Solution: $\left(\frac{1}{2},\frac{9}{2}\right)$

4. let x = smaller number and let y= larger number

$$\begin{cases}x+y=17\\3x^2-13=y\end{cases}$$
$$x+3x^2-13=17$$
$$3x^2+x-30=0$$
$$(3x+10)(x-3)=0$$
$$\cancel{x=-\frac{10}{3}} \text{ or } x=3$$

If $x=3$

$$y=3(3)^2-13=14$$

The two numbers are 3 and 14.

5.

a. $$\begin{cases}y-x^2=7-2x\\x^2-4x+y=3\end{cases}\rightarrow\begin{cases}y=x^2-2x+7\\y=-x^2+4x+3\end{cases}$$
$$x^2-2x+7=-x^2+4x+3$$
$$2x^2-6x+4=0$$
$$2(x^2-3x+2)=0$$
$$2(x-1)(x-2)=0$$
$$x=1 \text{ or } x=2$$

If $x=1$

$$y=(1)^2-2(1)+7=6$$

If $x=2$

$$y=(2)^2-2(2)+7=7$$

Solutions: $(1,6)$ and $(2,7)$

b. $$5x^2 + 3y = -3 - x$$
$$2x^2 - x = -4 - 2y$$

$$2(5x^2 + x + 3y + 3 = 0)$$
$$-3(2x^2 - x + 2y + 4 = 0)$$

$$10x^2 + 2x + 6y + 6 = 0$$
$$-6x^2 + 3x - 6y - 12 = 0$$

$$4x^2 + 5x - 6 = 0$$
$$(x+2)(4x-3) = 0$$
$$x = -2 \quad x = \frac{3}{4}$$

Using $2x^2 - x = -4 - 2y$:

So y = $\dfrac{2x^2 - x + 4}{-2}$

If $x = -2$, then $y = -7$

If $x = \dfrac{3}{4}$, then $y = \dfrac{-35}{16}$

Solutions: $(-2, -7)$ and $\left(\dfrac{3}{4}, \dfrac{-35}{16}\right)$

$$-5t^2 + 32t + 2 = 31.5t + 1$$

6. $$-5t^2 + 0.5t + 1 = 0$$
$$t = \frac{-0.5 \pm \sqrt{0.25 + 20}}{-10} = -0.4 \text{ and } 0.5$$

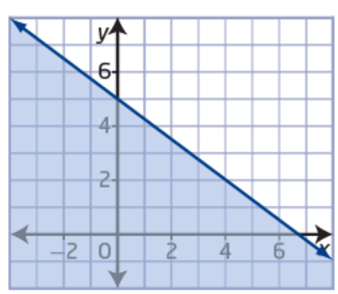
The skeet was shot at 0.5 seconds after release at a height of 16.75m

<u>Chapter 9 – Linear and Quadratic Inequalities</u>

1. 2 points on the graph are (4,2) and (0,5)

$$m = \frac{5-2}{0-4} = -\frac{3}{4}$$

$$y \le \frac{-3}{4}x + 5$$

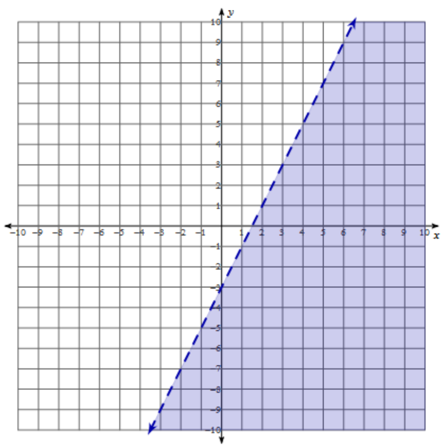


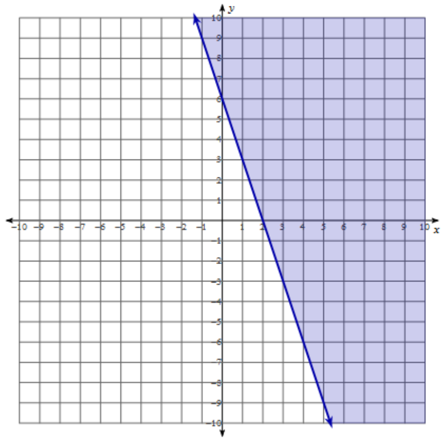
2.

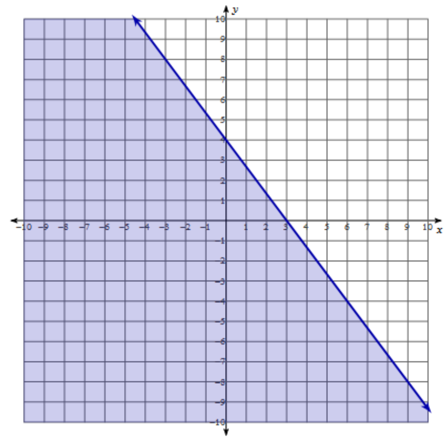
a. $y < 2x - 3$

b. $y \geq 6 - 3x$

c. $3x + 4y \leq 12$







3.

a. $(x+3)(2x-7) = 0 \to x-$intercepts @ $x = -3$ and $x = \frac{7}{2}$; $y = (x+3)(2x-7)$ opens up.

$(x+3)(2x-7) > 0$ so $x < -3$ or $x > \frac{7}{2}$

b. $-3x^2 = -9x + 6 \to -3x^2 + 9x - 6 = 0 \to -3(x-1)(x-2) = 0 \to$ roots @ $x = 1$ and $x = 2$.

$y = -3(x-1)(x-2)$ opens down, so $-3x^2 + 9x - 6 \geq 0$ when $1 \leq x \leq 2$.

c. $11x + 10 = 6x^2 \to 6x^2 - 11x - 10 = 0 \to (3x+2)(2x-5) = 0 \to$ roots @ $x = -\frac{2}{3}$ and $x = \frac{5}{2}$.

$y = 6x^2 - 11x - 10$ opens up, so $6x^2 - 11x - 10 \geq 0$ when $x \geq \frac{5}{2}$ or $x \leq -\frac{2}{3}$.

d. $2x^2 + 2 = 7x \to 2x^2 - 7x + 2 = 0 \to x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)} = \frac{7 \pm \sqrt{33}}{4}$

roots @ $x = \frac{7+\sqrt{33}}{4} \approx 3.19$ and $x = \frac{7-\sqrt{33}}{4} \approx 0.31$

$y = 2x^2 - 7x + 2$ opens up, so $2x^2 - 7x + 2 < 0$ when $\frac{7-\sqrt{33}}{4} < x < \frac{7+\sqrt{33}}{4}$

4.

$-16t^2 + 500t + 5 > 2000$

$-16t^2 + 500t - 1995 = 0 \to t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-500 \pm \sqrt{500^2 - 4(-16)(-1995)}}{2(-16)} = \frac{-500 \pm \sqrt{122320}}{-32}$

≈ 4.70 and 26.6

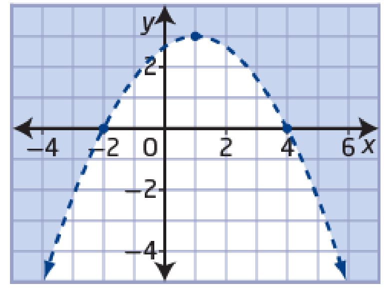
$y = -16x^2 + 500x - 1995$ opens down, so $-16t^2 + 500t - 1995 > 0$ when $4.70 < t < 26.6$

The firework is above 2000 ft between 4.70 and 26.6 seconds.

Extension : The firework is below 2000 ft at times less than 4.7 seconds and between 26.6 seconds and 31.26 sec.

(Firework hits the ground at 31.26 seconds)

5.



Equation of parabolic boundary: $y = a(x-p)^2 + q$

vertex at $(1,3)$ so: $y = a(x-1)^2 + 3$

substitute $(4,0)$: $0 = a(4-1)^2 + 3$

$$0 = 9a + 3$$

$$-3 = 9a$$

$$\frac{-1}{3} = a$$

Final equation of parabolic boundary: $y = \frac{-1}{3}(x-1)^2 + 3$

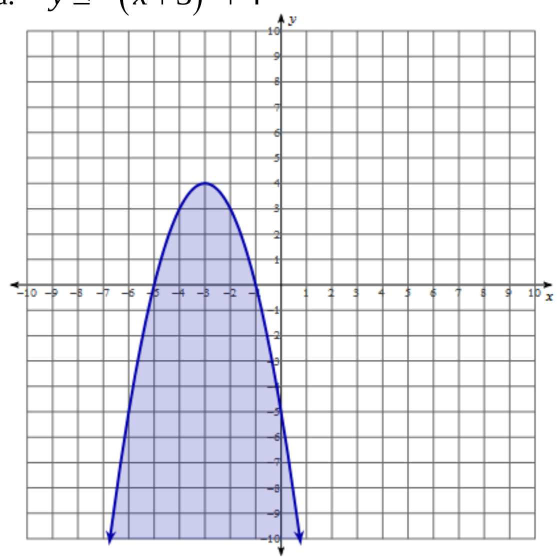
Inequality: The boundary is *broken* and the region *above* is shaded, so the inequality that describes the graph is:

In vertex form: $y > \frac{-1}{3}(x-1)^2 + 3$

In standard form: $y > \frac{-1}{3}x^2 + \frac{2}{3}x + \frac{8}{3}$

6.

a. $y \le -\left(x+3\right)^2 + 4$



parabolic boundary:

vertex $(-3,4)$

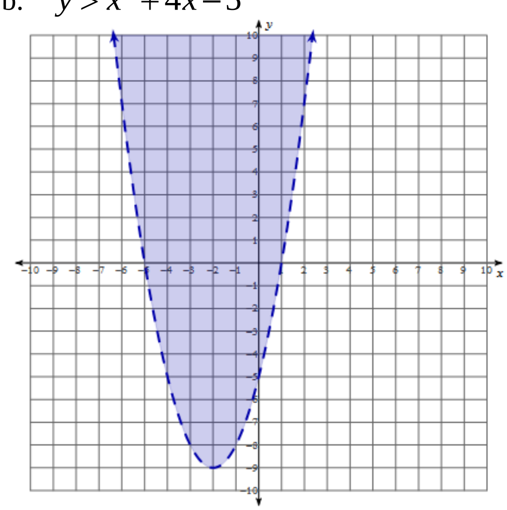
parabola opens downward

stretch factor = 1

$\le$ means *solid* boundary

and shade *below*

b. $y > x^2 + 4x - 5$



vertex $(-2,-9)$:

$x = \frac{-b}{2a} = \frac{-4}{2\left(1\right)} = -2$

$y = (-2)^2 + 4(-2) - 5$

$= -9$

parabola opens upward

stretch factor = 1

$>$ means *broken* boundary

and shade *above*

c. $y \geq -4x^2 - 4x + 3$

vertex $(-0.5, 4)$:

$$x = \frac{-b}{2a} = \frac{4}{2(-4)} = -0.5$$

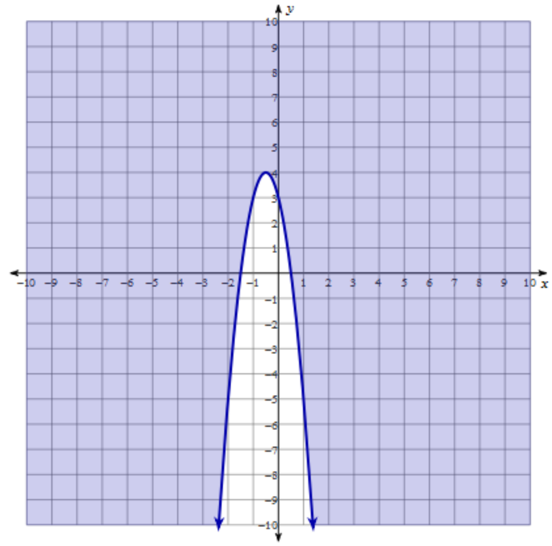
$$y = -4(-0.5)^2 - 4(-0.5) + 3$$
$$= 4$$

parabola opens downward

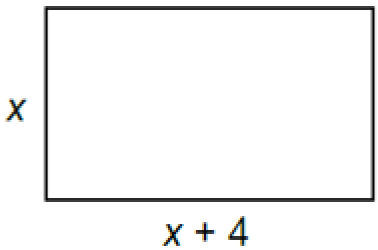
stretch factor = 4

$\geq$ means *solid* boundary

and shade *above*



7.



$x(x+4) \leq 480$

$x^2 + 4x - 480 \leq 0$

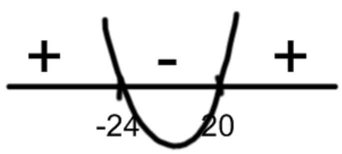
related equation:

$x^2 + 4x - 480 = 0$

$(x+24)(x-20) = 0$

$x = -24,\ 20$

graph of related function:



$\therefore$ solution to inequality:

$-24 \leq x \leq 20$

but, in this context:

$0 < x \leq 20$

So, width can be any value greater than 0 m but less than or equal to 20 m, and length would be 4 m more than the width.

Chapter 5 – Radical Expressions and Equations

1.

a. $5\sqrt{3}=\sqrt{5^2\cdot 3}=\sqrt{75}$

b. $b^3\sqrt{b}=\sqrt{\left(b^3\right)^2\cdot b}=\sqrt{b^6\cdot b}=\sqrt{b^7}$

c. $3k\sqrt{5k^3}=\sqrt{(3k)^2\cdot 5k^3}=\sqrt{3^2k^2\cdot 5k^3}=\sqrt{9k^2\cdot 5k^3}=\sqrt{45k^5}$

d. $2r\sqrt[3]{3r^4}=\sqrt[3]{(2r)^3\cdot 3r^4}=\sqrt[3]{2^3r^3\cdot 3r^4}=\sqrt[3]{8r^3\cdot 3r^4}=\sqrt[3]{24r^7}$

2.

a. $\sqrt{300}=\sqrt{100\cdot 3}=\sqrt{10^2\cdot 3}=10\sqrt{3}$

b. $\sqrt{450}=\sqrt{225\cdot 2}=\sqrt{15^2\cdot 2}=15\sqrt{2}$

c. $\sqrt[3]{160}=\sqrt[3]{8\cdot 20}=\sqrt[3]{2^3\cdot 20}=2\sqrt[3]{20}$

d. $\sqrt[4]{324}=\sqrt[4]{81\cdot 4}=\sqrt[4]{3^4\cdot 4}=3\sqrt[4]{4}$

3.

a. $6\sqrt{80}-2\sqrt{20}=6\cdot 4\sqrt{5}-2\cdot 2\sqrt{5}=24\sqrt{5}-4\sqrt{5}=20\sqrt{5}$

b. $5\sqrt{12}-2\sqrt{27}=5\cdot 2\sqrt{3}-2\cdot 3\sqrt{3}=10\sqrt{3}-6\sqrt{3}=4\sqrt{3}$

c. $\frac{5}{6}\sqrt{252}+\frac{\sqrt{175}}{8}=\frac{5}{6}\cdot 6\sqrt{7}+\frac{1}{8}\cdot 5\sqrt{7}=5\sqrt{7}+\frac{5}{8}\sqrt{7}=\frac{40}{8}\sqrt{7}+\frac{5}{8}\sqrt{7}=\frac{45}{8}\sqrt{7}$

d. $2\sqrt[3]{375}-3\sqrt[3]{648}=2\cdot 5\sqrt[3]{3}-3\cdot 6\sqrt[3]{3}=10\sqrt[3]{3}-18\sqrt[3]{3}=-8\sqrt[3]{3}$

4. $\sqrt{1573}=\sqrt{121\times 13}=11\sqrt{13}$ cm (side length of the square)

5. $\sqrt[4]{32x^9y^5}=\sqrt[4]{16\bullet 2\bullet x^8\bullet x\bullet y^4\bullet y}=2x^2y\sqrt[4]{2xy}, x\geq 0, y\geq 0$

6.

a. $\sqrt{3}\left(5\sqrt{6}-4\sqrt{24}\right)=5\sqrt{18}-4\sqrt{72}=5\cdot 3\sqrt{2}-4\cdot 6\sqrt{2}=15\sqrt{2}-24\sqrt{2}=-9\sqrt{2}$

b. $\left(2\sqrt{7}-3\sqrt{5}\right)\left(4\sqrt{5}+\sqrt{7}\right)=8\sqrt{35}-12(5)+2(7)-3\sqrt{35}=5\sqrt{35}-60+14=5\sqrt{35}-46$

c. $\left(2\sqrt[3]{4}-5\sqrt[3]{2}\right)\left(3\sqrt[3]{2}-\sqrt[3]{4}\right)=6\sqrt[3]{8}-15\sqrt[3]{4}-2\sqrt[3]{16}+5\sqrt[3]{8}=6(2)-15\sqrt[3]{4}-2\cdot 2\sqrt[3]{2}+5(2)$

$=12-15\sqrt[3]{4}-4\sqrt[3]{2}+10=22-15\sqrt[3]{4}-4\sqrt[3]{2}$

7.

a. $\frac{24\sqrt{14}}{8\sqrt{2}}=\frac{24}{8}\sqrt{\frac{14}{2}}=3\sqrt{7}$

b. $\frac{9\sqrt{5}}{4\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{9\sqrt{15}}{4(3)}=\frac{9\sqrt{15}}{12}=\frac{3\sqrt{15}}{4}$

c. $\frac{7\sqrt{5}}{4-2\sqrt{3}}\cdot\frac{4+2\sqrt{3}}{4+2\sqrt{3}}=\frac{28\sqrt{5}+14\sqrt{15}}{16-4(3)}=\frac{28\sqrt{5}+14\sqrt{15}}{16-12}=\frac{28\sqrt{5}+14\sqrt{15}}{4}=\frac{14\sqrt{5}+7\sqrt{15}}{2}$

d. $\dfrac{5\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}}\cdot\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}=\dfrac{10\cdot 3+5\sqrt{6}+2\sqrt{6}+2}{4\cdot 3-2}=\dfrac{30+2+7\sqrt{6}}{12-2}=\dfrac{32+7\sqrt{6}}{10}$

8. a. $2x+1\geq 0$

$x\geq\dfrac{-1}{2}$

b. $\dfrac{3}{\sqrt{9-x}}$

$9-x>0$

$-x>-9$

$x<9$

9. $\left(3-2\sqrt{x-5}\right)\left(3-2\sqrt{x-5}\right)$

$9-6\sqrt{x-5}-6\sqrt{x-5}+4(x-5)$

$9-12\sqrt{x-5}+4x-20$

$4x-12\sqrt{x-5}-11$

10.

a. $4\sqrt{2x-3}=12$

$\sqrt{2x-3}=3$

$\left(\sqrt{2x-3}\right)^2=3^2$

$2x-3=9$

$2x=12$

$x=6$

if $x=6$

$4\sqrt{2(6)-3}=4\sqrt{12-3}=4\sqrt{9}=4(3)=12$

$\therefore x=6$ is a solution

b. $x+2\sqrt{x-1}=9$

$2\sqrt{x-1}=9-x$

$\left(2\sqrt{x-1}\right)^2=(9-x)^2$

$4(x-1)=81-18x+x^2$

$4x-4=81-18x+x^2$

$x^2-22x+85=0$

$(x-5)(x-17)=0$

$x=5$ or $x=17$

if $x=5$

$5+2\sqrt{5-1}=5+2\sqrt{4}=5+2\cdot 2=5+4=9$

$\therefore x=5$ is a solution

if $x=17$

$17+2\sqrt{17-1}=17+2\sqrt{16}=17+2\cdot 4=17+8=25\neq 9$

$\therefore x=17$ is an extraneous root

c. $\sqrt{p+1}=\sqrt{p+6}-1$

$\left(\sqrt{p+1}\right)^2=\left(\sqrt{p+6}-1\right)^2$

$p+1=p+6-2\sqrt{p+6}+1$

$2\sqrt{p+6}=6$

$\sqrt{p+6}=3$

$\left(\sqrt{p+6}\right)^2=3^2$

$p+6=9$

$p=3$

if $p=3$

Left side $=\sqrt{3+1}=\sqrt{4}=2$

Right side $=\sqrt{3+6}-1=\sqrt{9}-1=3-1=2$

$\therefore$ $p=3$ is a solution

d. $1+\sqrt{6-2z}=\sqrt{5-4z}$

$\left(1+\sqrt{6-2z}\right)^2=\left(\sqrt{5-4z}\right)^2$

$1+2\sqrt{6-2z}+6-2z=5-4z$

$2\sqrt{6-2z}=-2-2z$

$\sqrt{6-2z}=-1-z$

$\left(\sqrt{6-2z}\right)^2=\left(-1-z\right)^2$

$6-2z=z^2+2z+1$

$z^2+4z-5=0$

$(z+5)(z-1)=0$

$z=-5$ or $z=1$

if $z=-5$

Left side $=1+\sqrt{6-2(-5)}=1+\sqrt{6+10}=1+\sqrt{16}=1+4=5$

Right side $=\sqrt{5-4(-5)}=\sqrt{5+20}=\sqrt{25}=5$

$\therefore$ $z=-5$ is a solution

if $z=1$

Left side $=1+\sqrt{6-2(1)}=1+\sqrt{6-2}=1+\sqrt{4}=1+2=3$

Right side $=\sqrt{5-4(1)}=\sqrt{5-4}=\sqrt{1}=1$

$\therefore$ $z=1$ is an extraneous root

<u>Chapter 6 – Rational Expressions and Equations</u>

1.

a. $g \neq 0;\ j \neq 0$

b. $x \neq 0; x \neq 5$

c. $p^2 - 3p - 10 = (p-5)(p+2) \rightarrow p \neq 5;\ p \neq -2$

2.

a. $\dfrac{3x-12}{32-8x} = \dfrac{3(x-4)}{8(4-x)} = -\dfrac{3}{8}; x \neq 4$

b. $\dfrac{r^2-2r-3}{2r^2-r-3} = \dfrac{(r-3)\cancel{(r+1)}}{(2r-3)\cancel{(r+1)}} = \dfrac{r-3}{2r-3}; r \neq -1; r \neq \dfrac{3}{2}$

c. $\dfrac{4x^2-y^2}{2x^2+5xy-3y^2} = \dfrac{\cancel{(2x-y)}(2x+y)}{\cancel{(2x-y)}(x+3y)} = \dfrac{2x+y}{x+3y}; y \neq 2x; x \neq -3y$

3.

a. $\dfrac{x^2-2x}{x+1} \times \dfrac{x^2-1}{x^2+x-6} = \dfrac{x\cancel{(x-2)}}{\cancel{x+1}} \times \dfrac{(x-1)\cancel{(x+1)}}{(x+3)\cancel{(x-2)}} = \dfrac{x(x-1)}{x+3}$ or $\dfrac{x^2-x}{x+3}; x \neq -3; x \neq -1; x \neq 2$

b. $\dfrac{x^2+2x-15}{x^3+4x^2} \div \dfrac{x^2+9x+20}{2x^3+7x^2-4x} = \dfrac{\cancel{(x+5)}(x-3)}{x^{\cancel{2}}(x+4)} \times \dfrac{\cancel{x}\cancel{(x+4)}(2x-1)}{\cancel{(x+5)}\cancel{(x+4)}}$

$= \dfrac{(x-3)(2x-1)}{x(x+4)}$ or $\dfrac{2x^2-7x+3}{x^2+4x}; x \neq 0; x \neq -4; x \neq -5; x \neq \dfrac{1}{2}$

c. $\dfrac{x^2-4}{x^2+2x} \div \dfrac{x-2}{x^2-3x} \times \dfrac{x^3}{x^3+3x^2} = \dfrac{\cancel{(x-2)}\cancel{(x+2)}}{\cancel{x}\cancel{(x+2)}} \times \dfrac{\cancel{x}(x-3)}{\cancel{x-2}} \times \dfrac{x^{\cancel{3}}}{\cancel{x^2}(x+3)}$

$= \dfrac{x(x-3)}{x+3}$ or $\dfrac{x^2-3x}{x+3}; x \neq 0; x \neq \pm 2; x \neq \pm 3$

4.

a. $\dfrac{3x+1}{2x^2-2} + \dfrac{2x+2}{2x^2-8x+6} = \dfrac{3x+1}{2(x-1)(x+1)} + \dfrac{2(x+1)}{2(x-3)(x-1)} = \dfrac{(3x+1)(x-3)}{2(x-1)(x+1)(x-3)} + \dfrac{2(x+1)(x+1)}{2(x-3)(x-1)(x+1)}$

$\dfrac{3x^2-8x-3+2x^2+4x+2}{2(x-1)(x+1)(x-3)} = \dfrac{5x^2-4x-1}{2(x-1)(x+1)(x-3)} = \dfrac{(5x+1)\cancel{(x-1)}}{2\cancel{(x-1)}(x+1)(x-3)} = \dfrac{(5x+1)}{2(x+1)(x-3)}; x \neq 3; x \neq \pm 1$

b. $\dfrac{2x+2}{x^2+4x-12} - \dfrac{x+1}{x^2-4} = \dfrac{2(x+1)}{(x+6)(x-2)} - \dfrac{x+1}{(x-2)(x+2)} = \dfrac{2(x+1)(x+2)}{(x+6)(x-2)(x+2)} - \dfrac{(x+1)(x+6)}{(x-2)(x+2)(x+6)}$

$\dfrac{2x^2+6x+4-x^2-7x-6}{(x+6)(x-2)(x+2)} = \dfrac{x^2-x-2}{(x+6)(x-2)(x+2)} = \dfrac{\cancel{(x-2)}(x+1)}{(x+6)\cancel{(x-2)}(x+2)} = \dfrac{x+1}{(x+6)(x+2)}; x \neq -6; x \neq \pm 2$

c. $\dfrac{5}{x^2-1}-\dfrac{2}{x^2+4x+3}+\dfrac{3}{x^2+2x-3}=\dfrac{5}{(x-1)(x+1)}-\dfrac{2}{(x+3)(x+1)}+\dfrac{3}{(x+3)(x-1)}$

$=\dfrac{5(x+3)}{(x-1)(x+1)(x+3)}-\dfrac{2(x-1)}{(x+3)(x+1)(x-1)}+\dfrac{3(x+1)}{(x+3)(x-1)(x+1)}=\dfrac{5x+15-2x+2+3x+3}{(x-1)(x+1)(x+3)}$

$=\dfrac{6x+20}{(x-1)(x+1)(x+3)};\ x\neq -3;\ x\neq \pm 1$

5.

a. $\dfrac{\frac{4}{3}+\frac{4}{r}}{\frac{r}{r+12}-\frac{1}{r}}=\dfrac{4r+12}{3r}\div\dfrac{r^2-(r+12)}{r(r+12)}=\dfrac{4(r+3)}{3r}\div\dfrac{r^2-r-12}{r(r+12)}=\dfrac{4(r+3)}{3\cancel{r}}\times\dfrac{\cancel{r}(r+12)}{(r-4)(r+3)}=\dfrac{4(r+12)}{3(r-4)}$

$r\neq 0,-12,-3,4$

b. $\dfrac{\frac{-2}{x-7}+\frac{4}{x+7}}{\frac{x}{x^2-49}-\frac{-2}{x-7}}=\left(\dfrac{-2}{x-7}+\dfrac{4}{x+7}\right)\div\left(\dfrac{x}{(x-7)(x+7)}+\dfrac{2}{x-7}\right)=\dfrac{-2(x+7)+4(x-7)}{(x-7)(x+7)}\div\dfrac{x+2(x+7)}{(x-7)(x+7)}$

$=\dfrac{-2x-14+4x-28}{(x-7)(x+7)}\div\dfrac{x+2x+14}{(x-7)(x+7)}=\dfrac{2x-42}{(x-7)(x+7)}\times\dfrac{(x-7)(x+7)}{3x+14}=\dfrac{2x-42}{3x+14};\ x\neq\pm 7,-\dfrac{14}{3}$

6.

a. $\dfrac{x-3}{x+2}+\dfrac{8x+4}{x^2+2x-35}\times\dfrac{x^2+4x-21}{2x^2-5x-3}=\dfrac{x-3}{x+2}+\dfrac{4\cancel{(2x+1)}}{\cancel{(x+7)}(x-5)}\times\dfrac{\cancel{(x+7)}\cancel{(x-3)}}{\cancel{(2x+1)}\cancel{(x-3)}}=\dfrac{x-3}{x+2}+\dfrac{4}{x-5}$

$=\dfrac{(x-3)(x-5)}{(x+2)(x-5)}+\dfrac{4(x+2)}{(x-5)(x+2)}=\dfrac{x^2-8x+15+4x+8}{(x+2)(x-5)}=\dfrac{x^2-4x+23}{(x+2)(x-5)};\ x\neq -2;\ x\neq -7;\ x\neq 5;\ x\neq 3;\ x\neq -\dfrac{1}{2}$

b. $\dfrac{x-1}{x+5}-\dfrac{x^3-4x}{x^2-6x-7}\div\dfrac{3x^3+5x^2-2x}{3x^2+2x-1}=\dfrac{x-1}{x+5}-\dfrac{\cancel{x}(x-2)\cancel{(x+2)}}{(x-7)\cancel{(x+1)}}\times\dfrac{\cancel{(3x-1)}\cancel{(x+1)}}{\cancel{x}\cancel{(3x-1)}\cancel{(x+2)}}=\dfrac{x-1}{x+5}-\dfrac{x-2}{x-7}$

$=\dfrac{(x-1)(x-7)}{(x+5)(x-7)}-\dfrac{(x-2)(x+5)}{(x-7)(x+5)}=\dfrac{x^2-8x+7-x^2-3x+10}{(x+5)(x-7)}=\dfrac{17-11x}{(x+5)(x-7)}$

$x\neq -5,-1,7,0,-2,\dfrac{1}{3}$

7.

d	r	t
120	x	$\frac{120}{x}$
300	$x + 20$	$\frac{300}{x+20}$

$$\frac{120}{x}+\frac{300}{x+20}=10$$
$$120(x+20)+300x=10x(x+20)$$
$$120x+2400+300x=10x^2+200x$$
$$0=10x^2-220x-2400$$
$$0=x^2-22x-240$$
$$0=(x-30)(x+8)$$
$$x=30 \quad x=-8$$

Elliott drove 30 mph while it was raining.

8.

a.

$$\frac{2x}{x+1}-\frac{5}{x-1}=\frac{x^2-17}{x^2-1}$$
$$\frac{2x}{x+1}-\frac{5}{x-1}=\frac{x^2-17}{(x-1)(x+1)}$$
$$2x(x-1)-5(x+1)=x^2-17$$
$$2x^2-2x-5x-5=x^2-17$$
$$x^2-7x+12=0$$
$$(x-3)(x-4)=0$$
$$x=3 \text{ or } x=4$$

$x \neq \pm 1$

So $x=3$ is a solution

and $x=4$ is a solution.

b. $$\frac{5}{m-1}+\frac{2}{m+1}=-6$$
$$\frac{5(m+1)}{(m-1)(m+1)}+\frac{2(m-1)}{(m+1)(m-1)}=\frac{-6(m+1)(m-1)}{(m+1)(m-1)}$$
$$5(m+1)+2(m-1)=-6(m+1)(m-1)$$
$$5m+5+2m-2=-6m^2+6$$
$$6m^2+7m-3=0$$
$$(2m+3)(3m-1)=0$$
$$m=-\frac{3}{2} \text{ or } m=\frac{1}{3}$$

$m \neq \pm 1$

So $m=-\frac{3}{2}$ is a solution

and $m=\frac{1}{3}$ is a solution.

c. $\dfrac{b-24}{b^2-8b}-\dfrac{5-b}{b-8}=\dfrac{2b+3}{b}$

$$\frac{b-24}{b(b-8)}-\frac{5-b}{b-8}=\frac{2b+3}{b}$$

$$\frac{b-24}{b(b-8)}-\frac{b(5-b)}{b(b-8)}=\frac{(2b+3)(b-8)}{b(b-8)}$$

$$\frac{b-24}{b(b-8)}+\frac{b^2-5b}{b(b-8)}=\frac{2b^2-13b-24}{b(b-8)}$$

$$b-24+b^2-5b=2b^2-13b-24$$

$$b^2-9b=0$$

$$b(b-9)=0$$

$b=0$ or $b=9$

$b\neq 0; b\neq 8$

so $b=0$ is extraneous

and $b=9$ is a solution.

9.

	Time to open hill	Fraction completed in 1 hour	Fraction of work completed in x hours
Machine-made snow	12	$\frac{1}{12}$	$\frac{x}{12}$
Natural snow	36	$\frac{1}{36}$	$\frac{x}{36}$
Together	x	$\frac{1}{x}$	1

$$\frac{x}{12}+\frac{x}{36}=1$$

$$\frac{3x}{36}+\frac{x}{36}=\frac{36}{36}$$

$$4x=36$$

$$x=9 \text{ hours}$$

10. Let n represent the larger positive integer.

Let $n-4$ represent the smaller positive integer.

$$\frac{1}{n}+\frac{1}{n-4}=\frac{10}{21}$$

$$21n(n-4)\cdot\left(\frac{1}{n}+\frac{1}{n-4}\right)=21n(n-4)\cdot\left(\frac{10}{21}\right)$$ *Multiply both sides by the LCD.*

$$1n(n-4)\cdot\frac{1}{n}+21n(n-4)\cdot\frac{1}{n-4}=21n(n-4)\cdot\left(\frac{10}{21}\right)$$ *Distribute and then cancel.*

$$21(n-4)+21n=10n(n-4)$$

$$21(n-4)+21n=10n(n-4)$$

$$21n-84+21n=10n^2-40n$$

$$42n-84=10n^2-40n$$

$$0=10n^2-82n+84$$

$$0=2(5n^2-41n+42)$$

$$0=2(5n-6)(n-7)$$

$$5n-6=0 \quad \text{or} \quad n-7=0$$

$$5n=6 \qquad n=7$$

$$n=\frac{6}{5}$$

The two positive numbers are 7 and 3.

11.

	Distance (miles)	Speed (mph)	Time (h)
With the current	5	x+1	$\frac{5}{x+1}$
Against the current	6	x–1	$\frac{6}{x-1}$
		Total	4

$$\frac{5}{x+1}+\frac{6}{x-1}=3$$

Now we solve.

$$(x+1)(x-1)\cdot\left(\frac{5}{x+1}+\frac{6}{x-1}\right)=(3)\cdot(x+1)(x-1)$$ Multiply by the LCD

$$\frac{5(x+1)(x-1)}{x+1}+\frac{6(x+1)(x-1)}{x-1}=3(x+1)(x-1)$$ Distribute the LCD

Reduce

$$5(x-1)+6(x+1)=3(x+1)(x-1)$$

Multiply out each side

$$5x-5+6x+6=3(x^2-1)$$

$$\begin{array}{rl} 11x+1 &= 3x^2-3 \\ -11x-1 & \quad -11x-1 \end{array}$$ Move everything to one side

$$3x^2-11x-4=0$$

$$3x^2-11x-4=0$$

$$(3x+1)(x-4)=0$$

$$\begin{array}{ll} 3x+1=0 & x-4=0 \\ \quad -1 \quad -1 & \quad +4 \quad +4 \end{array}$$

$$3x=-1 \qquad x=4$$

Mark paddles at 4mph.

$$\frac{3x}{3}=\frac{-1}{3}$$

$$x=-\frac{1}{3}$$

Chapter 7 – Absolute Value

1.

a. $\left|-5+6^2\right|-\left|8-(-9)\right|+\left|2-5\right|+\left|-4\right|=\left|31\right|-\left|17\right|+\left|-3\right|+\left|-4\right|=31-17+3+4=21$

b. $7\left|0.4-5\right|+\left|-2^3\right|=7\left|-4.6\right|+\left|-8\right|=7(4.6)+8=32.2+8=40.2$

c. $\dfrac{-7-\left|-7^2-(-6)\right|}{-2}=\dfrac{-7-\left|-49+6\right|}{-2}=\dfrac{-7-\left|-43\right|}{-2}=\dfrac{-7-43}{-2}=\dfrac{-50}{-2}=25$

2.

a. $3x+5=0\rightarrow$ invariant point @ $x=-\dfrac{5}{3}$; $y=\begin{cases}3x+5 \text{ if } x\geq -\dfrac{5}{3}\\ -3x-5 \text{ if } x<-\dfrac{5}{3}\end{cases}$

b. $-4x+7=0\rightarrow$ invariant point @ $x=\dfrac{7}{4}$; $y=\begin{cases}-4x+7 \text{ if } x\leq \dfrac{7}{4}\\ 4x-7 \text{ if } x>\dfrac{7}{4}\end{cases}$

c. $x^2-3x-10=(x-5)(x+2)=0\rightarrow$ invariant points @ $x=5$ and $x=-2$

$y=x^2-3x-10$ opens up, $y=\begin{cases}x^2-3x-10 \text{ if } x\geq 5 \text{ or } x\leq -2\\ -x^2+3x+10 \text{ if } -2<x<5\end{cases}$

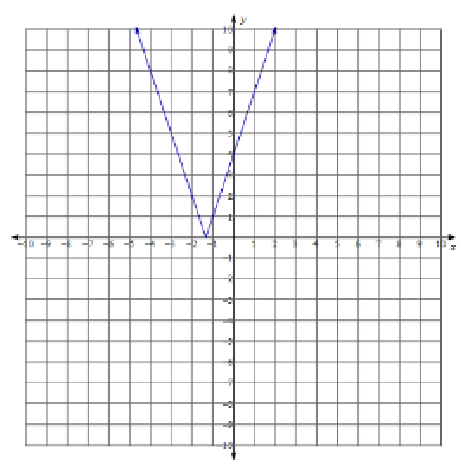
d. $6-x^2=\left(6-\sqrt{x}\right)\left(6+\sqrt{x}\right)=0\rightarrow$ invariant points @ $x=\pm\sqrt{6}$

$y=6-x^2$ opens down, y $=\begin{cases}6-x^2 \text{ if } -\sqrt{6}\leq x\leq \sqrt{6}\\ x^2-6 \text{ if } x>\sqrt{6} \text{ or } x>-\sqrt{6}\end{cases}$

3.

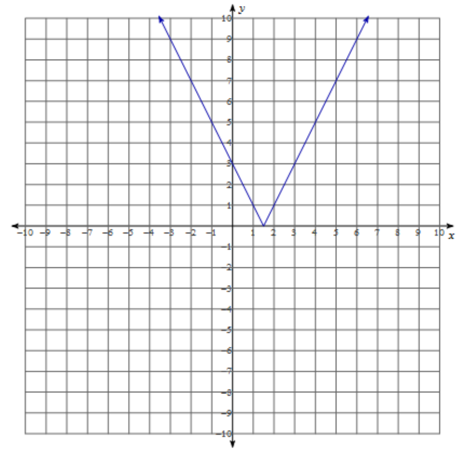
a. Sketch the graph of $y=3x+4$ and reflect the piece that is negative.

D: $\{x\in R\}$; R: $\{y\in R \mid y\geq 0\}$



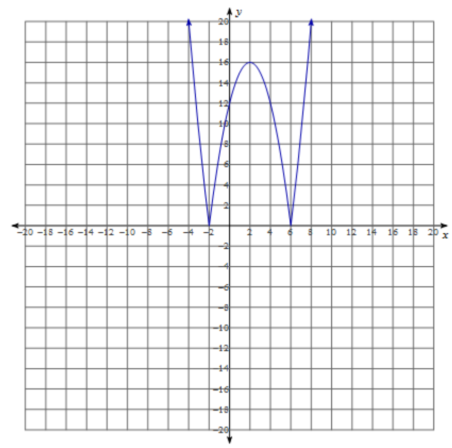
b. Sketch the graph of $y = -2x + 3$ and reflect the piece that is negative.

$$\text{D}: \{x \in R\};\ \text{R}: \{y \in R \mid y \geq 0\}$$



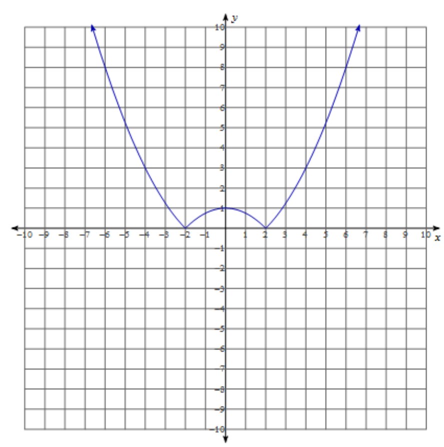
c. Sketch the graph of $y = x^2 - 4x - 12$ and reflect the piece that is negative.

$$\text{D}: \{x \in R\};\ \text{R}: \{y \in R \mid y \geq 0\}$$



d. Sketch the graph of $y = -\frac{1}{4}x^2 + 1$ and reflect the parts that are negative.

$$\text{D}: \{x \in R\};\ \text{R}: \{y \in R \mid y \geq 0\}$$



4.

a. $|3x-8|+9=0 \rightarrow |3x-8|=-9$ No solution

b. $|4x+8|=-8x+3$

$$4x+8=0 \rightarrow \text{ invariant point @ } x=-2; |4x+8| = \begin{cases} 4x+8 \text{ if } x \geq -2 \text{ (case 1)} \\ -4x-8 \text{ if } x < -2 \text{ (case 2)} \end{cases}$$

Case 1 $x \geq -2$

$4x+8=-8x+3$

$12x=-5$

$x=-\frac{5}{12}$

$x=-\frac{5}{12}$ is a solution

Case 2 $x < -2$

$-(4x+8)=-8x+3$

$-4x-8=-8x+3$

$4x=11$

$x=\frac{11}{4}$

$x=\frac{11}{4}$ is an extraneous root

$$\therefore x=-\frac{5}{12}$$

c. $|3x+7|=x^2+3x+3$

$$3x+7=0 \rightarrow \text{ invariant point @ } x=-\frac{7}{3}; |3x+7| = \begin{cases} 3x+7 \text{ if } x \geq -\frac{7}{3} \text{ (case 1)} \\ -3x-7 \text{ if } x < -\frac{7}{3} \text{ (case 2)} \end{cases}$$

Case 1 $x \geq -\frac{7}{3}$

$3x+7=x^2+3x+3$

$x^2-4=0$

$(x-2)(x+2)=0$

$x_1=2$

$x=2$ is a solution

$x_2=-2$

$x=-2$ is a solution

Case 2 $x < -\frac{7}{3}$

$-(3x+7)=x^2+3x+3$

$-3x-7=x^2+3x+3$

$x^2+6x+10=0$

$\mathrm{D}=b^2-4ac=6^2-4(1)(10)=-4<0$

since $\mathrm{D}<0$, no real solutions for Case 2

$$\therefore x=\pm 2$$

d. $\left|\frac{1}{2}x^2-2x-2\right|=2-x$

$$\frac{1}{2}x^2-2x-2=0\rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-2)\pm\sqrt{(-2)^2-4\left(\frac{1}{2}\right)(-2)}}{2\left(\frac{1}{2}\right)}=\frac{2\pm\sqrt{8}}{1}=2\pm2\sqrt{2}$$

invariant points @ $x=2-2\sqrt{2}\approx-0.83$ and $x=2+2\sqrt{2}\approx4.83$; $y=\frac{1}{2}x^2-2x-2$ opens up

$$\left|\frac{1}{2}x^2-2x-2\right|=\begin{cases}\frac{1}{2}x^2-2x-2 \text{ if } x\le(2-2\sqrt{2}) \text{ or } x\ge(2+2\sqrt{2}) \text{ (case 1)}\\ -\frac{1}{2}x^2+2x+2 \text{ if } (2-2\sqrt{2})<x<(2+2\sqrt{2}) \text{ (case 2)}\end{cases}$$

Case 1 $x \le -0.83$ or $x \ge 4.83$

$$\frac{1}{2}x^2 - 2x - 2 = 2 - x$$
$$x^2 - 4x - 4 = 4 - 2x$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x_1 = 4$$
$x = 4$ is an extraneous root
$$x_1 = -2$$
$x = -2$ is a solution

Case 2 $-0.83 \le x \le 4.83$

$$-\left(\frac{1}{2}x^2 - 2x - 2\right) = 2 - x$$
$$x^2 - 4x - 4 = 2x - 4$$
$$x^2 - 6x = 0$$
$$x(x-6) = 0$$
$$x_1 = 0$$
$x = 0$ is a solution
$$x_1 = 6$$
$x = 6$ is an extraneous root

$$\therefore x = -2; x = 0$$